QCD24 27th HIGH-ENERGY PHYSICS INTERNATIONAL CONFERENCE IN QUANTUM CHROMODYNAMICS

$a_\mu|^{hvp}_{l.o}$, power corrections and α_s from e^+e^-

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  - QCD condensates are used as inputs in some other approaches.

Based on the recent papers

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  Updated: SN, Nucl.Phys.A 1046 (2024) 122873 : ArXiv 2402.13983 [hep-ph]

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e^+e^- -> Hadrons

Total cross-sections

R= Hadrons /$\mu^+\mu^-$

History : Dispersion Relation

- Bridge between High AND Low energy QCD regions

$$\begin{aligned}\Pi_H(Q^2 \equiv -q^2) &\equiv i\int d^4x \, \langle 0|\mathcal{T} J_H(x) J_H^\dagger(0)|0\rangle \\ &= \int_{t<}^{\infty} \frac{dt}{t+Q^2+i\varepsilon} \mathrm{Im}\Pi(t) + \text{subtraction terms}\end{aligned}$$

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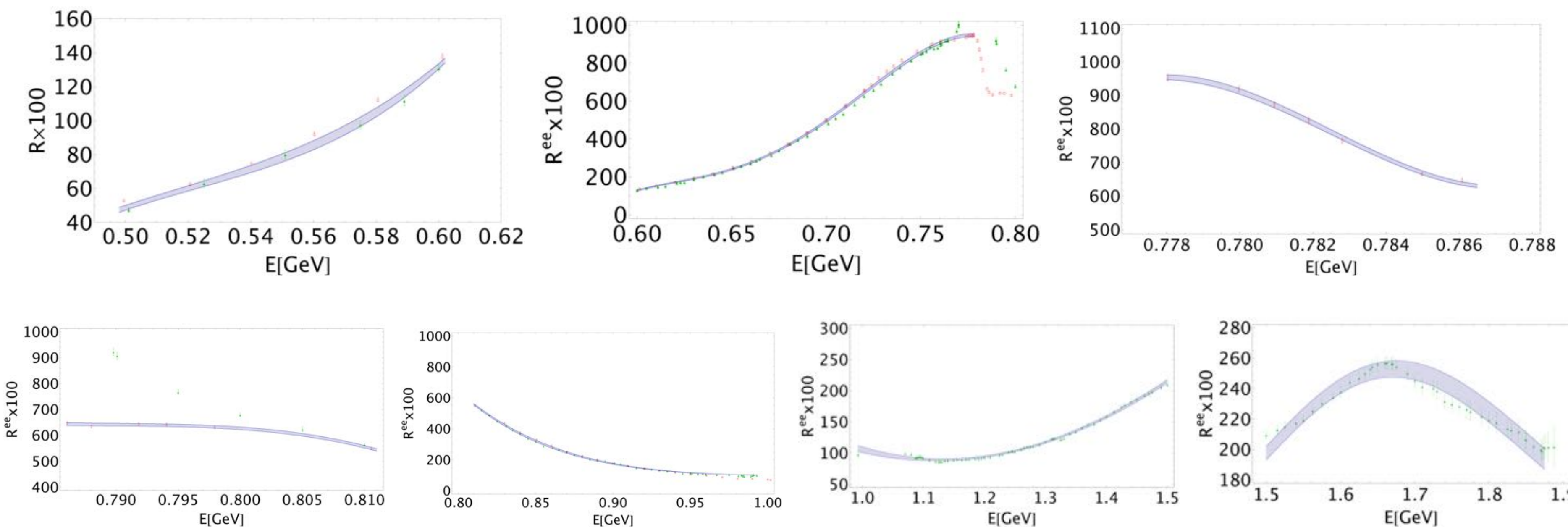
- $\frac{1}{\pi}\mathrm{Im}\Pi(t) \sim \sigma_{tot}(e^+e^- \to \text{ Hadrons})$: Optical theorem.

- ♣ Data compiled by PDG22 $\oplus$ New CMD3 ($E \leq 1.875$ GeV)

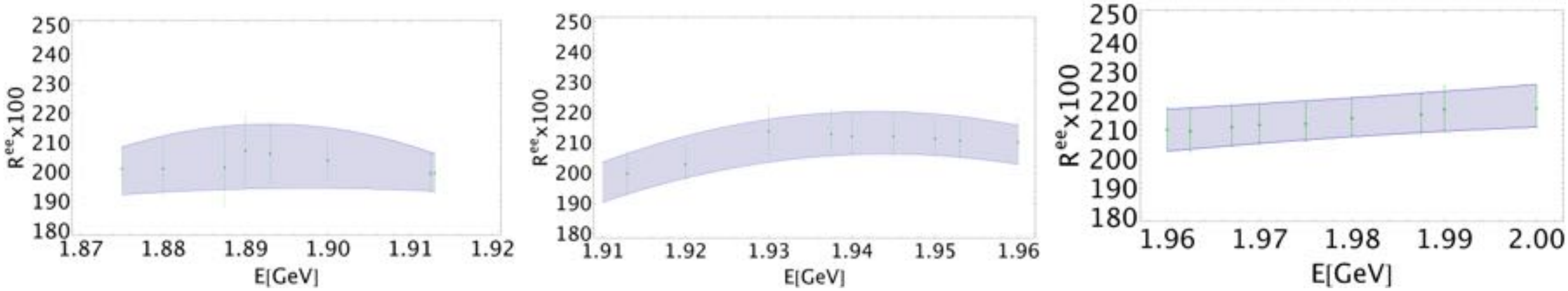
$$R^{ee} \equiv \frac{\sigma(e^+e^- \to \text{ Hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 12\pi \,\text{Im}\Pi(t) \;\; : \quad \text{Optical theorem}$$

We divide the region $0.5 \leq \sqrt{t} \leq 1.9$ GeV into 7 subregions

and use a Mathematica Interpolating Fitting program.



- Data fit from 1.875 to 2 GeV



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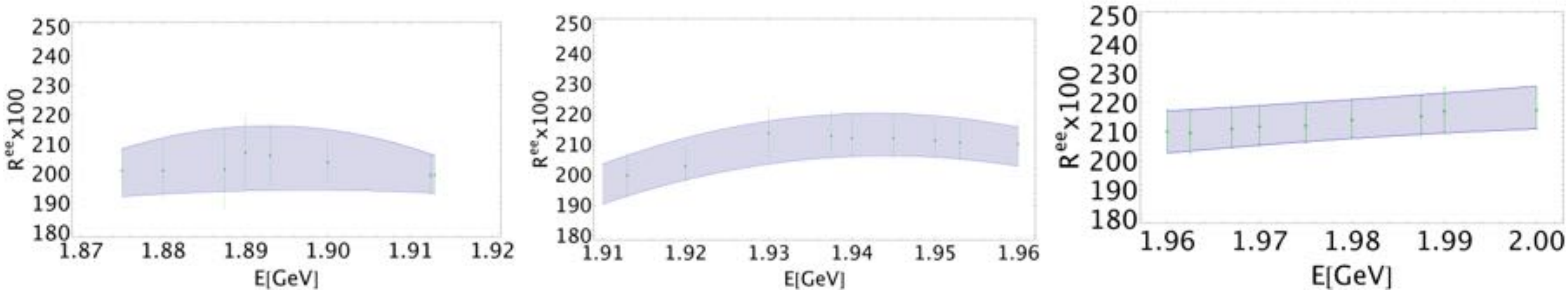
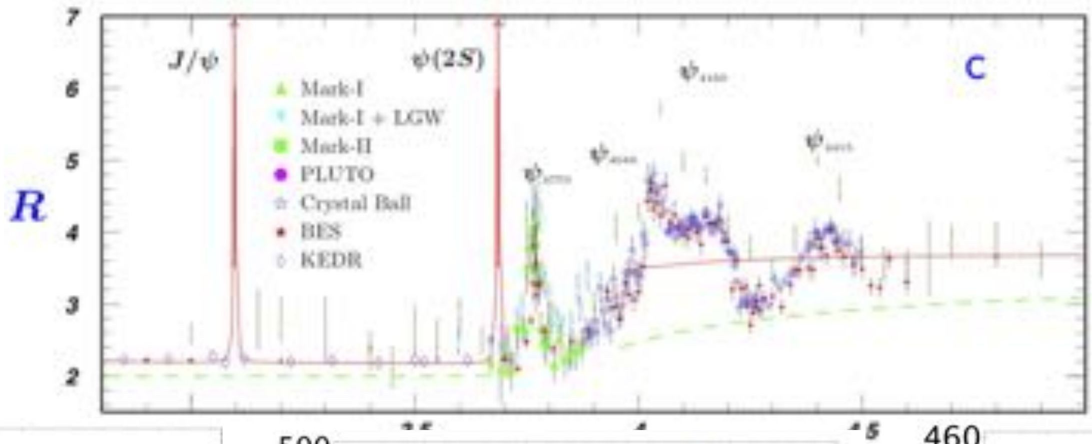
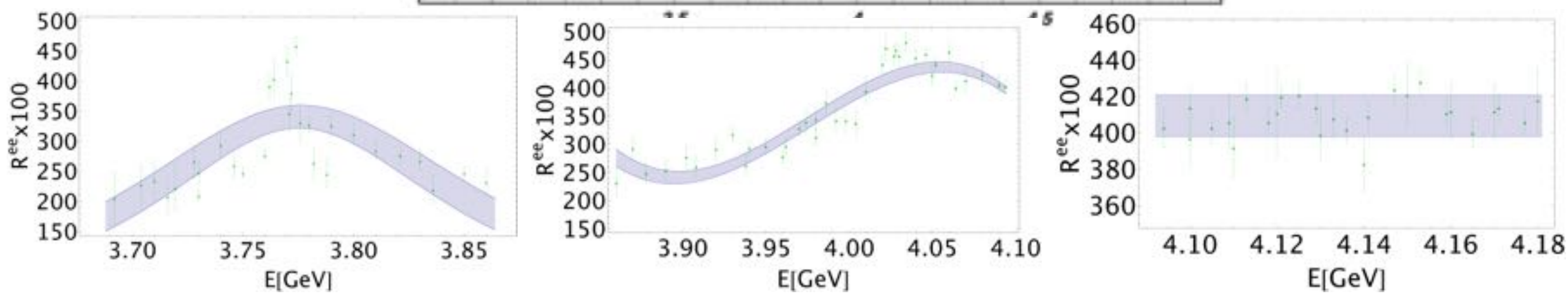


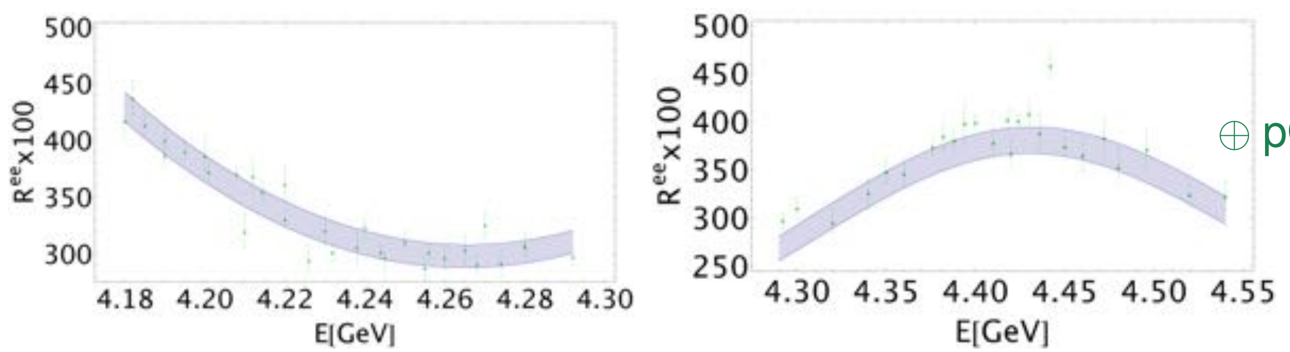
- pQCD - 3 flavours from 2 to 3.68 GeV

$$
\begin{aligned}
R &= 3\sum Q_i^2 \Big[1 + a_s + 1.6398 a_s^2 - 10.2839 a_s^3 - 106.8798 a_s^4 + O(a_s^5) \\
&+ d_{2m} = -\frac{\overline{m}^2}{t}\left(12\, as + 104.833\, as^2 + 541.735\, a_s^3\right), \\
&+ d_{4m} = \frac{\overline{m}^4}{t^2}\Big[-6\left(1 + \frac{11}{3} a_s\right) + a_s^2 \Big[n_f \left(\frac{\mathrm{L_m}}{3} - 1.841 \right) - \frac{11}{2} \mathrm{L_m} \\
&+ 136.693 + 12 - 0.475 - \mathrm{L_m} \Big]\Big]\Big] \quad : \quad L_m = Log(\overline{m}^2/E^2)
\end{aligned}
$$

- Detailed fit of charmonium data from 3.68 to 4.55 GeV

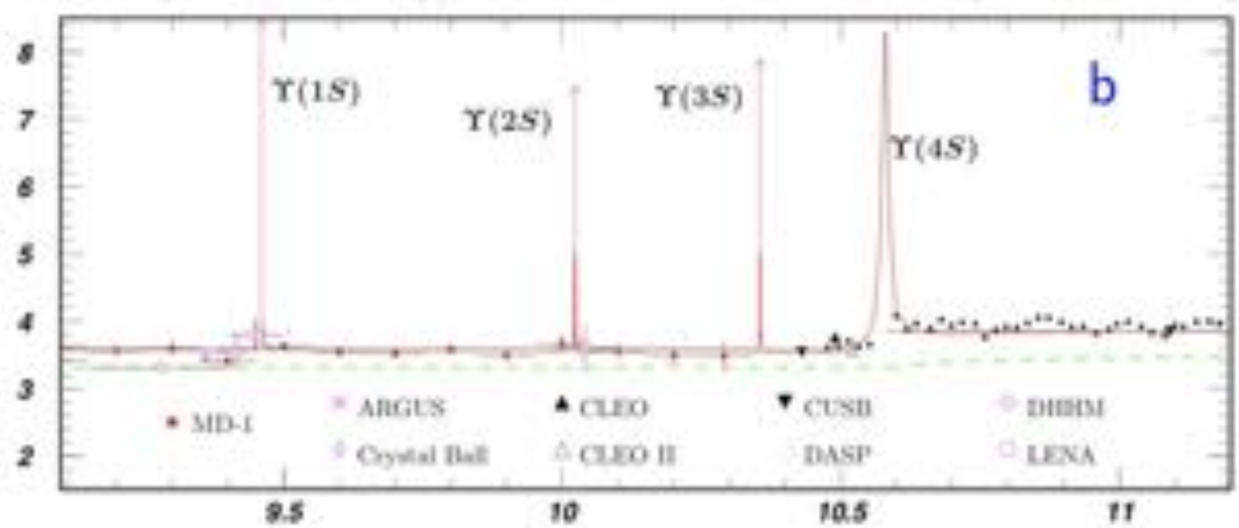






$\oplus$ pQCD - 4 flavours ($E \geq$ 4.58 GeV)

- Bottomium data from PDG 22



$\Upsilon(1S),\cdots\Upsilon(4S)$: Narrow Width Approximation

$\oplus$ pQCD - 5 flavours ($E \geq$ 10.6 GeV)

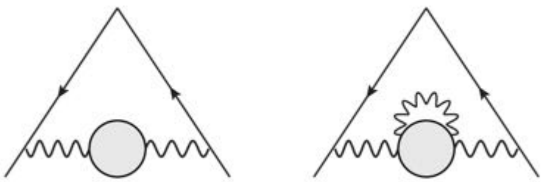
TEST OF THE SM

from $(g-2)_\mu$

using e^+e^- -> Hadrons

Test / Calibration of the Fit and Estimate of $a_\mu|^{hvp}_{l.o}$

- Hadronic Vacuum Polarisation of $a_\mu|^{hvp}_{l.o} \equiv \frac{1}{2}(g-2)_\mu|^{hvp}_{l.o}$



$\frac{1}{\pi}\mathrm{Im}\Pi(t)|^{hvp}_{l.o} \sim \sigma_{tot}(e^+e^- \to \mathrm{Hadrons})$: Optical theorem.

$$a_\mu|^{hvp}_{l.o} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} dt\, K_\mu(t/m_\mu^2)\,\sigma(e^+e^- \to \mathrm{hadrons})$$

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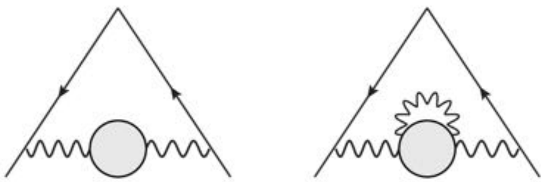

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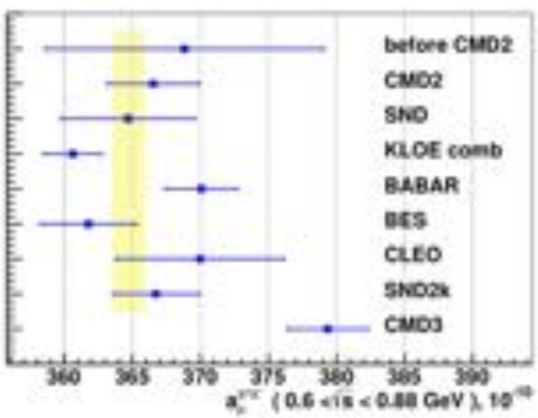
- Comparison of $\neq$ Results in the Low-Energy regions



Experiment	$a_\mu^{\pi^+\pi^-,LO}$, 10^{-10}
before CMD2	368.8 ± 10.3
CMD2	366.5 ± 3.4
SND	364.7 ± 4.9
KLOE	360.6 ± 2.1
BABAR	370.1 ± 2.7
BES	361.8 ± 3.6
CLEO	370.0 ± 6.2
SND2k	366.7 ± 3.2
CMD3	379.3 ± 3.0

$$a_\mu^\rho|^{hvp}_{l.o}[0.6 \to 0.88] = (377.4 \pm 3.1)\times 10^{-10}, \quad a_\mu^\rho|^{hvp}_{l.o}[0.6 \to 0.993] = (400.9 \pm 3.2)\times 10^{-10}.$$

About the same as CMD3.

$$\Longrightarrow \quad a_\mu^{I=1+0}|^{hvp}_{l.o}[2m_\pi \to 1.875] = (6492.3 \pm 38.8)\times 10^{-11},$$

$(100 \sim 136)\times 10^{-11}$ larger than Davier et al., Nomura et al. using KLOE data.

- Final results

$$a_\mu|^{hvp}_{lo} = (7036.5 \pm 36.9) \times 10^{-11}$$

SN1978 (3rd cycle thesis) :

$(7020 \pm 800) \times 10^{-11}$

Same central value but

22 $\times$ more precise ! $\Longrightarrow$

$$a_\mu^{exp} - a_\mu^{th} = (142 \pm 42_{th} \pm 22_{exp}) \times 10^{-11}$$

3σ deviation from SM : New BNL data 2023

$$a_\tau^{hvp} = (3494.8 \pm 24.7) \times 10^{-9}$$

SN 1978 : $(3700 \pm 400) \times 10^{-9}$: 1st estimate !

$$\Delta\alpha^{(5)}_{had}(M_Z) = (2766.3 \pm 4.5) \times 10^{-5}$$

$\sqrt{t}$ [GeV]	$a_\mu\vert^{hvp}_{l.o} \times 10^{11}$	$a_\tau\vert^{hvp}_{l.o} \times 10^{9}$	$\Delta\alpha^{(5)}_{had}(M_Z^2) \times 10^5$
Light I=1			
$\rho(2m_\pi \to 0.50)$	489.4 ± 3.1	85.1 ± 0.6	12.38 ± 0.08
$\rho(0.50 \to 0.60)$	524.5 ± 13.7	135.9 ± 3.5	22.84 ± 0.59
$\rho(0.60 \to 0.776)$	2712.0 ± 30.2	943.7 ± 10.3	182.87 ± 1.97
$\rho(0.776 \to 0.993)$	1297.3 ± 9.8	1797.3 ± 20.3	117.82 ± 0.98
$0.993 \to 1.5$	354.4 ± 6.7	228.2 ± 4.1	67.3 ± 1.14
$1.5 \to 1.875$	237.6 ± 5.7	206.8 ± 4.9	80.11 ± 1.9
Total Light I=1 ($\leq$ 1.875)	*5615.2 ± 36.0*	*2453.7 ± 11.9*	*483.7 ± 4.4*
Light I=0			
ω (NWA)	417.1 ± 13.7	163.3 ± 5.3	33.6 ± 1.1
ϕ (NWA)	389.6 ± 4.6	20.6 ± 0.2	51.2 ± 0.6
$0.993 \to 1.5$	44.3 ± 0.8	28.5 ± 0.5	8.4 ± 0.1
$\omega(1650)$(BW)	24.3 ± 0.1	16.7 ± 0.1	5.2 ± 0.1
$\phi(1680)$(BW)	1.8 ± 0.9	1.3 ± 0.6	0.4 ± 0.2
Total Light I=0 ($\leq$ 1.875)	*877.1 ± 14.5*	*230.4 ± 5.3*	*99.4 ± 1.3*
Light I=0 $\oplus$ 1			
$1.875 \to 1.913$	14.7 ± 0.7	14.4 ± 0.7	6.3 ± 0.3
$1.913 \to 1.96$	17.6 ± 0.6	17.5 ± 0.6	7.9 ± 0.3
$1.96 \to 2$	14.3 ± 0.5	14.5 ± 0.5	6.7 ± 0.2
$2 \to 3.68$: QCD (u,d,s)	247.2 ± 0.3	308.3 ± 0.5	202.8 ± 0.5
Total Light I=0 $\oplus$ 1 (1.875 $\to$ 3.68)	*293.8 ± 1.1*	*354.7 ± 1.2*	*223.7 ± 0.7*
Total Light I=0 $\oplus$ 1 ($2m_\pi \to 3.68$)	6786.1 ± 38.8	3038.4 ± 24.5	806.4 ± 4.6
Charmonium			
$J/\psi(1S)$ (NWA)	65.1 ± 1.2	92.7 ± 1.8	73.5 ± 1.4
$\psi(2S)$ (NWA)	16.4 ± 0.6	26.0 ± 0.9	26.1 ± 0.8
$\psi(3773)$ (NWA)	1.7 ± 0.1	2.7 ± 0.2	2.9 ± 0.2
Total J/ψ(NWA)	*83.2 ± 1.4*	*121.4 ± 2.0*	*102.5 ± 1.6*
$3.69 \to 3.86$	11.4 ± 1.0	18.3 ± 1.6	19.0 ± 1.6
$3.86 \to 4.094$	16.6 ± 0.5	27.5 ± 0.8	30.9 ± 0.9
$4.094 \to 4.18$	6.6 ± 0.2	11.2 ± 0.3	13.2 ± 0.4
$4.18 \to 4.292$	6.5 ± 0.2	11.2 ± 0.4	13.7 ± 0.5
$4.292 \to 4.54$	11.8 ± 0.6	20.7 ± 0.8	26.8 ± 1.1
$4.54 \to 10.50$: QCD (u,d,s,c)	92.0 ± 0.0	186.2 ± 0.0	$458.7.3 \pm 0.1$
Total Charmonium (3.69 $\to$ 10.50)	*145.1 ± 1.3*	*275.6 ± 2.0*	*564.9 ± 2.2*
Total Charmonium	228.1 ± 1.9	396.5 ± 2.8	664.8 ± 2.7
Bottomium			
$\Upsilon(1S)$ (NWA)	0.54 ± 0.02	1.25 ± 0.07	5.65 ± 0.29
$\Upsilon(2S)$ (NWA)	0.22 ± 0.02	0.51 ± 0.06	2.54 ± 0.29
$\Upsilon(3S)$ (NWA)	0.14 ± 0.02	0.33 ± 0.04	1.77 ± 0.23
$\Upsilon(4S)$ (NWA)	0.10 ± 0.01	0.23 ± 0.03	1.26 ± 0.16
$\Upsilon(10.86 \oplus 11)$ (NWA)	0.1 ± 0.06	0.20 ± 0.06	1.67 ± 0.39
Total Bottomium (NWA)	*1.0 ± 0.1*	*2.3 ± 0.1*	*11.2 ± 0.5*
Z − pole	-	-	29.2 [81]
$10.59 \to 2m_t$: QCD (u,d,s,c,b)	22.4 ± 0.3	57.5 ± 0.1	1282.9 ± 1.2
Total Bottomium	23.4 ± 0.3	59.8 ± 0.1	1323.3 ± 1.3
$2m_t \to \infty$: QCD (u,d,s,c,b,t)	0.03	0.08	-28.2
Total sum	**7036.5 ± 38.9**	**3494.8 ± 24.7**	**2766.3 ± 4.5**

Comparisons with Results reported @ QCD 24

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- BABAR (G. Vasseur)

$$\begin{aligned} a_\mu^{exp} - a_\mu^{th} &= (168 \pm 38 \pm 29) \ : \ \text{BABAR} \\ &= (142 \pm 42_{th} \pm 22_{exp}) \ : \ \text{This Work } (3\sigma) \end{aligned}$$

♢ SVZ sum rule 45 years ago !

Zakharov-fest Ringberg Castel, Tegernsee, Munich (19-21 May 2005).

From left to right: A.I. Vainshtein, Your Servitor, V.I. Zakharov, M.A. Shifman.

History : Dispersion Relation

- Bridge between High AND Low energy QCD regions

$$\Pi_H(Q^2 \equiv -q^2) \equiv i \int d^4x \, \langle 0|\mathcal{T} J_H(x) J_H^\dagger(0)|0\rangle$$
$$= \int_{t<}^{\infty} \frac{dt}{t + Q^2 + i\varepsilon} \mathrm{Im}\Pi(t) + \text{subtraction terms}$$

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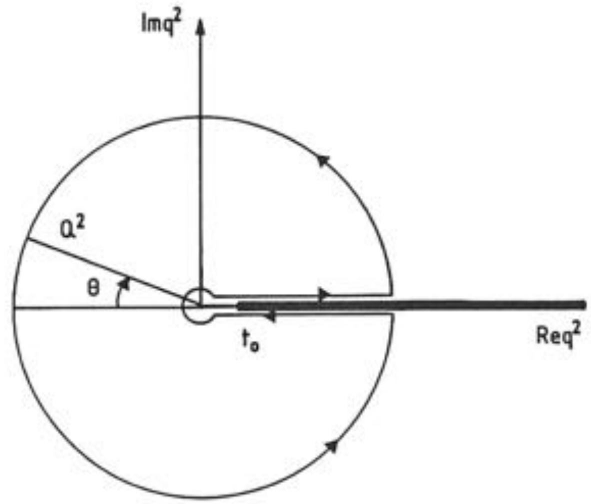

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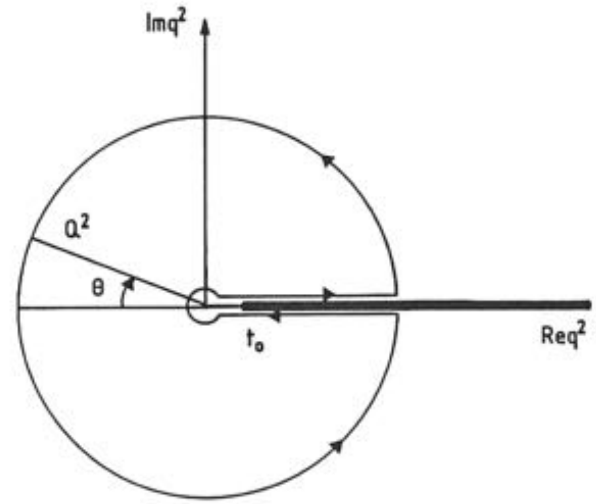

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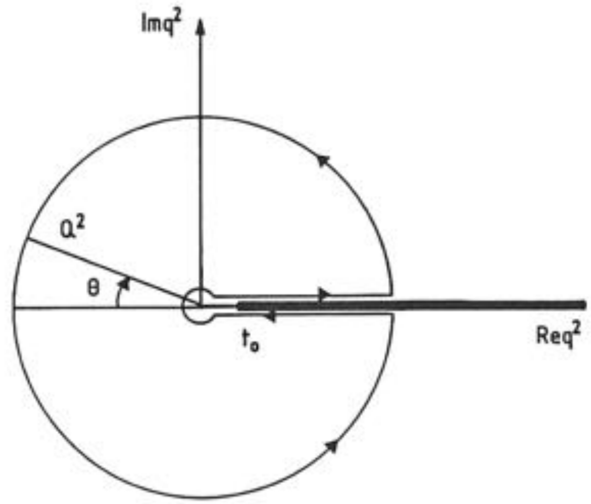

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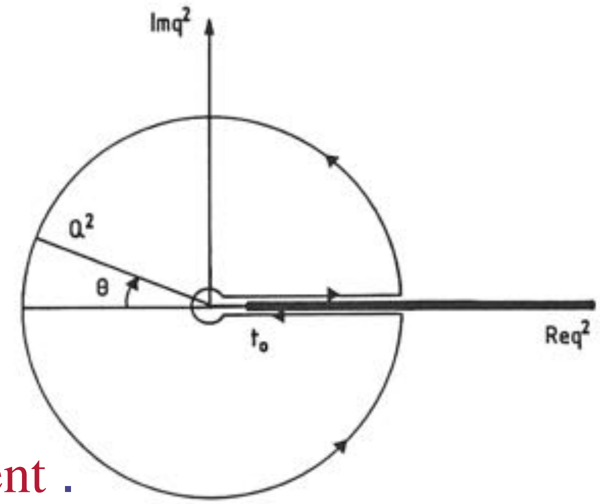

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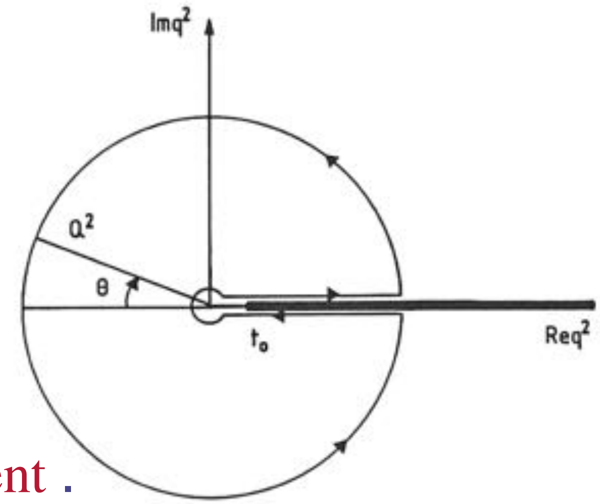

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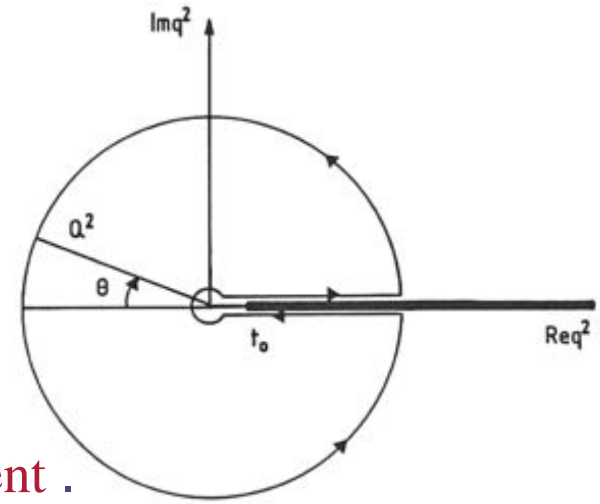

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  - No good control of condensates beyond $D=6$: violation of factorization, mixing under renormalization NT83, often some classes of diagrams are only computed,...

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  - $\mathcal{R}(\tau)$: less senstive to α_s-corrections $\Longrightarrow$
    Good tools for extracting the QCD condensates

    Launer-SN-Tarrach 84

QCD CONDENSATES

from

e^+e^- -> Hadrons

QCD condensates from Ratio of LSR

- Choice of the ratio of LSR Launer-SN-Tarrach 84

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- $\mathcal{R}_{10}(\tau)$ less sensitive to α_s than the moment $\mathcal{L}_0(\tau)$: starts to $\mathcal{O}(\alpha_s^2)$.

- Relative contributions of the condensates increased in the OPE !

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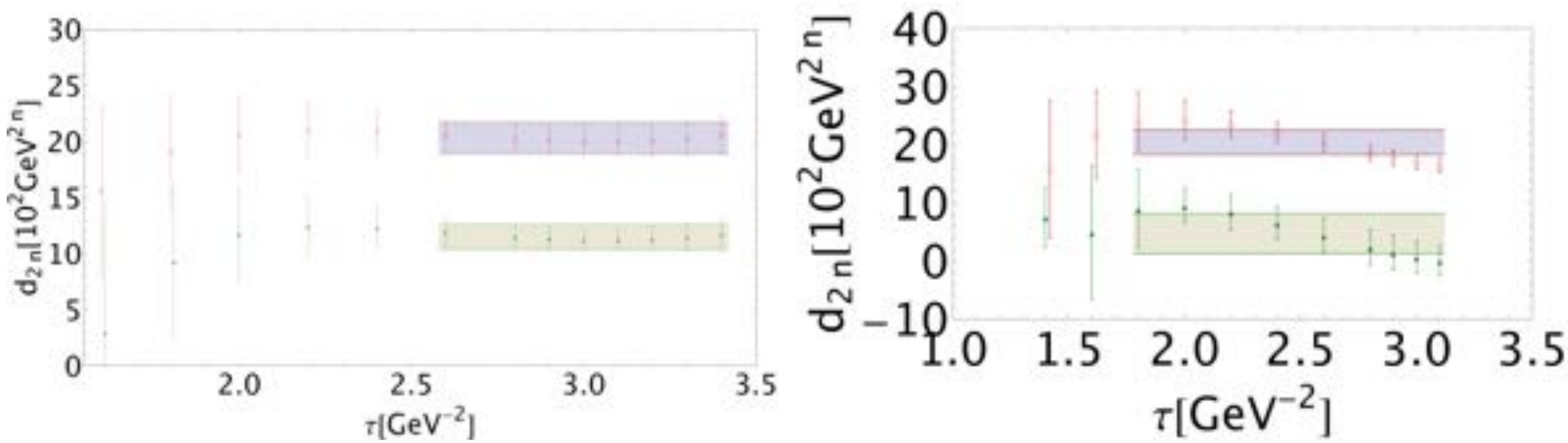
- $\langle \alpha_s G^2 \rangle$ from heavy quarkonia $\oplus$ 2-parameter fit of $(d_6,\ d_8)$:

$O(\alpha_s^2)$ $O(\alpha_s^4)$



d_6 stable for $\alpha_s^2 \to \alpha_s^4$ but not d_8. To order α_s^4 :

$$d_6 = -(20.5 \pm 2.0) \times 10^{-2}\,\mathrm{GeV}^6, \quad \mathrm{d}_8 = (4.7 \pm 3.5) \times 10^{-2}\,\mathrm{GeV}^8.$$

NB : $\tau \simeq (2 \sim 3)$ GeV^{-2} relatively big ! Results to be improved later on !

τ-*like decay moments* *BNP 92*

$$R_n^{ee} = \int_0^1 dx_0\,(1-3x_0^2+2x_0^3)x_0^n\,2R_{ee}^{I=1}(x_0)\ :\ x_0 \equiv (t/M_0^2)$$

- Perturbative corrections $\delta_n^{(0)}$ to order α_s^4 (FO) for $n \geq 1$, :

τ-like decay moments BNP 92

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- Perturbative corrections $\delta_n^{(0)}$ to order α_s^4 (FO) for $n \geq 1$, :
- Condensates to lowest order of α_s Cauchy Property :

$$\begin{aligned}
R_0^{ee} &= f(\delta_0^{(0)}, d_6, d_8), & d_{n\geq 10} = 0 \\
R_1^{ee} &= f(\delta_1^{(0)}, d_4, d_8, d_{10}), & d_{n\geq 12} = 0 \\
R_2^{ee} &= f(\delta_2^{(0)}, d_6, d_{10}, d_{12}), & d_{n\geq 14} = 0 \\
R_3^{ee} &= f(\delta_3^{(0)}, d_8, d_{12}, d_{14}), & d_{n\geq 16} = 0 \\
R_4^{ee} &= f(\delta_4^{(0)}, d_{10}, d_{14}, d_{16}), & d_{n\geq 18} = 0 \\
R_5^{ee} &= f(\delta_5^{(0)}, d_{12}, d_{16}, d_{18}), & d_{n\geq 20} = 0 \\
R_6^{ee} &= f(\delta_6^{(0)}, d_{14}, d_{18}, d_{20}), & d_{n\geq 22} = 0
\end{aligned}$$

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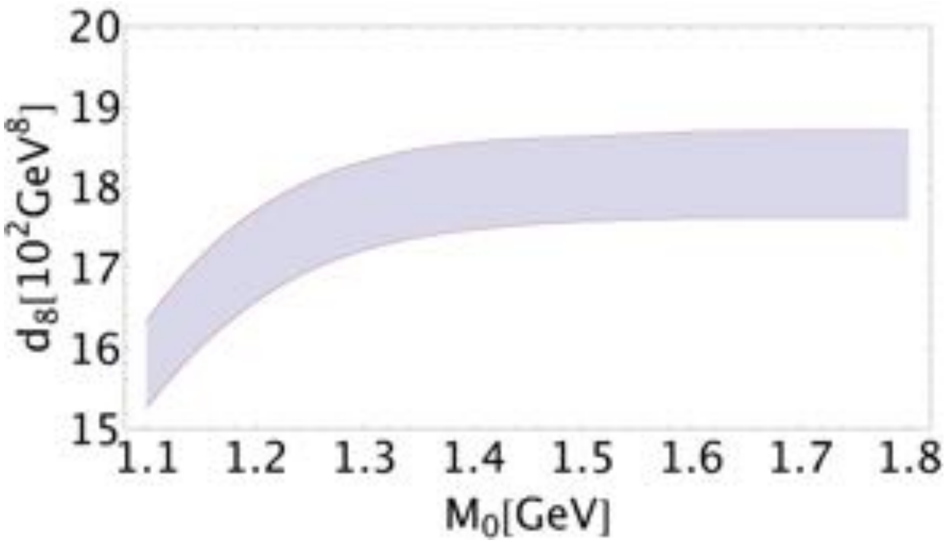
- We proceed iteratively to improve the results

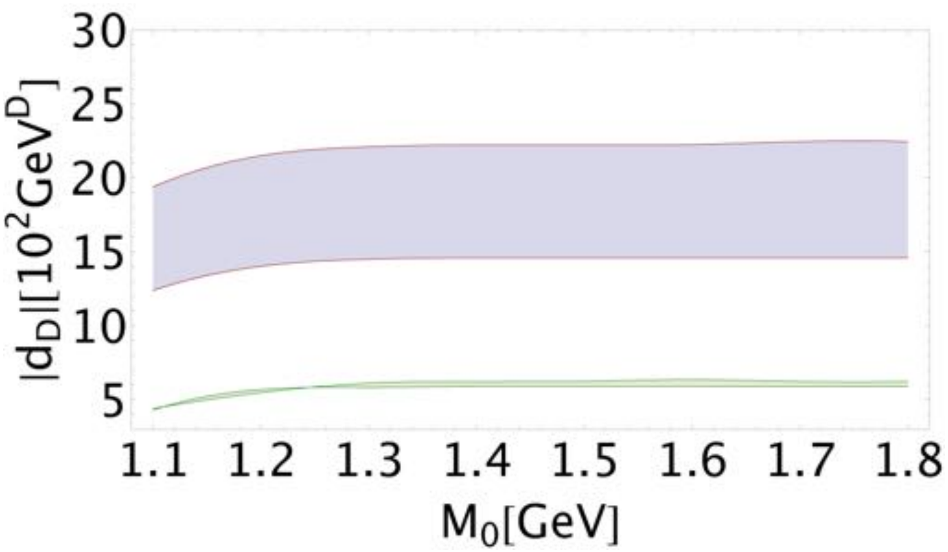
$$\underset{J/\psi,\Upsilon}{d_4} \underset{LSR}{\longleftrightarrow} d_6 \underset{R_0}{\longleftrightarrow} d_8 \underset{R_1}{\overset{(2)}{\longleftrightarrow}} d_{10} \underset{R_2}{\overset{(6)}{\longleftrightarrow}} d_{12} \underset{R_3}{\longrightarrow} d_{14} \underset{R_4}{\longrightarrow} d_{16}$$

where (2) and (6) indicate the number of iterations.

Analysis from τ-like decay moments

- d_8 from R_1 : inputs α_s, d_4, d_{10} — d_{10}, d_{12} from R_2 : inputs α_s, d_6





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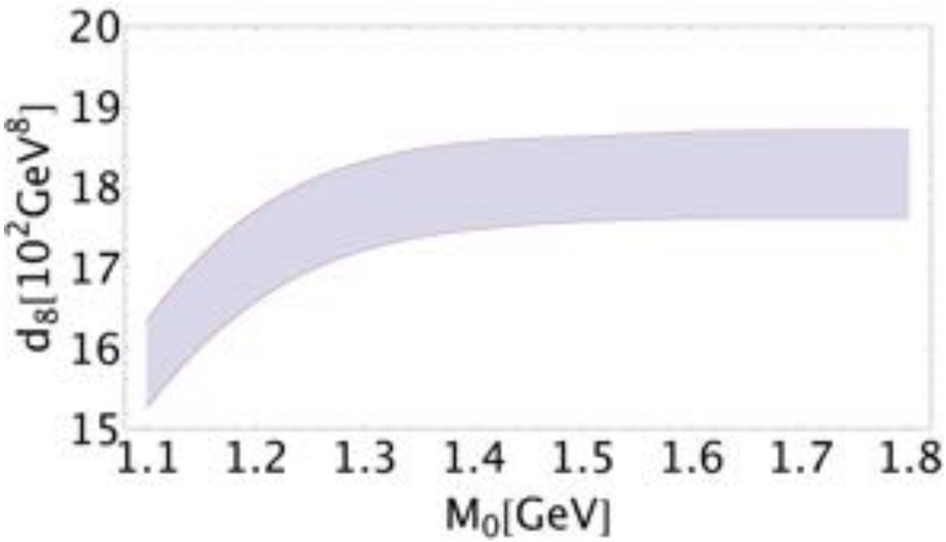
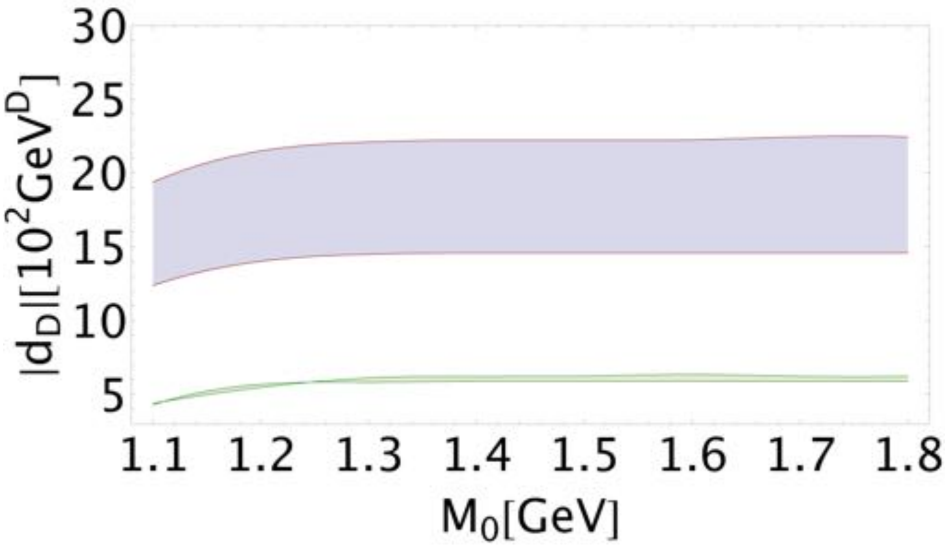
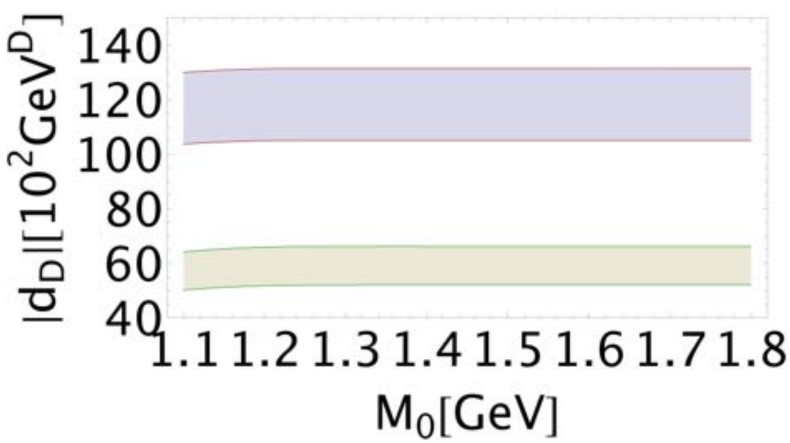
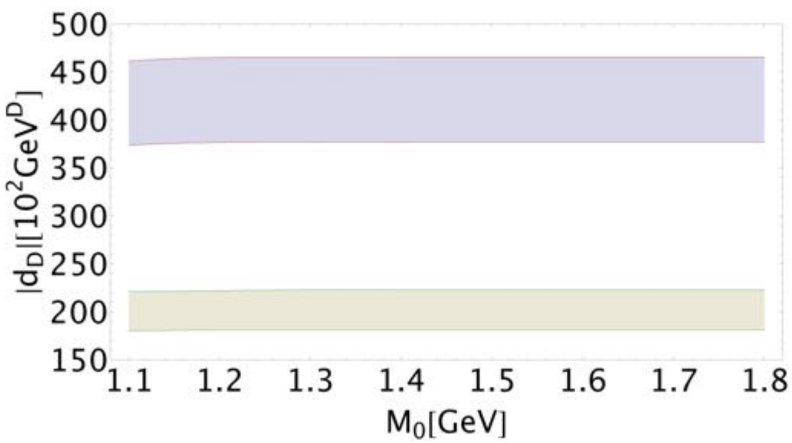




- d_{14}, d_{16} from R_3 and R_4 — d_{18}, d_{20} from R_5 and R_6.





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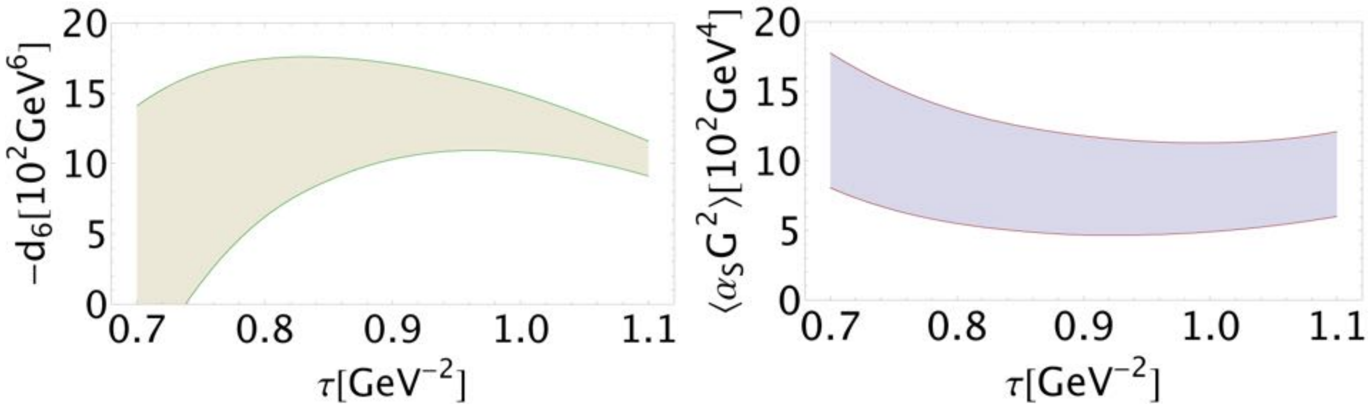

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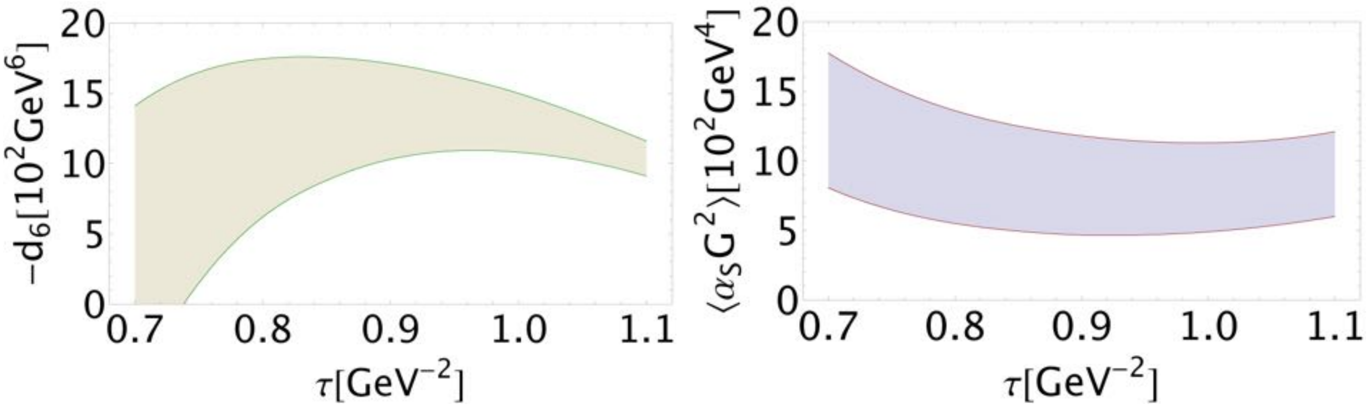

- $d_6 = -(23.6 \pm 3.7) \times 10^{-2}$ GeV^6 : for $\tau \simeq (0.85 \sim 0.95)$ GeV^{-2}
  – Agrees with $d_6 = -(20.5 \pm 2.2) \times 10^{-2}$ GeV^6 (initial inputs obtained at larger τ-values).
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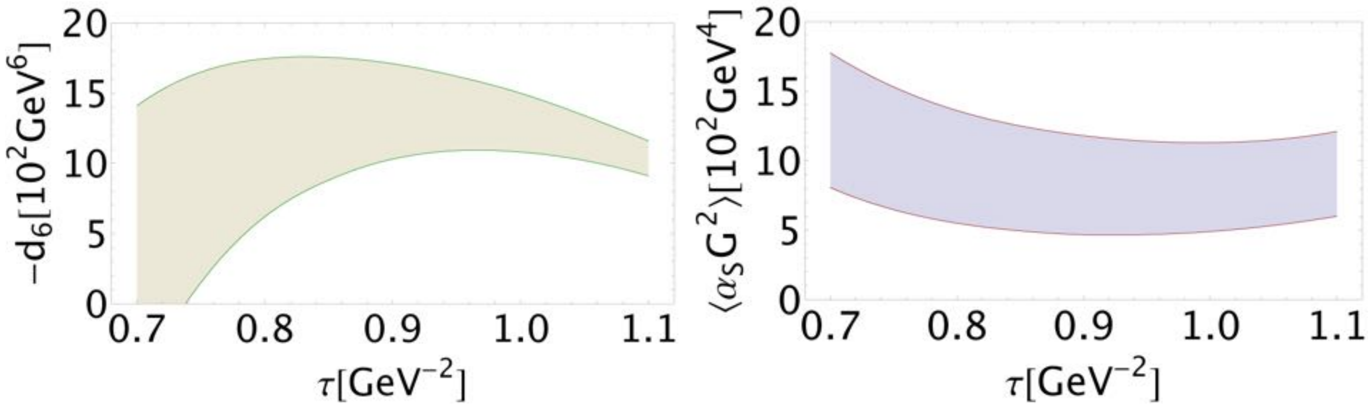

- $d_6 = -\,(23.6 \pm 3.7) \times 10^{-2}$ GeV^6 : for $\tau \simeq (0.85 \sim 0.95)$ GeV^{-2}
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- $\langle \alpha_s G^2 \rangle = (7.8 \pm 3.3) \times 10^{-2}$ GeV^4 : for $\tau \simeq (0.9 \sim 1.0)$ GeV^{-2}
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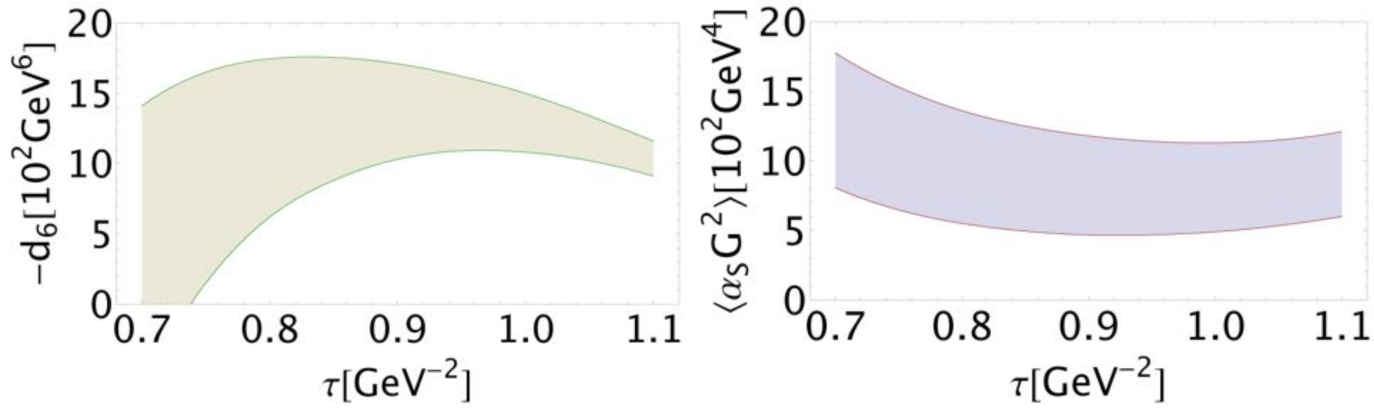

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- Good convergence of the OPE at the optimization scale :

$$\tau R_{10}(\tau) = 1 + \beta_1 a_s^2 + \cdots - 0.123\tau^2 + 0.164\tau^3 - .140\tau^4 - 0.015\tau^5 + 0.050\tau^6 - 0.034\tau^7 + 0.193\tau^8$$

$$= 1 + 10^{-3} + \sum_{n=2}^{3} d_{2n}\tau^n = 0.127(0.091) + \sum_{n=4}^{8} d_{2n}\tau^n = 0.017(-0.023) \text{ for } \tau = 0.95\ (0.85)\ \text{GeV}^{-2}$$

Summary : QCD condensates from $e^+e^- \to$ Hadro

- This work at (FO) : $\langle\alpha_s G^2\rangle = (6.49 \pm 0.35)\times 10^{-2}$ GeV^4 input from $J/\psi, \Upsilon$ for $d_n \geq 6$. Units : 10^{-2} GeV^D : D dimension of the condensates. Systematic Errors not Inlcuded.

$\langle\alpha_s G^2\rangle$	$-d_6$	d_8	$-d_{10}$	$-d_{12}$	d_{14}	$-d_{16}$	$-d_{18}$	d_{20}
7.8 ± 3.5	23.6 ± 3.7	18.2 ± 0.6	6.0 ± 0.2	18.4 ± 3.8	59.2 ± 7.1	118.4 ± 13.2	202.0 ± 20.7	421.3 ± 44.3

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– d_6 confirms a huge violation of factorization: factor (7 ± 1) LNT 84, SN96,

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- QCD condensates from some other τ-like decay moments at (FO)

$\langle \alpha_s G^2 \rangle$	$-d_6$	d_8	$-d_{10}$	$-d_{12}$	Refs.
0.67 ± 0.89	15.2 ± 2.2	22.3 ± 2.5			ALEPH 99
5.34 ± 3.64	14.2 ± 3.5	21.3 ± 2.5			OPAL 99
$3.5^{+2.2}_{-3.8}$	$19.7^{+11.8}_{-7.9}$	$23.7^{+11.8}_{-15.8}$	11.8 ± 19.7	7.9 ± 19.7	Pich-Rodriguez 16 ($d_{14,16} = 0$)

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QCD COUPLING α_s

from e^+e^- -> Hadrons

and τ -> Vector + ν_τ

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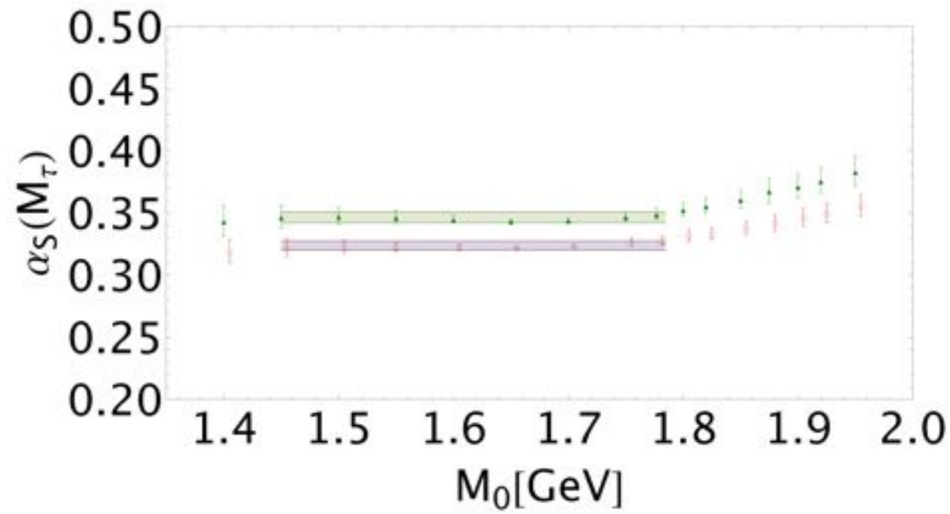

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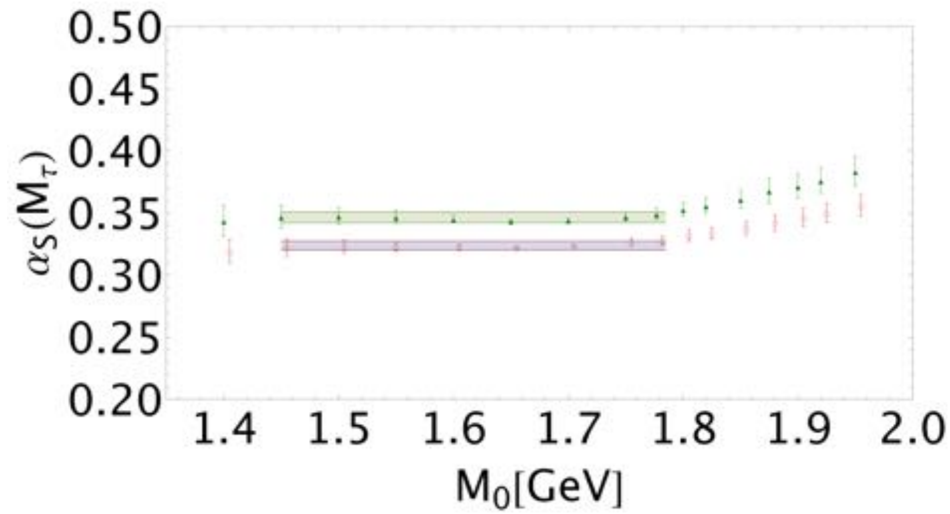

- One obtains to order $O(\alpha_s^4)$ [$M_0 = 1.675(25)$ GeV (stability point)] :

$$\alpha_s(M_\tau) = 0.3238(36) \Longrightarrow \alpha_s(M_Z) = 0.1190(2) \quad \text{FO}$$
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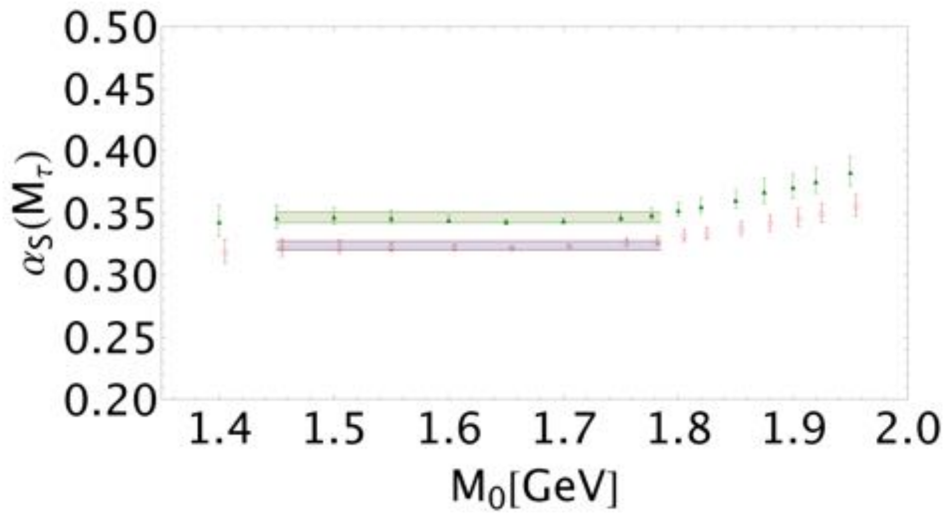

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- Sum of NP terms :

$$\sum \delta_{NP}(M_\tau) = (3.8 \pm 0.8) \times 10^{-2} \qquad \approx \alpha_s^3 \ (FO), \quad \alpha_s^2 \ (CI).$$

Estimate of the error due to α_s^5

- Geometric growth of different PT series SN-Zakharov 09 :

$$
\begin{aligned}
D(Q^2) &= \sum_n a_s^n c_n \ : \ c_0 = c_1 = 1,\ \ c_2 = 1.656,\ c_3 = 6.37,\ c_4 = 49.09 \\
&\Longrightarrow \quad c_4 \approx c_3^2 \Longrightarrow c_5 \simeq (c_3/c_2)\, c_4^2 \approx (228 \pm 114). \\
R_0 &= \sum_n a_s^n (g_n + c_n) : \quad g_n \ \mathrm{from\ RG-resummation} \\
& \quad (\mathrm{see\ e.g:\ Pich-Lediberder\,92,\ Kataev-Starshenko\,95}) \\
& \quad g_5 = -780\ (FO), \qquad 0(CI) \\
&\Longrightarrow \quad \Delta\alpha_s(M_\tau) = \pm 71 \times 10^{-4}\ (FO),\ \ \pm 62 \times 10^{-4}\ (CI),
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- $\alpha_s(M_\tau)$ to order α_s^4 including error due to α_s^5

$$\alpha_s(M_\tau) = 0.3238(36)_{fit}(71)_{\alpha_s^5} \Longrightarrow \alpha_s(M_Z) = 0.1190(9)(3)_{evol} \quad \text{FO}$$

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- Mean : assume that $\Delta\alpha_s^5$ absorbs the $\neq$ between FO and CI at H.O :

$$\alpha_s(M_\tau) = 0.3358(55)(107)_{syst} \Longrightarrow \alpha_s(M_Z) = 0.1204(7)(3)_{evol}$$

where syst comes from the distance of the mean central value to the ones of FO or CI.

$\alpha_s(M_\tau)$ from $e^+e^- \to I=1$ Hadrons & $\tau \to$ Vector

- Comparison of $\neq$ Determinations

	THIS WORK		ALEPH 98	OPAL 99	PR 16	ALEPH 14
	e^+e^-	τ-decay				
FO	0.3081(86)	0.3128(79)	0.3200(220)	0.3230(160)	0.3200(150)	–
CI	0.3260(78)	0.3291(70)	0.3400(230)	0.3470(220)	0.3370(200)	0.3460(110)
FO $\oplus$ CI	0.3179(100)	0.3219(105)				
$\delta^{(NP)} \times 10^{-2}$	3.8(8)	3.7(1.0)	2.0(3)	3.6(4)	1.7(3)	2.0(3)

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FO $\oplus$ CI	0.3179(100)	0.3219(105)				
$\delta^{(NP)} \times 10^{-2}$	3.8(8)	3.7(1.0)	2.0(3)	3.6(4)	1.7(3)	2.0(3)

- Average of our result for FO $\oplus$ CI :

$$\alpha_s(M_\tau)|^{SVZ}_{e^+e^-} = 0.3179(58)(81)_{syst} \longrightarrow \alpha_s(M_Z)|^{SVZ}_{e^+e^-} = 0.1182(12)(3)_{evol}$$

$$\alpha_s(M_\tau)|_{\tau,V} = 0.3219(52)(91)_{syst} \longrightarrow \alpha_s(M_Z)|_{\tau,V} = 0.1187(13)(3)_{evol}$$

$\alpha_s(M_\tau)$ from $e^+e^- \to I=1$ Hadrons & $\tau \to$ Vector

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FO	0.3081(86)	0.3128(79)	0.3200(220)	0.3230(160)	0.3200(150)	–
CI	0.3260(78)	0.3291(70)	0.3400(230)	0.3470(220)	0.3370(200)	0.3460(110)
FO $\oplus$ CI	0.3179(100)	0.3219(105)				
$\delta^{(NP)} \times 10^{-2}$	3.8(8)	3.7(1.0)	2.0(3)	3.6(4)	1.7(3)	2.0(3)

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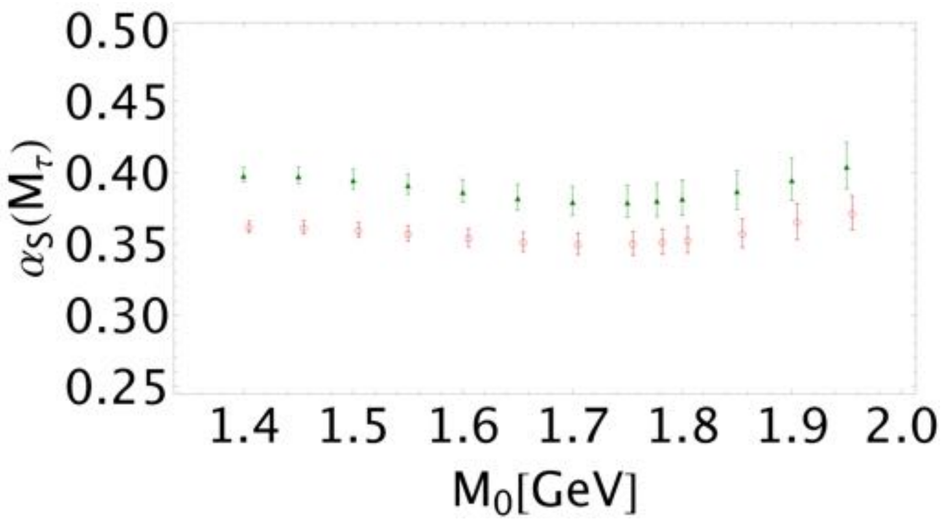

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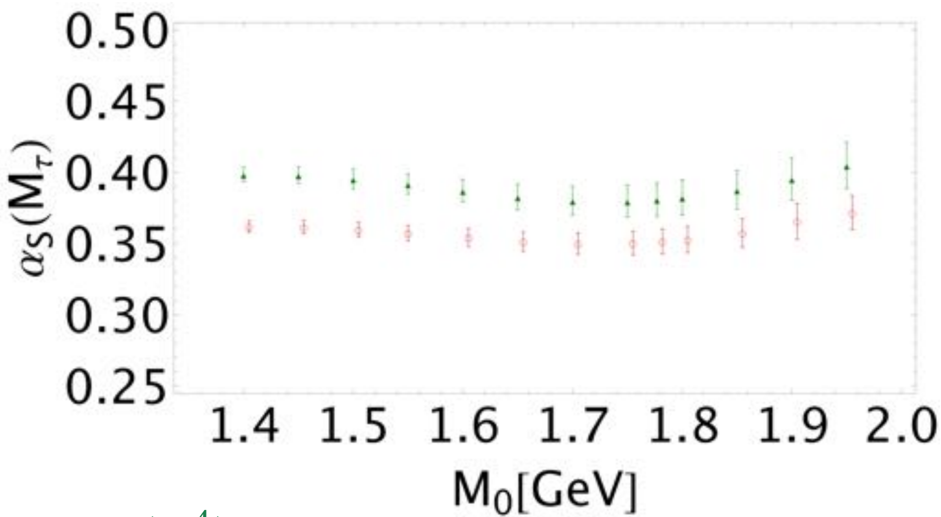

- One obtains to order $O(\alpha_s^4)$:

$$\alpha_s(M_\tau) = 0.3514(68) \Longrightarrow \alpha_s(M_Z) = 0.1221(7) \quad \text{FO}$$
$$= 0.3827(90) \Longrightarrow \alpha_s(M_Z) = 0.1252(9) \quad \text{CI}$$

Relatively High compared to the PDG 24 average !

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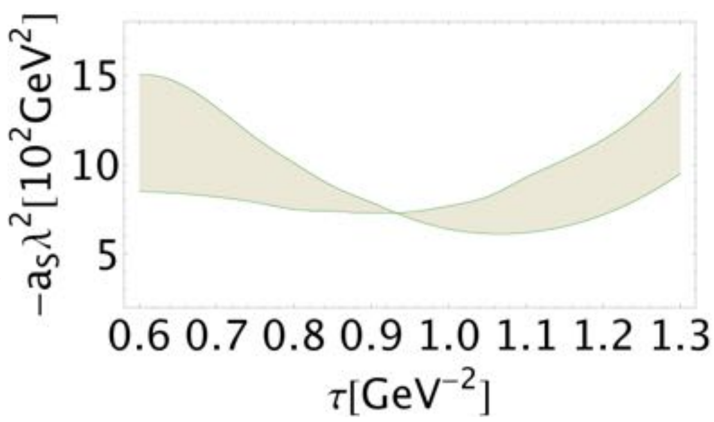

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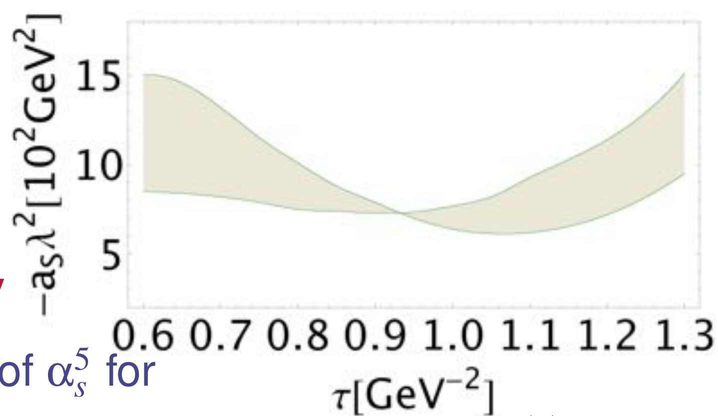

  - To avoid a double counting with the estimate of α_s^5 for HO terms, we do not include by duality $\alpha_s\lambda^2$ but give an upper bound : $\delta^{(2)}_{\lambda^2} \leq 4\%$.

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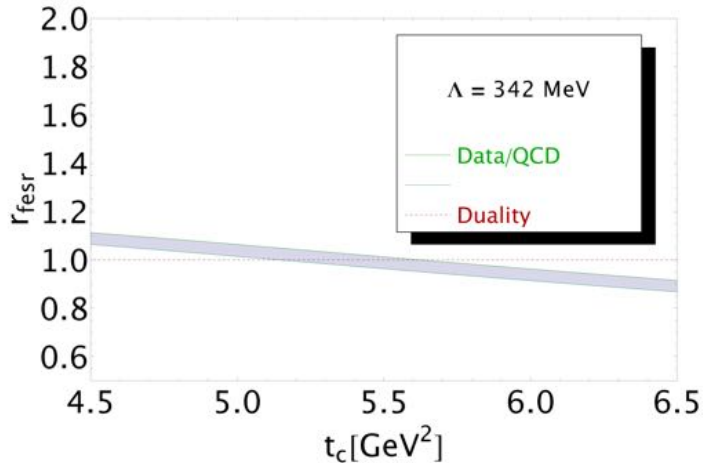
$\Longrightarrow$ Huge Exponential suppression !



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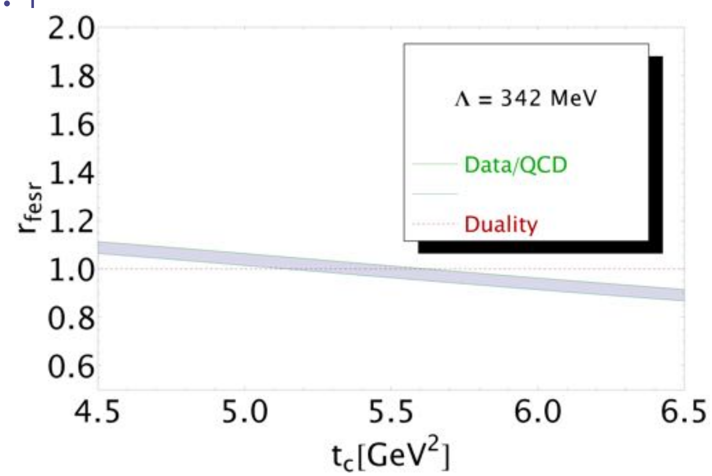

- Oscillating DV term in Minkowski space DUAL to Exponential behaviour of Condensates (Not seen from our analysis !) in Euclidean region SHIFMAN10.

Summary

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  - Average of $e^+e^- \to \mathrm{I} = 1\mathrm{Hadrons}$ results for FO $\oplus$ CI :

$$\alpha_s(M_\tau)|^{SVZ}_{e^+e^-} = 0.3179(58)(81)_{syst} \longrightarrow \alpha_s(M_Z)|^{SVZ}_{e^+e^-} = 0.1182(12)(3)_{evol}$$

$$\alpha_s(M_\tau)|_{\tau,V} = 0.3219(52)(91)_{syst} \longrightarrow \alpha_s(M_Z)|_{\tau,V} = 0.1187(13)(3)_{evol}$$

  - Average from $e^+e^- \to \mathrm{I} = 1\mathrm{Hadrons}$ and $\tau \to \mathrm{Vector} + \nu_\tau$:

$$\langle \alpha_s(M_\tau) \rangle = 0.3198(72) \Longrightarrow \langle \alpha_s(M_Z) \rangle = 0.1185(9)(3)_{evol}$$

  - Good agreement with PDG24 : $\alpha_s(M_Z)_{PDG}$ = 0.1178(5)
- Beyond the Standard SVZ OPE : Negligible effects !

Misaotra Anareo @ ny Faharetana !

Merci pour Votre Patience !

Thanks for Your Patience !

https://www.lupm.in2p3.fr/users/qcd/agmm