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ABSTRACT

Basics of scalar and vector Finite Quantum Field Theories are recalled, stressing the importance of the quantization of classical physical fields as Operator-Valued- Distributions with specific fast decreasing test functions of the coordinates. The procedure respects full Lorentz and symmetry invariances and, due to the presence of test functions, leads to finite Feynman diagrams directly at the physical dimension D = 2.4. In dimension 2 it is only with such test function that the canonical quantization of the massless scalar field is found to be fully consistent with the most successfull Conformal Field Theoretic approach, pioneered by Belavin, Polyakov and Zamolodchikov in the early 1980's. The question is then raised how Poliakov's wordline path integral representation of the relativistic string could possibly lead tofinite Feynmann diagrams. The natural way of inquiries is through the extension of the string formalism with classical convoluted coordinates leading then to Operator-Valued-Distributions and thereby to Finite Quantum Field Theories. It is shown that in the process some age-old certitudes about quantized strings are somewhat jostled.

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Finite Quantum-Field Theory and the Bosonic String Formalism: A Critical Point of View

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ABSTRACT

Basics of scalar and vector Finite Quantum Field Theories are recalled, stressing the importance of the quantization of classical physical fields as Operator-Valued- Distributions with specific fast decreasing test functions of the coordinates. The procedure respects full Lorentz and symmetry invariances and, due to the presence of test functions, leads to finite Feynman diagrams directly at the physical dimension D = 2..4. In dimension 2 it is only with such test function that the canonical quantization of the massless scalar field is found to be fully consistent with the most successfull Conformal Field Theoretic approach, pioneered by Belavin, Polyakov and Zamolodchikov in the early 1980's. The question is then raised how Poliakov's wordline path integral representation of the relativistic string could possibly lead tofinite Feynmann diagrams. The natural way of inquiries is through the extension of the string formalism with classical convoluted coordinates leading then to Operator- Valued-Distributions and thereby to Finite Quantum Field Theories. It is shown that in the process some age-old certitudes about quantized strings are somewhat jostled.

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I. INTRODUCTION

Finite Quantum Field Theories (FQFT) originate from the early causal and finite approach of Bogoliubov-Epstein-Glaser ($BEG{-}CSFT$) [1–7]. The inital steps are based on the early recognition that, in general, fields are not regular functions in the usual sense but distributions [8,9]. However the setting up of a Lagrangian formalism in the QFT context encounters products of fields as distributions at the same space-time point, which are ill-defined and the later sources of crippling divergences. Past QFT history essentially deals with the search for counter-terms cancelling these anoying divergences. On the opposite the $BEG - CSFT$ approach under the forms of Refs. [6,7] aims from the start at a Lagrangian formulation in keeping with the basic underlying classical differentiable structure of the space-time manifold. The taming of these divergencies involves regularization procedures which ought to preserve, to start with, the symmetry principles of the Lagrangian. Using a naïve cut-off for instance is known to violate Lorentz and gauge invariances, whereas Dimensional Regularization (DR) [10] and that of Ref. [7] -dubbed $TLRS$ here after- do preserve these fundamental symmetries. The two procedures have in common the distinctive aspect of their implementation prior to the construction of the Lagragian density. The use of DR does not however address directly to the origin of these divergencies but just avoids them in going to an hypothetical space in $D = 4-\epsilon$ dimensions. $TLRS$ was developped in Ref. [11,12]. Since the early applications of this scheme [13,14] the calculation of radiative corrections to the Higgs mass [15] and the treatment of the axial anomaly [16,17] are relevant illustrations of the practical use of the $TLRS$ procedure in the $D = 4$ context. It was shown recently how $TLRS$ solves the long-standing consistency problem [18] encountered between EqualTime (EQT) and Light-Front-Time (LFT) quantizations of bosonic two-dimensional massless fields. Our purpose here is to confront the findings of [18] with the standard bosonic string theory approach of [19,20] and elaborate on the values of the critical dimension for the cancelation of the conformal anomaly.

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II THE MATHEMATICAL SETTING

2.1. Classical wave equations

To the original classical field-distribution $\phi(x^0, x^1)$ is associatted a translation-convolution product $\Phi(\rho)$ built on a rapidly decreasing test functions $\rho(x^0, x^1)$, symmetric under reflexion in the variables x^0 and x^1. In Fourier-space variables this linear functional can be written as an integral with the proper bilinear form $\ll p, x \gg = p^a g_{a,\nu} x^\nu$ $(g_{a,\nu} = \text{diag}\{1, -1\})$

$$(\Phi * \rho)(x^0, x^1) = \int \frac{dp_0 dp_1}{(2\pi)^2} e^{-\imath \ll p,x \gg} \tilde{\phi}(p_0, p_1) f(p_0^2, p_1^2),$$

where $\tilde{\phi}(p_0, p_1)$ (resp. $f(p_0^2, p_1^2)$) is the Fourier-space transform of $\phi(x^0, x^1)$ (resp. of $\rho(x^0, x^1)$). Hereafter $\mathbf{\Phi}(x^0, x^1)$ will stand for $(\Phi * \rho)(x^0, x^1)$.

The wave-equation for the classical convoluted distribution in space-time variables is obtained from the hyperbolic partial differential equation (HPDE)

$$\Box \mathbf{\Phi}(x^0, x^1) = \left[\partial^2_{x^0} - \partial^2_{x^1}\right] \mathbf{\Phi}(x^0, x^1) = 0. \tag{2.1}$$

A solution of the Cauchy problem in the sense of convolution of tempered distributions is nothing else than D'Alembert's $(1717 - 1783)$ solution. It can be written as

$$\mathbf{\Phi}(x^0, x^1) = \frac{1}{2\pi} \int d^2p \delta(p_0^2 - p_1^2) \chi(p_0, p_1) \mathrm{e}^{-\imath \ll p,x \gg} f(p_0^2, p_1^2), \tag{2.2}$$

with $\chi(\pm|p_1|, p_1) = \chi_\pm(p_1)$. Canonical quantization of the zero mass scalar quantum operator valued-distribution (OPVD) field $\hat{\Phi}(x^0, x^1)$ proceeds from Eq.(2.2) via the correspondance, in terms of creation and annihilation operators, $\{\chi_-(p) \curvearrowright a^\dagger(p), \chi_+(p) \curvearrowright a(p)\}$, with commutator algebra $[a(p), a^+(q)] = 4\pi p \delta(p-q)$ and a vacuum $|\mathbf{0}>$ such that $a(p)\,|\mathbf{0}>= 0\ \forall p$. That is

$$\hat{\Phi}(x^0,x^1)=\frac{1}{4\pi}\int_0^{\infty}\frac{dp}{p}[a(p)e^{-\imath p(x^0-x^1)}+a^{\dagger}(p)e^{\imath p(x^0+x^1)}]f(p^2). \quad (2.3)$$

Then, one easily evaluates the commutator of two free scalar OPVD to

$$\left[\hat{\Phi}(x),\hat{\Phi}(0)\right]\equiv \imath\Delta(x)=-\frac{\imath}{\pi}\int_0^{\infty}\frac{dp}{p}\sin(px^0)\cos(px^1)f^2(p^2). \quad (2.4)$$

This integral is finite without the test function and the limiting procedure where $f^2(p^2)\equiv f(p^2)=1$ refers to important mathematical properties of metric spaces (whether Minskowskian or Euclidean) [18].

2.2. The ET-LFT consistency problem

Going to light-cone (LC) variables $x^0\pm x^1=x^{\pm}$ is motivated by Dirac's early observation that the LC-stability group is maximal: LC-dynamics has much to share with gallilean dynamics (*e.g.*relative motion of LC-interacting particles decouples from global center of mass motion...). However in the LC-variables the nature of the initial Klein-Gordon equation in Eq.(2.1) is changed to a characteristic initial value problem (CIVP) relative to the partial-differential equation

$$\partial_+\partial_-\Phi(x^+,x^-)=0 \quad (2.5)$$

with initial data on characteristic surfaces

$$\Phi(x^+,x_0^-)=\mathfrak{f}(x^+),\quad \Phi(x_0^+,x^-)=\mathfrak{g}(x^-), \quad (2.6)$$

and the continuity condition

$$\Phi(x_0^+,x_0^-)=\mathfrak{f}(x_0^+)=\mathfrak{g}(x_0^-). \quad (2.7)$$

At first sight the LC-Lagrangian is singular[1]: $W(x,y)=\frac{\delta^2 L}{\delta[\partial^-\Phi(x)]\delta[\partial^-\Phi(y)]}=0$, but the appearence of a primary contrainst is known to be of no physical significance [21].

Nevertheless the consistency of the solutions in the two reference frames cannot be established without further insight. This is just the content of Ref. [18], with two main conclusions:

-On the one hand, full consistency of EQT and LFT quantizations can only be achieved when fields are considered as OPVD with partition of unity test-functions $f(p^{+2})$ such that, for the light-cone momentum p^+ , $\lim_{p^+\to 0^+}\frac{f(p^{+2})}{p^+}=0$.
-On the other hand operator series in the Discretized-LC-Quantization (DLCQ) find their natural handling of divergences in the substraction scheme embedded in the OPVD formulation. The net effect of the PU-test function is the appearence of its inherent RG-scale parameter (η).

[1] The Hessian is indentically null

Then the LF-formulation and CFT analysis of $2d$-massless models are in complete agreement in their representation of the energy-impulsion tensor in term of infinite dimensional Virasoro Lie-algebras.

III. THE QUANTUM BOSONIC STRING [19, 23_27]

3.1 Equations of motion of the scalar bosonic string in the LC-gauge

The motion under consideration here is taking place on a $2d$-worksheet embedded in a D-dimensionnal space. The initial field variables are then $x_a(\sigma,\tau), p_a(\sigma,\tau)$ elevated to OPVD. A well-defined Lagrangian is then obtained in terms these regular field variables $X_a(\sigma,\tau), P_a(\sigma,\tau)$. After dealing with the LC-gauge conditions the equation of motion for $X_a(\sigma,\tau)$ is just that of Eq.(2.1) with appropriate position and time variables. Accordingly the sum of the zero-point energies of the first quantized string is just $\frac{(D-2)}{2}\sum_{n=0}^{\infty} n$. The well-known conventional evaluation of this sum is given by the Zeta-function $\zeta(s) = \sum_{n=0}^{\infty}\frac{1}{n^s}$ with $\zeta(-1) = -\frac{1}{12}$. The critical dimension for the absence of the overall conformal anomaly must then be such as to suppress that one with the cental charge $c = 1$ coming from the $2d$ worksheet analysis and thus obeys $\frac{(D-2)}{2}\zeta(-1) = -1$, that is $D = 26$! However, even though at the same time this reasoning based on Zeta-function was already under scruteny [24], this critical value survived the long haul!

3.2. TLRS and the Renormalization Group

In the advocated $2d$ QFT treatment the key role is in the pseudo-function distribution extension $\mathcal{P}f(\frac{1}{p^2})$ of $\frac{1}{p^2}$ at the origin. It is defined by the integral

$$I_N = \int_0^\infty d(p^2)\mathcal{P}f(\frac{1}{p^2})f(p^2) \stackrel{\text{def}}{=} \lim_{\epsilon\to 0}\,[\int_{\xi\epsilon}^{\Lambda}\frac{d(p^2)}{p^2} + \int_{\frac{\epsilon}{\eta^2}}^{\frac{1}{\Lambda}}\frac{d(p^2)}{p^2} + 2\ln(\epsilon)] = \ln(\frac{\eta^2}{\xi}) \quad (3.1)$$

where η is the dilatation-scale inherent to the construction of the test function $f(p^2)$ [7, 14]. The term in $\ln(\epsilon)$ corresponds to the general Hadamard substraction procedure to generate a Finite part (F.p.).

The factor ξ is arbitrary[2] with no physical meaning unless explicit symmetry violations need enforcement. Consider now the identity

$$\begin{aligned} IPf(\eta) &= \int \frac{d^2(p)}{(2\pi)^2)} \frac{f(p^2)}{p^2} \equiv \int \frac{d^2(p)}{(2\pi)^2} \frac{(p+q)^2}{p^2(p+q)^2} f(p^2), \\ &= \int_0^1 dx \int \frac{d^2(\mathbf{p})}{(2\pi)^2} \frac{(\mathbf{p^2}+q^2(1-x)^2)}{[\mathbf{p^2}+q^2x(1-x)]^2} f(\mathbf{p^2}), \\ &= \frac{1}{4\pi}(\ln(\frac{\eta^2}{\xi})-1). \end{aligned} \tag{3.2}$$

This is easy to understand due to the identity in the UV limit of the $\mathbf{p}$-integration where $f[(\mathbf{p}+\mathbf{q})^2]\mathbf{f}(\mathbf{p^2}) \equiv \mathbf{f^2}(\mathbf{p^2}) \equiv \mathbf{f}(\mathbf{p^2})$. Moreover the overall $\mathcal{O}(2)$ $\mathbf{p}$-invariance implies that terms linear in $\mathbf{p}$ do not contribute to the integral.

Consider then the one loop Feynman diagram in relation to the energy-momentum tensor of the X-field and in the same UV limit[3]

$$\begin{aligned} \Pi_{ab|cd}(q) &= \frac{\mathcal{D}}{8}\int \frac{d^2p}{(2\pi)^2} \frac{t_{a,b}(p,q)t_{c,d}(p,q)}{p^2(p+q)^2} f[p^2]f[(p+q)^2], \\ &= \frac{\mathcal{D}}{8}\int_0^1 dx \int \frac{d^2\mathbf{p}}{(2\pi)^2} \frac{t_{a,b}(\mathbf{p},q,x)t_{c,d}(\mathbf{p},q,x)}{[\mathbf{p^2}+q^2x(1-x)]^2} f[\mathbf{p^2}], \end{aligned} \tag{3.3}$$

with

$$\begin{aligned} t_{a,b}(p,q) &= p_a(p+q)_b + p_b(p+q)_a - \delta_{a,b}(p.(p+q)), \\ t_{a,b}(\mathbf{p},q,x) &= (\mathbf{p}-q(1-x))_a(\mathbf{p}+qx)_b + (\mathbf{p}+qx)_a(\mathbf{p}-q(1-x))_b \\ &\quad -\delta_{a,b}[\mathbf{p^2}-\mathbf{p}q(1-2x)-q^2x(1-x)]. \end{aligned}$$

The presence of the test-function $f[\mathbf{p^2}]$ ensures the existence of this phase-space integral, which otherwise would exibit divergences when $\mathbf{p}\to\infty$. The common pratice in the far past was to consider their cancelations by appropriate counter terms. In that case the only surviving regular contribution to $\Pi_{ab|cd}(q)$ is[4]

$$\begin{aligned} \Pi^{reg}_{ab|cd}(q) &= \frac{\mathcal{D}}{8}(2q_aq_b - q^2\delta_{a,b})(2q_cq_d - q^2\delta_{c,d})\int_0^1 dx x^2(1-x)^2 \int \frac{d^2\mathbf{p}}{(2\pi)^2[\mathbf{p^2}+q^2x(1-x)]^2} \\ &= -\frac{\mathcal{D}q_M^2}{192\pi}(\delta_{a,b}-2\frac{q_aq_b}{q^2})(\delta_{c,d}-2\frac{q_cq_d}{q^2}) \end{aligned} \tag{3.4}$$

---

[2] For Gauge Theories ξ is related to the gauge fixing parameter [12].

[3] This is the 2-points-function, eq.(9.158), of Poliakov's monograph. A coupling vertex factor would be $\imath\frac{g^2}{2}f^{acd}f^{bcd} = \imath\frac{g^2}{2}\mathbf{C}_A\delta^{ad}$.

[4] Here q_M is with Minkowski's signature opposite to Euclid's one.

Here, from the embedding of the $2-d$ worksheet, $\mathcal{D}$ does stand for $\mathcal{D}-2$. Follwing $sect(3.1)$ what is at sake is the sum (e.g. Trace) of the eigen-modes of this matrix. It can be diagonalized by a unitary transformation with a preserved Trace equal to 4. The result [5] is then just the same critical dimension for the absence of the conformal anomaly obtained in the first quantization framework, that is $\mathcal{D}_{cr}=26$. It is clear then that the elimination of diverging contributions by counter-terms just leaves the evaluation of (3.4) in keeping with the findings of [19].

However our TLRS formalism shows that this is not the end of the story.Indeed from examples (3.1,3.2) we observe that diverging integrals in $\mathbf{p^2}$ and $\mathbf{p^4}$ carry essential dependencies on the RG-parameter η. Then the complete η-dependence governing the RG-analysis of the critical equation is concerned with the behaviour of the central charge under the flow of the renormalization group (RG). Zamolodchikov realized this as early as 1986 with his c-theorem [29]:

"There is a function C on the space of unitary 2d-field theories that monotonically decreaes along the RG-flows and which coincides with the Virasoro central charge c at fixed points."

It takes the form

$$\mu\frac{d}{d\mu}C(\mu,\Lambda)\equiv\frac{\mu}{\Lambda}\frac{d}{d(\frac{\mu}{\Lambda})}C(\frac{\mu}{\Lambda},1)=\eta\frac{d}{d\eta}C(\eta,1)=-\beta(i,\eta)g(i,j)\beta(j,\eta)$$

where the Calan-Symanzik β-function at fixed point is independent of η and takes the primitive value [30] $\frac{6}{LambertW(6)}$.

With the stress energy-tensors $\Theta(z)\equiv T_{z,z}$ and $\bar{\Theta}(\bar{z})\equiv T_{\bar{z},\bar{z}}$ the C-function and the metric write [31, 33]

$$C \;=\; -\frac{1}{2\imath}\int_{\text{real surface}} dz\wedge d\bar{z} < \Theta(z)\bar{\Theta}(\bar{z}) >_c |_{IR(TLRS\ limit)} \tag{3.5}$$

and

$$g_{(z,\bar{z})} \;=\; \frac{6\pi^2}{\mu^4} < \phi(z)\bar{\phi}(\bar{z}) >_c |_{IR(TLRS\ limit)},$$

[5] In the perpective of the analytic continuation of $sect.(3.1)$ it is instructive to note how here this decomposes as $-\frac{q_M^2}{4\pi}\frac{(\mathcal{D}-2)}{2*8}\frac{4}{6}$,4 from the trace itself and $\frac{1}{6}$ from the final x-integration $\int_0^1 dx x(1-x)=\frac{1}{6}$ *cf* Appendix B

where the subscript c at the bracket indicates connected collerator contributions. μ is an arbitrary inverse distance inherent to the construction of the TLRS test function as a partition of unity with a dimensionless argument (*cf* footnote 5). The fields $\phi^i(x)$ originate from local coupling sources $\lambda^i(x)$.

Let us consider the correlator of two stress tensors on the plane in the TLRS context [31]

$$< T_{\alpha,\beta}(x)T_{\rho,\sigma}(0) >= \frac{\pi}{3}\int_0^\infty d\mu C(\mu)\int\frac{d^2pf(p^2)}{(2\pi)^2}\exp(\imath px)\frac{(g_{\alpha\beta}p^2 - p_\alpha p_\beta)(g_{\rho\sigma}p^2 - p_\rho p_\sigma)}{p^2+\mu^2}.$$

We are only left with the unknown scalar function of the mass scale μ, the spectral density [32] $C(\mu)$. Its properties have to comply to the following requirements:

(i) Reflexion positivity of the euclidean field theory, i.e. unitarity of the Hibert space, implies $C(\mu) \geq 0$,
(ii) Due to $\dim(T_{\alpha\beta}) = 2$ the spectral density is a dimensionless measure of degrees of freedoom,
(iii)The form of $C(\mu)$ in a scale invariant field theory is completely fixed by it dimensionality. Since $d\mu C(\mu)$ is dimensionless one may not exclude $C(\mu) \sim \frac{c}{\mu}$.
This IR divergence at $\mu = 0$ is fully understood in the TLRS context [7,12] as long as the scaling limit to 1 of the test fuctions is not taken too early.

Indeed the correlator is[6]

$$\begin{aligned} < \Theta(x)\Theta(0) > &= \frac{c\pi}{3}\partial^4{}_{|x|}\int_0^\infty\frac{d\mu}{\mu}f(\mu^2)\int\frac{d^2pf(p^2)}{(2\pi)^2}\frac{\exp(\imath p.x)}{p^2+\mu^2}, \\ &= -\frac{c}{12\pi}\ln(\eta^2)\partial^4{}_{|x|}[\gamma_E + \ln(\frac{\Lambda|x|}{2})], \\ &= \frac{1}{4\pi}\ln(\eta^2)\frac{2c}{|x|^4} \end{aligned}$$

(iv) Conformity with conformal invariance is exibited through the $\frac{1}{|x|^4}$ dependence in agreement with the results of [18](Eq.(56)) for $< 0|T(z)T(w)|0 >$. The study of the central charge C from Eq.(3.5) on a $2d-$curved manifold [34] has established the general validity of Zamolodchikov c-theorem. It is sufficent, for our purpose, to consider only a flat real surface with coordinate parametrization $\{z,\bar{z}\} = \rho\exp(\pm\imath\theta)$ which leads to[7,8]

[6] It is always possible to write the initial PU-test function regulating the p-integral as $f^2(p^2) \sim f(p^2)f(p^2+\mu^2) \sim f(p^2)f(\mu^2)$, for, in the UV-limit, $f(p^2)f(p^2+\mu^2) \equiv f^2(p^2) \sim f(p^2)$, whereas in the IR-limit the remaining $f(\mu^2)$ function just validates the corresponding integral.

[7] Note that in the initial $\{z,\bar{z}\}$-integrals the factor is $\frac{1}{|z-\bar{z}|^4}$ so that the ρ-integral is on the variable $v = \rho^2\sin^2(\theta)$, hence the independent factorization of the remaining θ-integrals with the appearance the ubiquitous $\frac{1}{12}$ factor [18]($eq.$56).

[8] The TLRS analytic evaluation of $g(v^2)$ is proportional to the difference of step-functions $[\theta(v - x11) - \theta(v - x12)]$, with $x11 = (\eta^2)^{(\frac{1}{2})}$, $x12 = (2\eta^2)^{(\frac{1}{2})}$ [16,32].The final v-integration is then trivial, after Hadamard substractions of diverging contributions in $\ln(\epsilon)$, leaving the $\ln(\eta^2)$ factor.

$$\begin{aligned} C(\eta) &= -\frac{1}{32}\int_0^{2\pi}\frac{d(\theta)}{\sin^2(\theta)}\int_0^\infty d(v)\frac{f(v^2)}{v^2} = \frac{1}{32}\int_0^{2\pi}\frac{d(\theta)}{\sin^2(\theta)}\int_0^\infty d(v)\frac{d}{dv}(\frac{1}{v})f(v^2) \\ &= -\frac{1}{32}\int_0^{2\pi}\frac{d(\theta)}{\sin^2(\theta)}\int_0^\infty \frac{dv}{v}g(v^2) \quad \text{with } g(v^2) = \frac{d}{dv}f(v^2) \\ &= -\frac{1}{32}\ln(\eta^2)\lim_{\epsilon\to 0}\{\frac{1}{\epsilon}\int_\epsilon^{\frac{\pi}{2}-\epsilon} d(\theta)[\frac{1}{\sin^2(\theta)}+\frac{1}{\cos^2(\theta)}]\} \\ &= \frac{1}{12}\ln(\eta^2) \end{aligned} \tag{3.6}$$

It is plain to see that this result is in agreement with the observation about the unicity of the solution, up to to an arbitrary constant (here $\ln(\eta^2)$), of "Cayley's identity" known as the "Schwarz derivative" [18].

Recently J.F. Mathiot established that, within general arguments valid in the TLRS framework, the trace of the energy-momentum tensor in 4-dimensions does not show any anomalous contribution even though quantum corrections are considered [35]. It is then our concern to turn now to the determination of the critical dimension $\mathcal{D}_{cr}$ for the absence of the overall conformal anomaly with $\mathbf{p^2}$ and $\mathbf{p^4}$ divergences of the Poliakov-tensor treated in the TLRS formalism(*cf* Appendix *A*). As mentioned after Eq.(3.4) the elimination of diverging contributions by counter-terms just leads to the evaluation in keeping with the findings of [19], that is $\mathcal{D}_{cr} = 26$. However with TLRS the situation is different as shown in Appendix *A*. The surviving initial Poliakov-term comes with extra TLRS η-independent components. The immediate issue is then the fate of the $\mathcal{D}_{cr} = 26$ value under these additional TLRS terms[9].Following Poliakov's analysis [19] a direct calculation of $\Pi^{(4)}_{--|--}(q,\eta)$ shows explicitly the critical value $\mathcal{D}_{cr} = 4$, as detailed in Appendix B. Consider now the diagonalization of the normalized matrix $\Pi_{ab|cd}(q)$ with a Lagrange parameter ξ in relation to the stress-energy constraint $T_{ab} = 0$. At the value $\mathcal{D}_{cr} = 4$ ξ is completely fixed, indicating that reparametrizations of the world-sheet and conformal rescaling allow to fully fix g_{ab} to anything wanted.

IV. FINAL REMARKS

As a final additional observation it is instructive to consider the string description for the VVA-anomaly [22] versus its direct calculation with TLRS [16, 17]. In the string treatment of the massless case (*cf Eq.*(6.44) of [22]) "explicit divergences are made of a difference of two tadepoles type and hence do not contribute in dimensional regularization, whereas for the remaining terms integrations are elementary, and the result is, using Γ-function identities, easily identified to the standard result for the massless QED vacuum polarization". In TLRS the calculation is directly in dimension $D = 4$ with the usual γ_5 and all contributions are either null or finite: a simple bookeeping leads then to the standard VVA-anomaly without further ado. The TLRS procedure does provide

[9] given by Eq.(*A*.9) of Appendix *A*.

a very clear and coherent picture. All known invariance properties, besides those of the VVA-anomaly, are preserved [13–15]. It is a direct consequence of the fundamental properties of TLRS. As an "a-priori" regularization procedure, it provides a well defined mathematical meaning to the local Lagrangian we start from in terms of products of OPVD at the same space-time point. It also yields a well defined unambiguous strategy for the calculation of elementary amplitudes, which are all finite in strictly 4-dimensional space-time and with no new non-physical degrees of freedom nor any cut-off in momentum space.

In summary the strategy developped here was based on the passage from first-quantization to second quantization of the bososnic string. It is characterized by the introduction of the notion of L.Schwartz's Pseudo-Functions [8](*cf* Eq.(3.1)) with their dilatation scale dependences. This result is at variance wih the usual dilatation-scale independant Zeta-fuction evaluation of the discrete sum on inverse quantum n of first-quantized space-time objects. Actually it is easy to see that the standard evaluation of the Zeta-function through normal Eulers'integral in the integration interval $(0, \infty)$ should be considered as the limit $\epsilon \to 0$ of the same integral in the interval $(\xi\epsilon, \frac{\eta^2}{\epsilon})$, thereby collecting first from the logarithmic term the contribution $\ln(\frac{\eta^2}{\xi})$ and not the value $\zeta(-1) = -\frac{1}{12}$.

The main conclusion is then that String Theory in the OPVD picture reduces to Finite Quantum Field Theory, *directly in 4-dimensions with no trace anomaly of the energy-momentum tensor* , and in the limit where the tension along the string becomes infinite.

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