



Electric Corrections to π - π Scattering lengths in the Linear Sigma Model

M. Loewe

Universidad San Sebastián, Santiago, Chile

In collaboration with R. Cádiz and R. Zamora

27th High-Energy Physics International Conference in QCD
(8-12 July, 2024, Montpellier)

This work has been supported by Fondecyt under grants:
1220035, 1241436.

This presentation is based on the article
Electric corrections to π - π scattering lengths in the linear sigma
model.

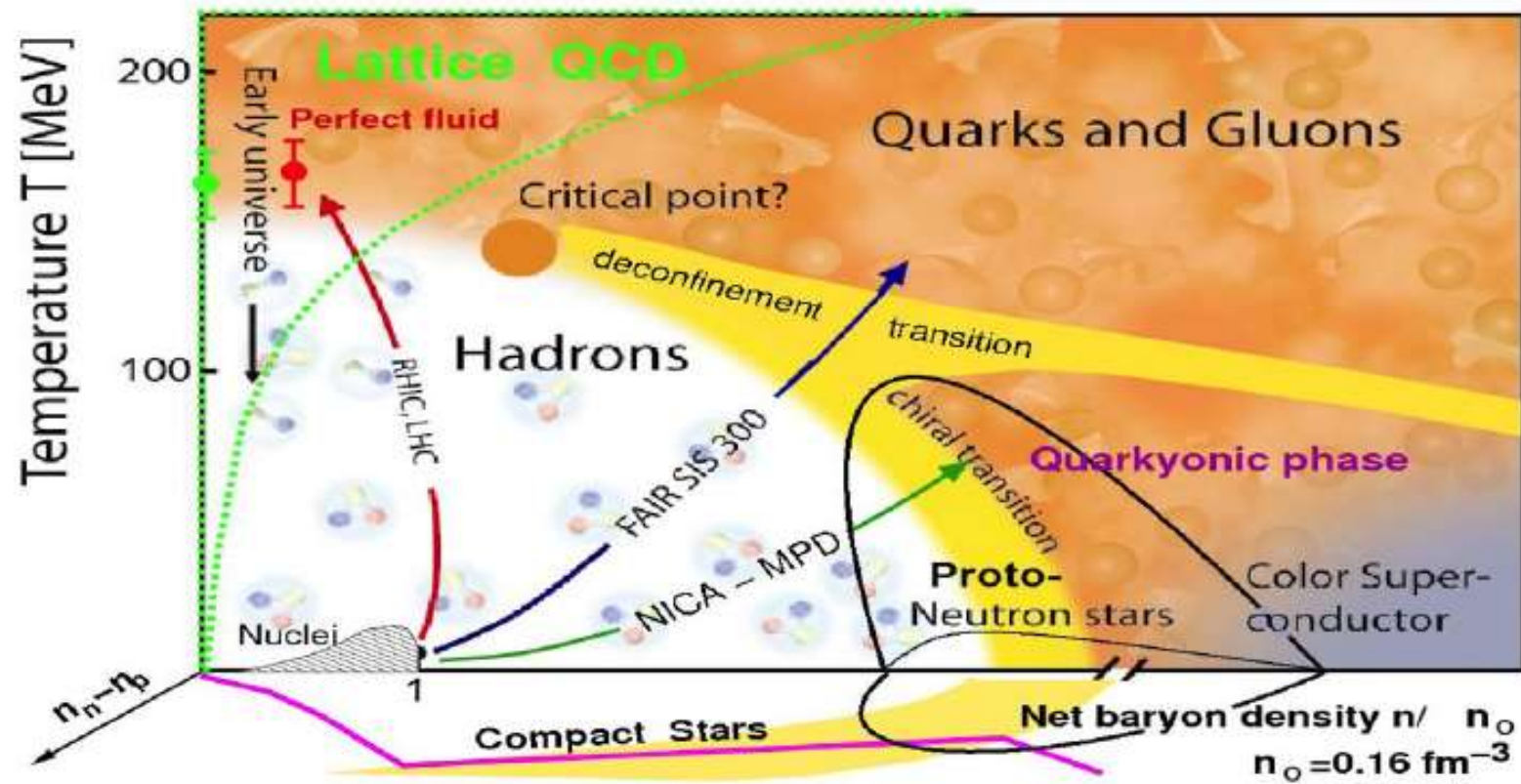
R. Cádiz, M. Loewe and R. Zamora
Phys. Rev. D 109, 116004 (2024)

Relativistic heavy ion collision experiments (RHIC, LHC, FAIR and NICA in the near future) open a window to explore initial stages of the universe.

A high temperature regime, huge magnetic fields, together with density effects, affect in a dramatic way the physics of strong interactions.

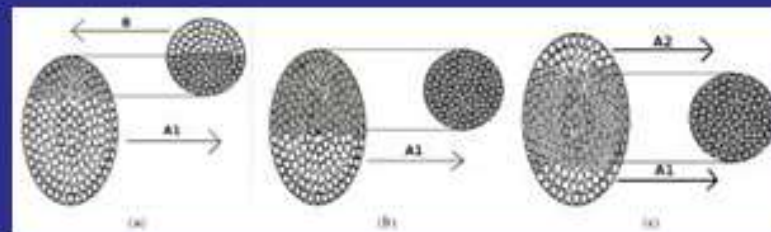
Several phase transitions occur: Deconfinement, Chiral Restoration, transition to a Quarkyonic phase

QCD phase diagram



Today I want to mention a different scenario where also huge electric fields appear: Asymmetric peripheral relativistic heavy ion collisions as, for example, Cu+Au, Cu+Pb collisions.

Due to the imbalance in the number of charges in each nuclei a strong electric field, \perp to B , appears



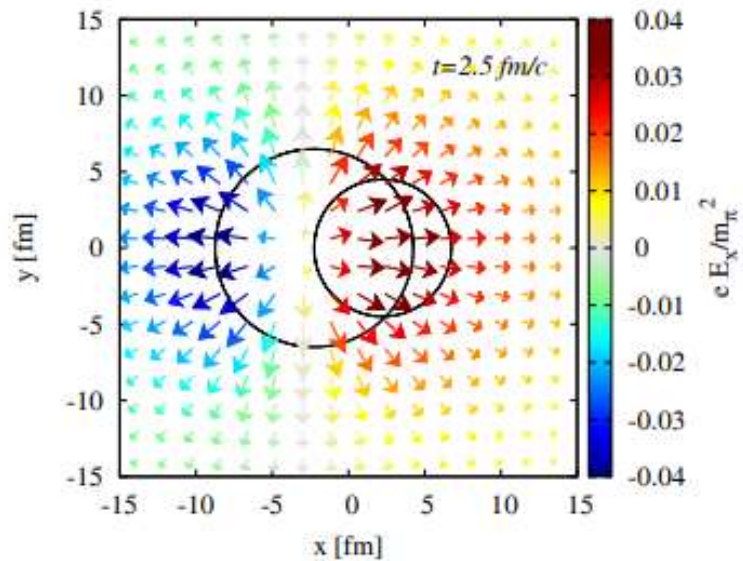


Fig. 1. Electric field generated in the transverse plane of Cu+Au collisions at $\sqrt{s_{NN}} = 9$ GeV, $b = 4.5$ fm and $t = 2.5$ fm/c. The direction of the arrows indicates the field direction projected onto the reaction plane and the length is proportional to the electromagnetic strength shown in color.

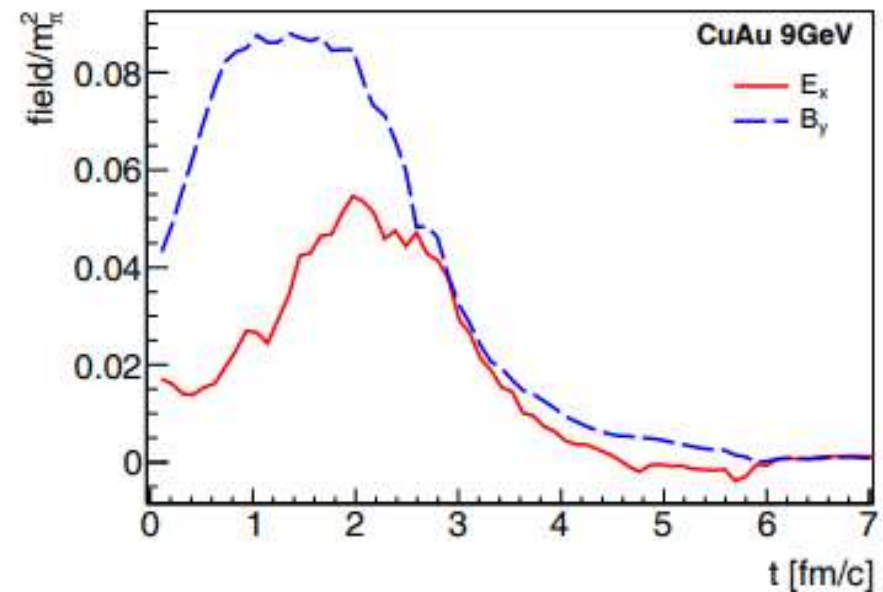


Fig. 2. The time dependent components of the average electric E_x and magnetic B_y field generated in the center of Cu+Au collision at $b = 4.5$ fm and $\sqrt{s_{NN}} = 9$ GeV.

Extracted from V. Toneev, O. Rogachevsky and V. Voronyuk: Eur. Phys. J. A. (2016) 53: 264

In general, a π - π scattering amplitude has the form (greek indices refer to Isospin indices)

$$T_{\alpha\beta\gamma\delta} = A(s, t, u)\delta_{\alpha\beta}\delta_{\gamma\delta} + A(t, s, u)\delta_{\alpha\gamma}\delta_{\beta\delta} + A(u, t, s)\delta_{\alpha\delta}\delta_{\beta\gamma}$$

Using the following projectors

$$P_0 = \frac{1}{3}\delta_{\alpha\beta}\delta_{\gamma\delta},$$

$$P_1 = \frac{1}{2}(\delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma}),$$

$$P_2 = \frac{1}{2}\left(\delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma} - \frac{2}{3}\delta_{\alpha\beta}\delta_{\gamma\delta}\right)$$

In this way we find the following Isospin dependent amplitudes

$$T^0 = 3A(s, t, u) + A(t, s, u) + A(u, t, s),$$

$$T^1 = A(t, s, u) - A(u, t, s),$$

$$T^2 = A(t, s, u) + A(u, t, s).$$

The elastic regime refers to the region $4M_\pi^2 < s < 16M_\pi^2$

See J. Gasser and H. Leutwyler: Ann. Phys. 158, 142 (1984); J. Donoghue, E. Golowich, and B. Holstein: The Dynamics of the Standard Model, Cambridge 1996

Scattering lengths were introduced as appropriate parameters for the description of nucleon-nucleon and pion nucleon scattering in the good old times.

Consider a partial wave expansion, projected into some isospin channel I.

$$T^I(s, t, u) = 32\pi \sum_{\ell=0}^{\infty} (2\ell + 1) P_{\ell}(\cos \theta) T_{\ell}^I(s)$$

Coming close to the threshold

$$T_{\ell}^I = \left(\frac{s}{s - 4m_{\pi}^2} \right)^{1/2} \frac{1}{2i} \left(e^{2i\delta_{\ell}^I(s)} - 1 \right)$$

In fact, the previous expression can be expanded as

$$\Re (T_\ell^I) = \left(\frac{p^2}{m_\pi^2} \right)^\ell \left(a_\ell^I + \frac{p^2}{m_\pi^2} b_\ell^I + \dots \right)$$

$$p^2 \equiv (s - 4m_\pi^2)/4$$

Where a_ℓ^I are the so called scattering lengths. The next coefficients, b_ℓ^I are the scattering slopes. It can be shown that

$$a = \lim_{p \rightarrow 0} \frac{\tan \delta_0}{p} = - \lim_{p \rightarrow 0} \frac{\delta_0}{p}$$

This definition can be extended to the other angular momentum channels

$$a_\ell = \lim_{p \rightarrow 0} \frac{\tan \delta_\ell}{p} = - \lim_{p \rightarrow 0} \frac{\delta_\ell}{p}$$

In general we have the following order

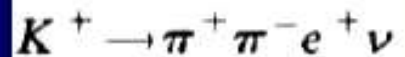
$$|a_0| > |a_1| > |a_2| > \dots$$

At very low energies, scattering is dominated by the s-channel amplitude (isotropic scattering) and the total cross section is given by $\sigma = 4\pi a_0^2$. Notice that the Martin inequalities are satisfied

A. Martin, Nuovo
Cimento A47 (1967)
265

$$a_{l+2}^I \leq a_l^I \frac{(l+1)(l+2)}{4(2l+3)(2l+5)}$$

π - π scattering lengths were measured first by L. Rossellet et al, PRD 15, 574 (1977) using pions coming from the decay



The interaction occurs between two real pions, the only hadrons in the final state. The quantum states of the di-pion in the decay are $L = 0, I = 0$ and $L = 1, I = 1$. The invariant mass distribution of the di-pions has a maximum relatively close to the π - π threshold.

Later: B. Adeva et al, Phys. Lett. B 704 (2011) 24 (Dirac experiment)

The DIRAC experiment at CERN has achieved a sizeable production of $\pi^+\pi^-$ atoms and has significantly improved the precision on its lifetime determination. From a sample of 21227 atomic pairs, a 4% measurement of the S-wave $\pi\pi$ scattering length difference $|a_0 - a_2| = (0.2533^{+0.0080}_{-0.0078} |_{\text{stat}} \ ^{+0.0078}_{-0.0073} |_{\text{syst}}) M_{\pi^+}^{-1}$ has been attained, providing an important test of Chiral Perturbation Theory.

A ponium atom, with a radius of 378 fm, decays into a pair of neutron pions.

$$\Gamma_{2\pi^0} = \frac{2}{9}\alpha^3 p^* (a_0 - a_2)^2 (1 + \delta) M_{\pi^+}^2$$

$$p^* = \sqrt{M_{\pi^+}^2 - M_{\pi^0}^2 - (1/4)\alpha^2 M_{\pi^+}^2}$$

More recently: X.-H Liu, F.-K. Guo and E. Epelbaum, Eur. Phys. J. C73, 2284, (2013) were able to extract the scattering lengths from heavy quarkonium decays

Charge-exchange rescattering $\pi^+\pi^- \rightarrow \pi^0\pi^0$ leads to a cusp effect in the $\pi^0\pi^0$ invariant mass spectrum of processes with $\pi^0\pi^0$ in the final state which can be used to measure $\pi\pi$ S -wave scattering lengths. Employing a non-relativistic effective field theory, we discuss the possibility of extracting the scattering lengths in heavy quarkonium $\pi^0\pi^0$ transitions. The transition $\Upsilon(3S) \rightarrow \Upsilon(2S)\pi^0\pi^0$ is studied in details. We discuss the precision that can be reached in such an extraction for a certain number of events.

$$\psi'(P_{\psi'}) \rightarrow \pi^0(p_1)\pi^0(p_2)J/\psi(p_3),$$

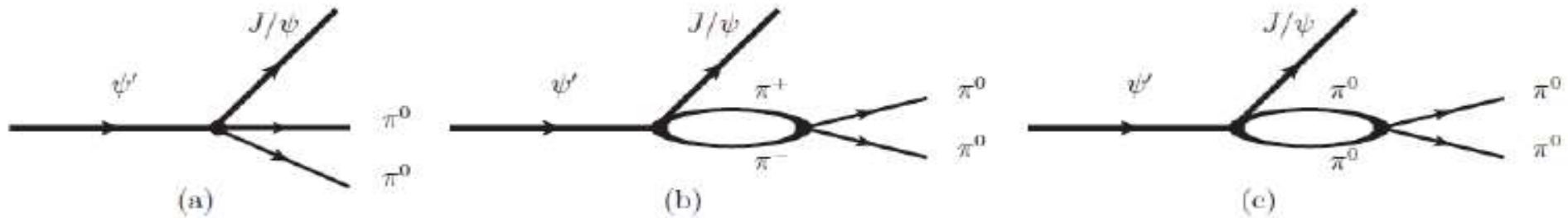


Figure 1: $\psi' \rightarrow J/\psi \pi^0 \pi^0$ via tree diagram and $\pi\pi$ rescattering diagrams.

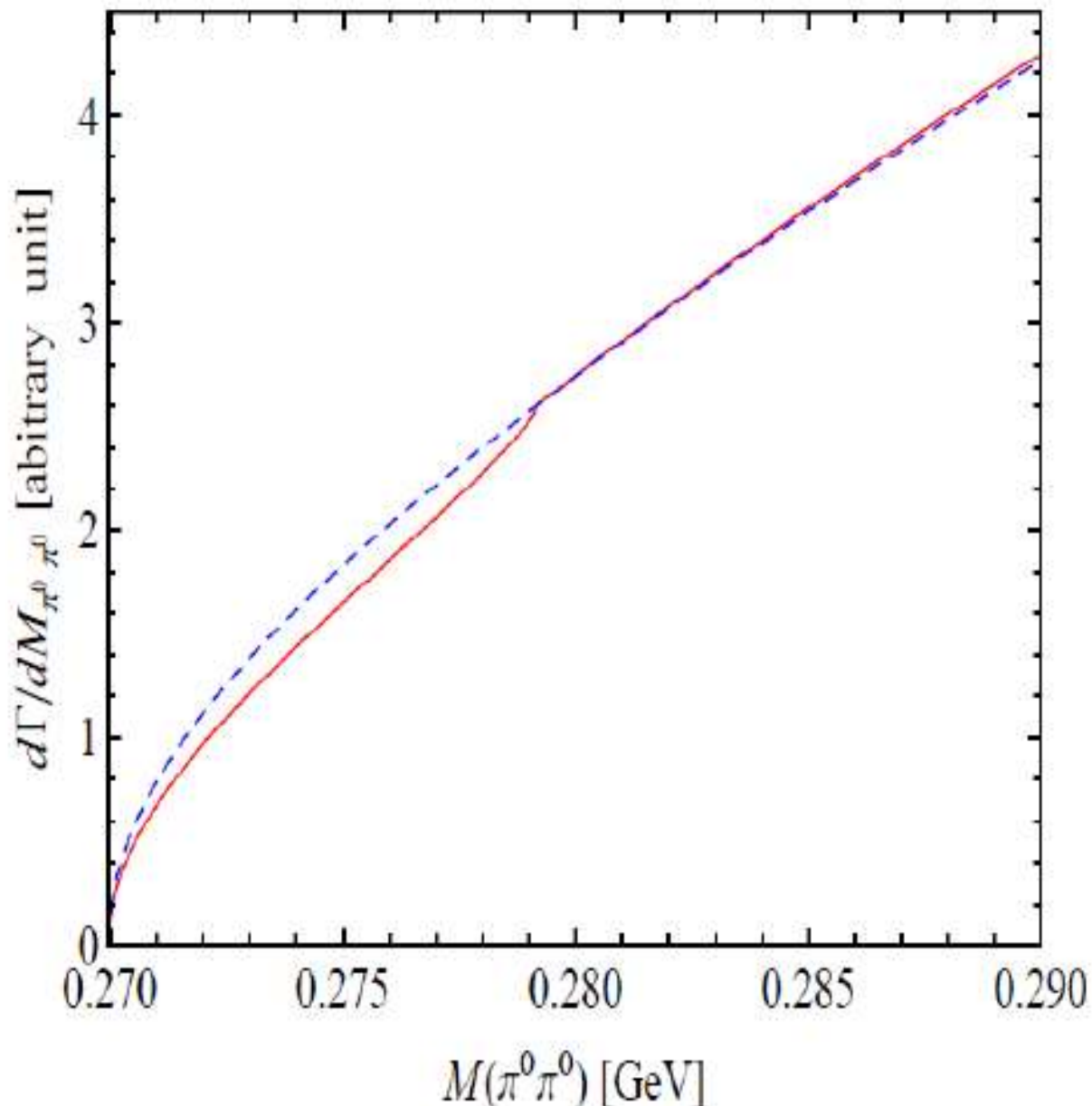


Figure 4: The cusp effect at the $\pi^+\pi^-$ threshold in the reaction $\Upsilon(3S) \rightarrow \Upsilon(2S)\pi^0\pi^0$ calculated in the NREFT framework (solid line). The dashed line shows the result without charge-exchange rescattering.

Linear sigma model. Gell-Mann-Levy 1960

$$\begin{aligned}\mathcal{L} = & \bar{\psi} [i\gamma^\mu \partial_\mu - m_\psi - g(s + i\vec{\pi} \cdot \vec{\tau}\gamma_5)]\psi \\ & + \frac{1}{2} [(\partial\vec{\pi})^2 + m_\pi^2 \vec{\pi}^2] + \frac{1}{2} [(\partial\sigma)^2 + m_\sigma^2 s^2] \\ & - \lambda^2 v s (s^2 + \vec{\pi}^2) - \frac{\lambda^2}{4} (s^2 + \vec{\pi}^2)^2 + (\epsilon c - v m_\pi^2) s.\end{aligned}$$

The σ field has been shifted

$$\sigma = s + v.$$

$$\langle s \rangle = 0$$

$c\sigma$ is the term that breaks the
SU(2)xSU(2) symmetry explicitly

$$v = \langle \sigma \rangle$$

All masses are determined by v , the expectation value of the σ field

$$m_\psi = gv$$

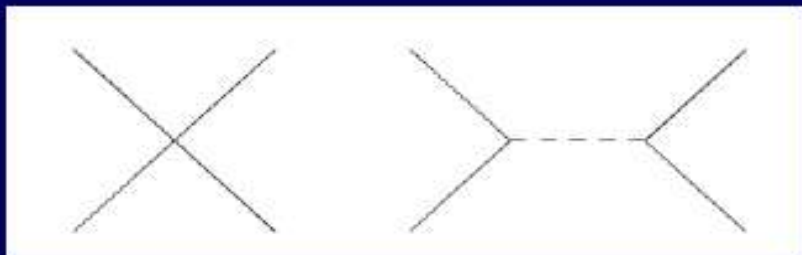
$$m_\pi^2 = \mu^2 + \lambda^2 v^2$$

$$m_\sigma^2 = \mu^2 + 3\lambda^2 v^2$$

Perturbation theory at the tree level allows to identify

$$f_\pi = v$$

Tree level diagrams for the π - π scattering lengths



$$T^0(s, t, u) = -10\lambda^2 - \frac{12\lambda^4 v^2}{s - m_\sigma^2} - \frac{4\lambda^4 v^2}{t - m_\sigma^2} - \frac{4\lambda^4 v^2}{u - m_\sigma^2}$$

$$T^1(s, t, u) = \frac{4\lambda^4 v^2}{u - m_\sigma^2} - \frac{4\lambda^4 v^2}{t - m_\sigma^2}$$

$$T^2(s, t, u) = -4\lambda^2 - \frac{4\lambda^4 v^2}{t - m_\sigma^2} - \frac{4\lambda^4 v^2}{u - m_\sigma^2}$$

	Experimental results	Chiral perturbation theory	Linear sigma model
a_0^0	0.218 ± 0.02	$\frac{7m_\pi^2}{32\pi f_\pi^2} = 0.16$	$\frac{10m_\pi^2}{32\pi f_\pi^2} = 0.22$
b_0^0	0.25 ± 0.03	$\frac{m_\pi^2}{4\pi f_\pi^2} = 0.18$	$\frac{49m_\pi^2}{128\pi f_\pi^2} = 0.27$
a_0^2	-0.0457 ± 0.0125	$\frac{-m_\pi^2}{16\pi f_\pi^2} = -0.044$	$\frac{-m_\pi^2}{16\pi f_\pi^2} = -0.044$
b_0^2	-0.082 ± 0.008	$\frac{-m_\pi^2}{8\pi f_\pi^2} = -0.089$	$\frac{-m_\pi^2}{8\pi f_\pi^2} = -0.089$
a_1^1	0.038 ± 0.002	$\frac{m_\pi^2}{24\pi f_\pi^2} = 0.030$	$\frac{m_\pi^2}{24\pi f_\pi^2} = 0.030$
b_1^1	...	0	$\frac{m_\pi^2}{48\pi f_\pi^2} = 0.015$

The (charged) bosonic propagator in the present of a constant electric field

$$D(p) = \int_0^\infty ds \frac{e^{-s(\frac{\tanh(qiEs)}{qiEs} p_\parallel^2 + p_\perp^2 + m^2)}}{\cosh(qiEs)}$$

For small values of the electric field the propagator goes into

$$D(p) \approx \frac{1}{p^2 + m^2} - (qE)^2 \left(-\frac{1}{(p^2 + m^2)^3} + \frac{2p_{\parallel}^2}{(p^2 + m^2)^4} \right).$$

We will use this propagator for the charged pions in our calculation

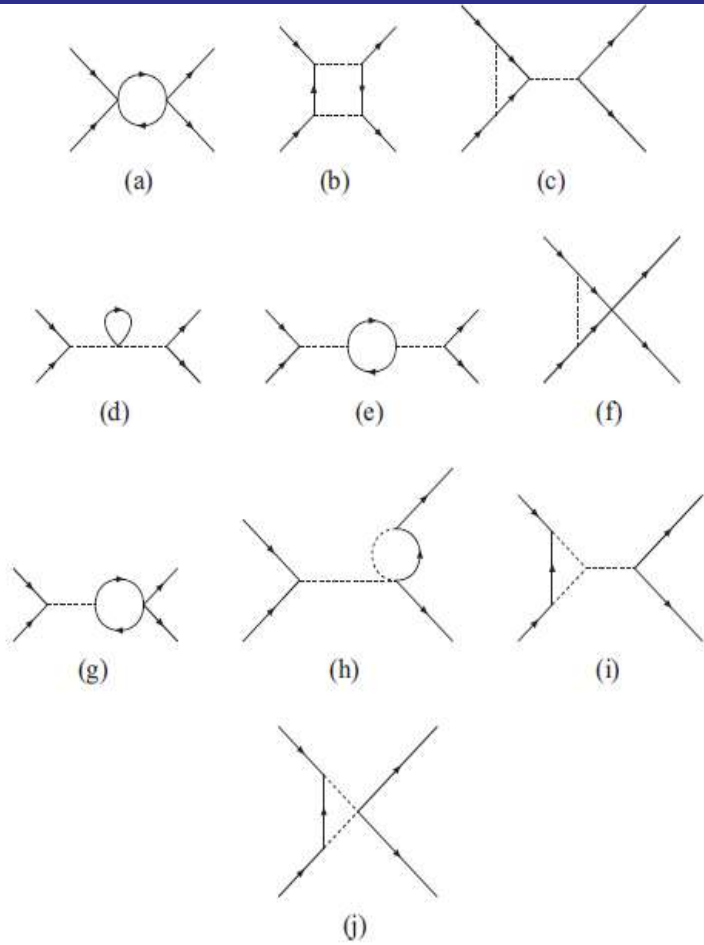


FIG. 1. (a)–(j) s channel diagrams.

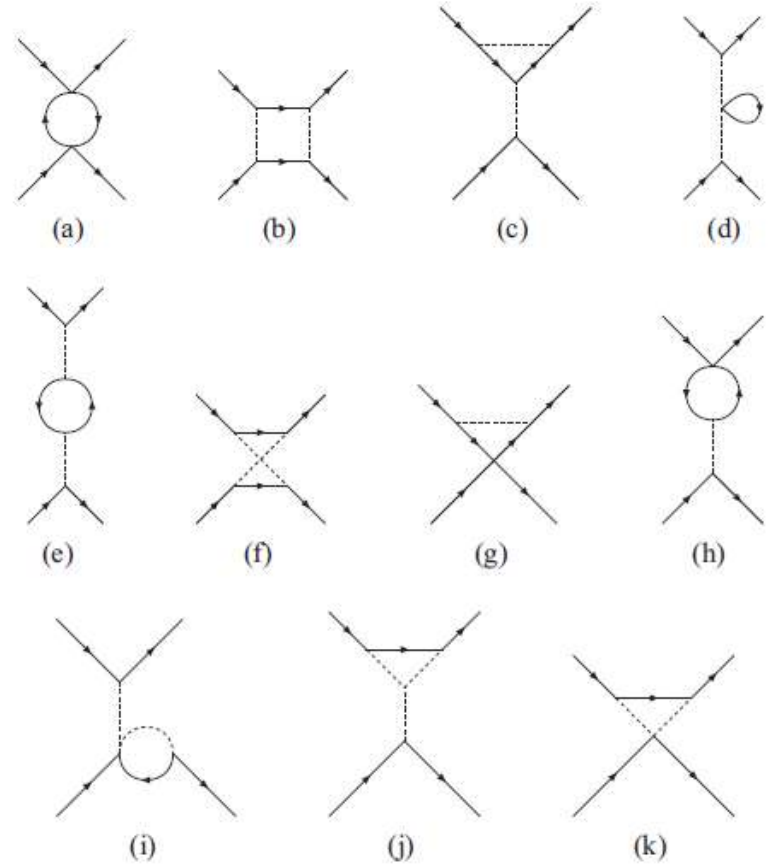


FIG. 2. (a)–(k) t channel diagrams.

The relevant diagrams

$$T_{1L}^0 = 3A(s, t, u) + 2A(t, s, u),$$

$$T_{1L}^1 = 0,$$

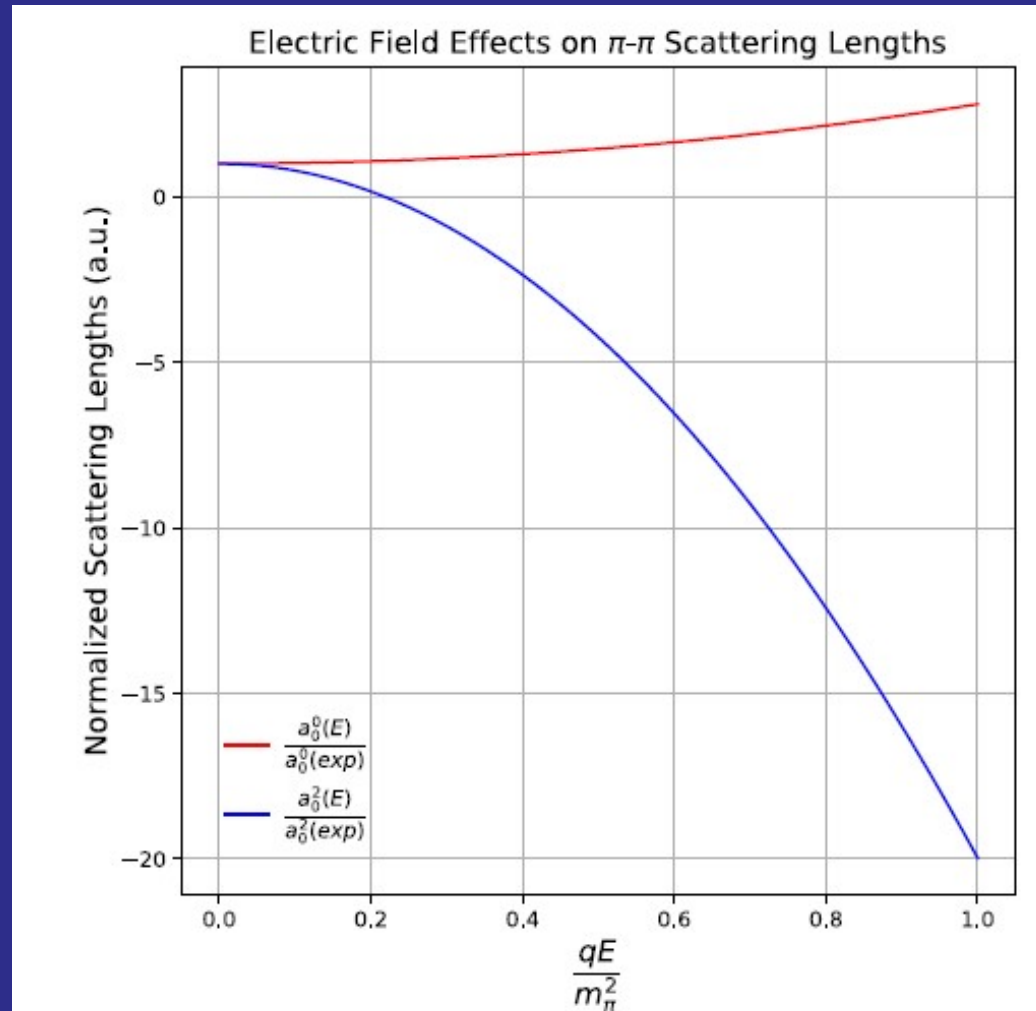
$$T_{1L}^2 = 2A(t, s, u).$$

$$\begin{aligned} T_{1L}^0 = & 32\lambda^8 v^4 \left(\frac{9}{2} I_{B1} + I_{B2} + I_9 \right) + I_3 \left(\frac{144\lambda^8 v^4}{4m_\pi^2 + m_\sigma^2} + 120\lambda^6 v^2 \right) + I_4 \left(\frac{32\lambda^8 v^4}{m_\sigma^2} + 80\lambda^6 v^2 \right) + I_6 \left(\frac{432\lambda^8 v^4}{4m_\pi^2 + m_\sigma^2} + 72\lambda^6 v^2 \right) \\ & + I_5 \left(\frac{96\lambda^8 v^4}{m_\sigma^2} + 16\lambda^6 v^2 \right) + I_7 \left(\frac{216\lambda^6 v^3}{(4m_\pi^2 + m_\sigma^2)^2} + \frac{48\lambda^6 v^3}{m_\sigma^4} \right) + I_8 \left(\frac{72\lambda^6 v^2}{4m_\pi^2 + m_\sigma^2} + \frac{16\lambda^6 v^2}{m_\sigma^2} \right) \\ & + I_2 \left(300\lambda^4 + \frac{432\lambda^8 v^4}{(4m_\pi^2 + m_\sigma^2)^2} + \frac{360\lambda^6 v^3}{4m_\pi^2 + m_\sigma^2} \right) + I_1 \left(120\lambda^4 + \frac{96\lambda^8 v^4}{m_\sigma^4} + \frac{80\lambda^6 v^3}{m_\sigma^2} \right), \end{aligned} \quad (2)$$

$$\begin{aligned} T_{1L}^2 = & 160\lambda^8 v^4 (I_{B2} + I_9) + I_4 \left(\frac{160\lambda^8 v^4}{m_\sigma^2} + 160\lambda^6 v^2 \right) + I_5 \left(\frac{480\lambda^8 v^4}{m_\sigma^2} + 80\lambda^6 v^2 \right) + \frac{240\lambda^6 v^3}{m_\sigma^4} I_7 + \frac{80\lambda^6 v^2}{m_\sigma^2} I_8 \\ & + I_1 \left(360\lambda^4 + \frac{480\lambda^8 v^4}{m_\sigma^4} + \frac{400\lambda^6 v^3}{m_\sigma^2} \right). \end{aligned} \quad (2)$$

In the previous expresión all I'_i denote certain integrals you might find in the original article

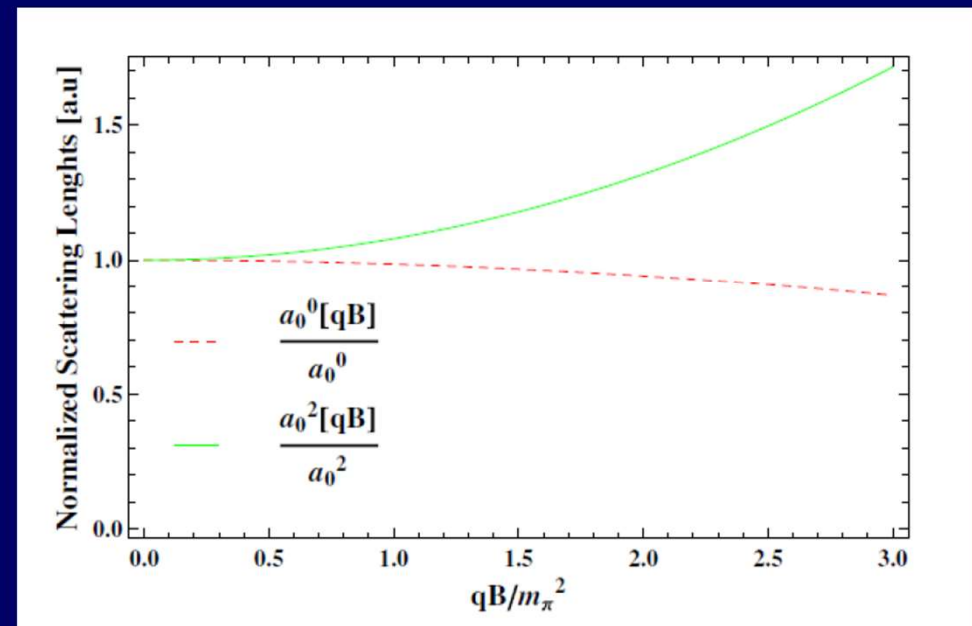
Behavior of the scattering lengths: we have used $m_\pi = 140 \text{ MeV}$, $m_\sigma = 550 \text{ MeV}$
 $V = 89 \text{ MeV}$, λ



It is interesting to see that this result is opposite respect to the same calculation in the presence of a constant magnetic field. Also, the effects of the electric field are much bigger than the magnetic effects.

Temperature and electric field effects go in opposite direction respect to magnetic effects. Thermo- magnetic effects on Pi-pi scattering lengths were calculated in
M. Loewe, E. Muñoz, and R. Zamora,
Phys. Rev. D 100 (2019) 116006

Using $m_\sigma = 550$ MeV and $m_\pi = 140$ MeV



In general there is an agreement in the community that it will be extremely difficult to disentangle magnetic from thermal effects.

Collisions between asymmetric nuclei, due to the important role of the electric field, might be an interesting window for exploring this kind of physics.