Hydro-Gravity Duality
and limit of Newton constant $G_N \rightarrow 0$

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QCD 24
27-th Internationnal Conference

Montpellier, 8-12 July 2024 . +

11 июля 2024 г.

Introductory remarks. Terminology used in title

DUALITY: Two different theories,
(as a rule, one is IR sensitive, the other -UV sensitive)
with same predictions for a certain class of variables

Example (from old days): quark-hadron duality

HYDRO :Hydrodynamics in equilibrium,
NON-inertial frames, appropximation of NO dissipation

GRAVITY: quasiclassical approximation,
or quantum particles in external gravitational field

Motivation

Quark-gluon plasma in heavy-ion collisions:

is produced in accelerated and rotated state (compare above “non-inertial frames”)

Equilibrates itself very fast (“in equilibrium”)

With smallest ratio viscosity/entropy (“neglecting dissipation")

Simplest example of bulk-boundary duality, discussed actively in literature

References

References in the text are scarce.

Orignal papers in collaboration with
O.V. Teryaev and G. Yu. Prokhorov (Dubna)

Generalities: Density Operator

In quatum statistics matrix elements are averaged with $\hat{\rho}$

$$\hat{\rho} \ = \ \exp\left(-\hat{H}_{eff}/T\right)$$

where $\hat{H}_{eff}$ is built on conserved quantities:
charges $\hat{Q}_i$, angular momentum $\hat{\vec{J}}$ (Landau-Lifshitz)
+ boost $\hat{\vec{K}}$ (F. Becattini (2017))

$$\hat{H}_{eff} \ = \ \hat{H}_0 \ - \ \Sigma_i \mu_i \hat{Q}_i - \vec{\Omega}\hat{\vec{J}} - \vec{a}\hat{\vec{K}}$$

where $\vec{\Omega}$ angular velocity, $\vec{a}$ is acceleration
$\hat{H}_{eff}$ picks up maximum entropy state
while in case of $\hat{H}_0$ we look for for minimum of energy

Generalized Equivalence Principle

Equivalence principle: physics in accelerated frame is imitated by a nontrivial gravitational field
Two basic non-inertial frames, considered by Einstein:
accelerated and rotated frames
(corresponding $h_{\mu\nu}$ are easy to identify)
But only considering acceleration brought a new principle.
Rotation is reduced to Lorentz

Plasma produced in heavy-ion collisions
both accelerated and rotated
as if echoing the Einstein's thoughts

Duality of statistical and gravitational approaches

Properties of fluids in equilibrium are evaluated statistically in terms of density operator, or effective interaction:

$$\hat{H}_{eff} = -\vec{\Omega}\hat{\vec{J}} - \vec{a}\hat{\vec{K}}$$

where $\hat{\vec{K}}, \hat{\vec{J}}$ are operators of boost and angular momentum and $\vec{a}$, $\vec{\Omega}$ are acceleration and angular velocity

Note that here $\vec{a}, \vec{\Omega}$ are pure kinematical:

$$a_\mu = u^\alpha \partial_\alpha u_\mu, \quad \Omega_\mu = (1/2)\epsilon_{\mu\nu\rho\sigma} u^\nu \partial^{rho} u^\sigma \ , \tag{1}$$

where u^μ is 4-velocity of fluid

Cnt'd

In field theory, gravitational interaction is described by fundamental interaction Lagrangian:

$$\delta L = \frac{1}{2} T^{\alpha\beta} h_{\alpha\beta}$$

where $T^{\alpha\beta}$ is the energy-momentum tensor of matter, $h_{\alpha\beta}$ is the gravitational potential, also accommodating $\vec{\Omega}_{grav}, \vec{a}_{grav}$:

$$\vec{a}_{grav} \sim \vec{\nabla} g_{00}, \quad \vec{\Omega}_{grav} \sim \vec{\nabla} \times \vec{h}$$

where $h_i = g_{0i}$; g_{00}, g_{0i} are components of metric tensor

Cnt'd

Furthermore, one evaluates "external probes",

$$< \hat{O} > = < T^{\alpha\beta} >, < J_5^{\alpha} > \quad \ldots$$

within both approaches, statistical and gravitational. The results compared for the same values of a, Ω. Note that

within statistical approach $\vec{a}, \vec{\Omega}$ are kinematic,
In gravity $\vec{a}_{grav}, \vec{\Omega}_{grav}$ are dynamic.

As a generalization of the equivalence principle,
for external probes the results are to be the same.

An example of duality confirmation

Concentrate on evaluation of energy density, T_0^0. Take a simplest model, that is ideal gas of massless fermions with spin $S = 1/2$. Determine ebergy levels for effective Hamiltonian

$$\hat{H}_{eff} = \hat{H}_0 - \vec{a}\hat{\vec{K}}$$

where $\hat{H}_0$ is Hamiltonian of free particles, $\hat{\vec{K}}$ is boost operator. And then average over the Fermi distribution. The result is:

$$(T_0^0)_{s=1/2} = \frac{7\pi^2 T^4}{60} + \frac{a^2 T^2}{24} - \frac{17a^4}{960\pi^2} \qquad (2)$$

Note that energy vanishes at so called Unruh temperature, $T_{Unruh} = a/(2\pi)$.

Cnt'd

So far, thermodynamical way to evaluate $< T_0^0 >$.
No UV divergencies since integrate with $\exp(-E_{eff}/T)$:

$$< T_0^0 > \sim \int p^2 E_{eff} dE_{eff} exp(-E_{eff}/T)$$

,
Effective interaction, temperature T as exponential cut off.
The other way is geometric, manifold is cone C_ν^2 $(\times R^2)$:

$$ds^2 = \frac{\rho^2}{\nu^2} d\phi^2 + d\rho^2 + dx^2 + dy^2$$

where $\rho = a^{-1}$, $\nu = \frac{2\pi T}{a}$, $2\pi(1 - \nu^{-1}) = angular\ deficit$
Fundamental interaction, UV sensitive. But results coinside.

Summary I: A new limit $G_{Newton} \to 0$

Thus, equilibrium conditions and motion in grav. field give the same results. But equilibrium does not know anything about Newton constant G_N

How to define "noval" duality formally?

Singled out by

$$G_N \to 0, \quad G_N \cdot M_{source} \to const \quad (M_{source} \to \infty)$$

Unifies one-loop theory of equilibrium and one-loop motion of particles in external gravitational field

Quasiclassics has been used already by S. Hawking

Unification with theory of equilibrium is new

Proceed to more subtle details

Unruh effect (a reminder)

Observer moving with acceleration a with respect to Minkowskian vacuum sees thermal distribution of particles with temperature

$$T_{Unruh} = \frac{a}{2\pi} \qquad (\textit{quantum effect})$$

while an observer at rest sees no particles.

For many, it sounds disappointing that the Unruh effect is a kind of observer-dependent. However, it can be viewed as dynamical: By virtue of the equivalence principle, accelerated frame equivalent to vacuum in gravitational field resulting in the same acceleration a. Naturally, such a field produces (massless) particles.

Removing ultraviolet

Currents in absence of external gravity refer in fact to accelerated frames describing results of measurements on the Unruh sample of particles

We start with Minkowski space, have no particles, no currents. Then, go to accelerated frame and work with UV safe sample created through Unruh effect.

Calculate divergence of the current obtained and reproduce the standard anomaly

Anomalous currents without anomaly

In more detail: Evaluate axial current J_5^α which is anomalous in presence of certain external grav. field:

$$\nabla^\alpha J_{\alpha,5} \ = \ C_5^{grav} R\tilde{R}$$

Alternatively,use equilibrium th. in absence of external field

Consider third order one- loop perturbation theory in a, Ω:

$$\delta L \sim (\vec{a}\vec{K}), (\vec{\Omega}\vec{J})$$

Find finite, non-vanishing J_5^α:

$$\vec{J}_5 \ \sim \ (\lambda_1 a^2 + \lambda_2 \Omega^2)\vec{\Omega}$$

where λ_1, λ_2 are calculable constants.

Kinematical Vortical Effect

Evaluate covariant derivative of the axial current obtained

$$J_5^{\alpha,kinematic} = -\frac{1}{24\pi^2}\left(\lambda_1\Omega_\mu^2 + \lambda_2 a_\mu^2\right)\Omega^\alpha$$

difference of the coefficients λ_1 and λ_2 is related to the coefficient C_5^{grav} in front of the gravitational anomaly:

$$\frac{\lambda_1 - \lambda_2}{32} = C_5^{grav} .$$

This prediction is called kinematical vortical effect effect (KVE) (G. Prokhorov, O. Teryaev, V.Z.)

Cnt'd

By a direct clculation, the relation has been verified both in case of spin-1/2 and spin-3/2 constituents. Namely:

$$\lambda_1 = -1 \quad \lambda_2 = -3 \quad C_5^{grav} = 1/(384\pi^2) \qquad (spin\, 1/2)$$

$$\lambda_1 = -53, \quad \lambda_2 = -15, \quad C_5^{grav} = -19/(384\pi^2) \quad (spin\, 3/2)$$

Significance of all this

First step: evaluation of "kinematical" $J_5^{\alpha,kinematical}$
Start from empty Minkowski vacuum $\vec{J}_5^{tot} \equiv 0$
Go to accelerated frame. This is kind of fixation of gauge.

In accelerated frame we have $T = T_{Unruh}$ and we evaluated $J_5^{kinematic}$ on this *Unruh sample of particles*.
Then we found $\nabla_\alpha J^\alpha =$ *standard anomaly*
Everything was UV stable

Chiral gravitational anomaly of field theory is replaced by Unruh effect of thermodynamics in non-inertial frames (!).

Summary II

One could try to develop a new type of UV regulariation by introducing temperature at an intermediate step and then removing it to obtain finite answers.
Works in case of gravitational chiral anomaly but has not been explored further.