Light scalar quarkonia from Laplace SR at NLO

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iHEPMAD Antananarivo

Motivation : Scalar meson

- ♣ Identification difficult experimentally [1]
- ♣ Nature not well established theoretically [2-12]
  - ⊛ eg. from naïve quark model :
    - Mass degeneracy of $f_0(980)$ and $a_0(980)$? ?
    - σ and κ broader than $f_0(980)$ and $a_0(980)$? ?

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    - σ and κ broader than $f_0(980)$ and $a_0(980)$? ?
- ♣ Theoretical interpretation of light scalar states (among others) :
  - ♢ Ordinary $\frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$, $\bar{u}d$ [3,4,9]
  - ♢ Ordinary $\bar{u}q$ from divergence of vector current [13-17]
  - ♢ Molecule and four-quark states [18-28] :

∘ $\pi^+\pi^-,\ K^+\pi^-,\ K^+K^-,\ \eta\pi^0$

∘ $\mathcal{O}^{S/P}$: $\epsilon_{abc}\epsilon_{dec}\left[\left(\bar{u}_a\,\gamma_5 C\,\bar{q}_b^T\right)\left(q_d'^T\,C\gamma_5 q_e\right) + r\left(\bar{u}_a\,C\,\bar{q}_b^T\right)\left(q_d'^T\,C\,q_e\right)\right]$

∘ $\mathcal{O}^{V/A}$: $\frac{1}{\sqrt{2}}\left[\left(\bar{u}_a\,\gamma_\mu\gamma_5 C\,\bar{q}_b^T\right)\left(q_a'^T\,C\gamma_\mu\gamma_5 q_b - q_b'^T\,C\gamma_\mu\gamma_5 q_a\right)\right.$

$$\left. + r\left(\bar{u}_a\,\gamma_\mu C\,\bar{q}_b^T\right)\left(q_a'^T\,C\gamma_\mu\,q_b + q_b'^T\,C\gamma_\mu\,q_a\right)\right]$$

$[q' \equiv u, d, \quad q \equiv d, s]$

QCD inverse Laplace sum rule (LSR) aproach

Evaluation of two point function $\rightsquigarrow$ Hadron parameters

$$\psi_S(q^2) = i \int d^4x \; e^{-iqx} \langle 0 | \mathcal{T} \mathcal{O}_S(x) \left(\mathcal{O}_S(0)\right)^\dagger | 0 \rangle$$

▷ Local hadronic operators introduce the characteristics of the hadron H

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- ♣ QCD side : quark and gluon fields, OPE
- ♣ Phenomenological side : Hadron parameters

- ♣ Quark Hadron duality principle :
  QCD side $\simeq$ PHEN side

- ♣ Inverse Laplace Transform

▷ Finite Energy inverse Laplace transform sum rule

$$\mathcal{L}_n^c(\tau,\mu) = \int_{t_>}^{t_c} dt\ t^n\ e^{-t\tau}\frac{1}{\pi}\text{Im}\ \psi_S(t,\mu)$$

Ansatz :

$$\frac{1}{\pi}\mathrm{Im}\,\psi_S(t) = 2f_S^2 M_S^{2(d-2)}\,\delta(t-M_S^2) \oplus \theta(t-t_c)\text{"QCD Continuum"}$$

⊙ At the **optimization point** :

$$M_S^2 \simeq \mathcal{R}_{10}^c(\tau_0) \equiv \frac{\mathcal{L}_1^c}{\mathcal{L}_0^c};$$

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⊛ **OPE convergence** obtained for condensates up to $d \leq 6$

⊛ To prevent the **violation of factorization** the inclusion of higher dimension condensates is not suggested

⊛ At LO : On-shell/MS running mass ? ?

⇒ Inclusion of **PT correction** solve this ambiguity and justify the use of MS running quark mass [NPA 1039 (2023) 122743, IJMPA 31 (2016) 1650093]

Stability criteria for extracting the optimal results

♣ (τ, t_c) free external parameters $\Rightarrow$ minimum sensitivity of (M_S, f_S) vs (τ, t_c)

τ stability

- Optimal result extracted at the **minimum or inflexion** point
(Harmonic oscillator of QM & J/ψ LSR) [PRD 105 (2022) 114035 and ref. therein]
- OPE convergence satisfied at τ extremum

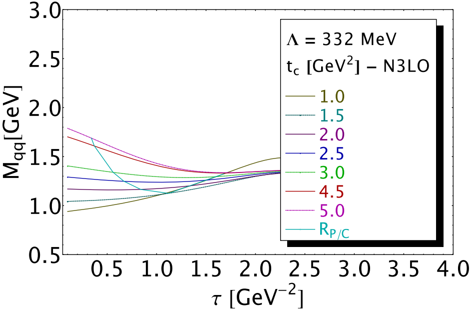
t_c stability

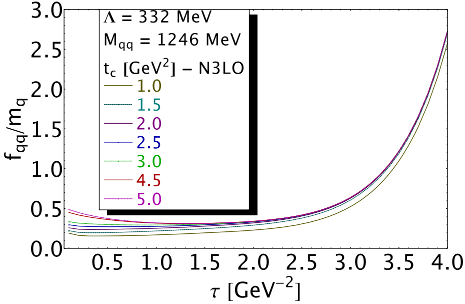
- From the beginning of τ stability $\leftrightsquigarrow$ the beginning of t_c stability
- Large range of t_c value (choice of t_c inside this range confirmed by FESR : [PRD 105 (2022) 114035])

♣ Pole contribution $>$ Continuum contribution

$$R_{P/C} \equiv \frac{\text{Lowest Pole}}{\text{QCD Continuum}} \geqslant 1$$

Ordinary $(\bar{u}u + \bar{d}d)$





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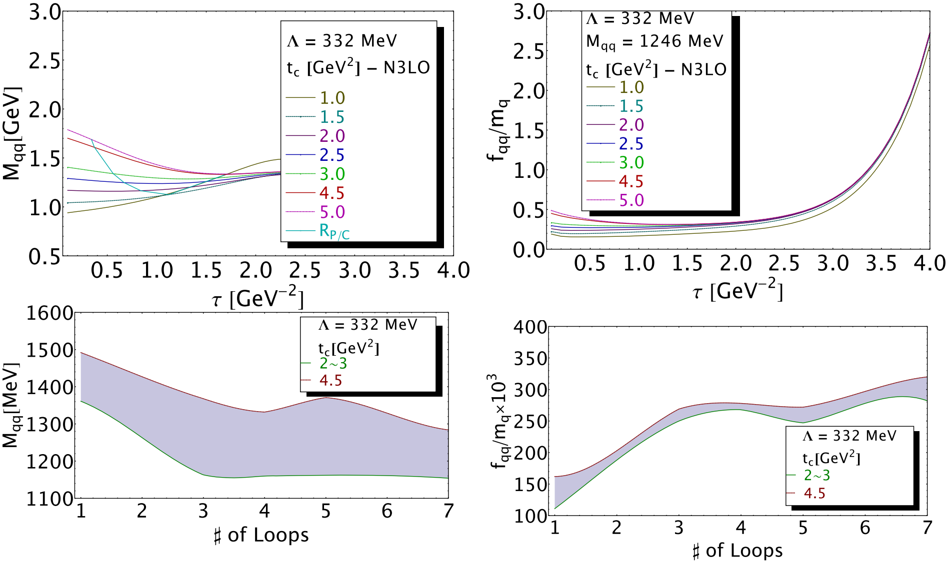


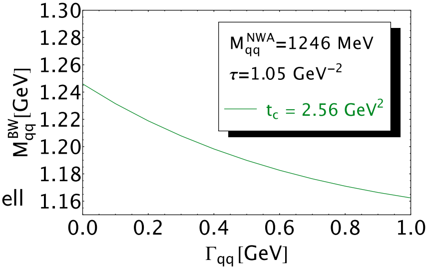
$$M_{\bar{q}q} = 1246(94)\ \text{MeV},\ f_{\bar{q}q}/\overline{m}_q(\tau) = 274(38) \times 10^{-3}$$

Ordinary $(\bar{u}u + \bar{d}d)$

▷ Finite width correction

$$\pi\delta(t - M_\sigma^2) \to BW(t) = \frac{M_\sigma \Gamma_\sigma}{(t-M_\sigma^2)^2 + M_\sigma^2\Gamma_\sigma^2}$$

$$\Delta M_{qq}^{BW} = -22|_{\text{vertex SR}},\ -60|_{\text{pole}},\ -70|_{\text{on-shell}}$$



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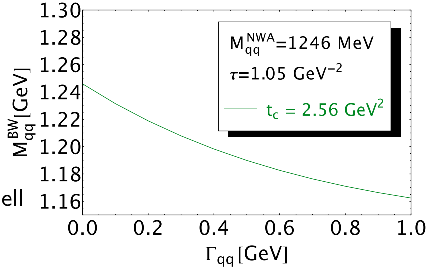
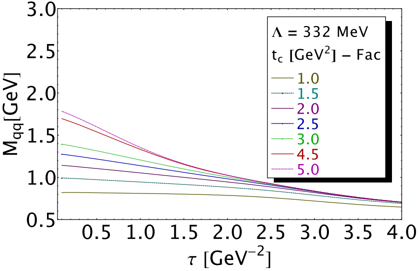
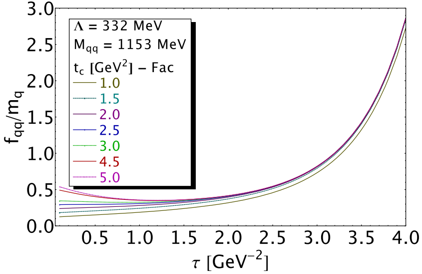


▷ Factorization of four-quark condensate : $\langle 0|\bar{q}q\bar{q}'q'|0\rangle \simeq \rho\langle 0|\bar{q}q|0\rangle\,\langle 0|\bar{q}'q'|0\rangle$





$$M_{\bar{q}q} = 1131(90)\text{ MeV}, \quad f_{\bar{q}q}/\overline{m}_q(\tau) = 338(24)\times 10^{-3}$$

Ordinary $\bar{u}s$ & $\bar{s}s$: $J_{\bar{u}s} = \partial_\mu V^\mu_{\bar{u}s} = (m_u - m_s)\bar{u}s$ and $J_{\bar{s}s} = m_s\bar{s}s$

$M_{\bar{u}s} = 1276(61)$ MeV, $\quad f_{\bar{q}q}/(\overline{m}_u - \overline{m}_s)(\tau) = 264(25)\times 10^{-3}$

$M_{\bar{s}s} = 1288(65)$ MeV, $\quad f_{\bar{q}q}/\overline{m}_s(\tau) = 256(19)\times 10^{-3}$

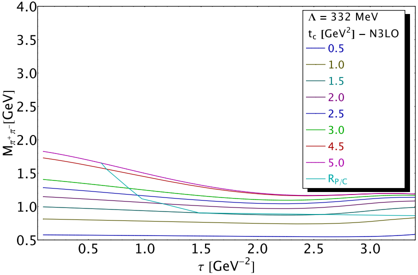
$$\frac{M_{\bar{s}s}}{M_{\bar{u}s}} = 1.01 \text{ (Direct determination : } 1.03 \pm 0.02 \text{ [3])}$$

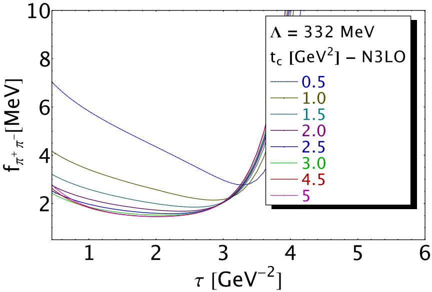
▷ Comments

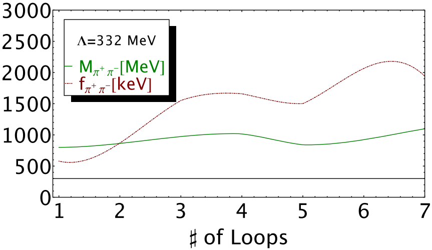
Currents	t_c	τ	$\Delta\Lambda$	PT	m_q	$\bar{q}q$	κ	G^2	$\bar{q}Gq$	G^3	$\bar{q}q^2$	OPE	Value
Ordinary $\bar{q}q$													
Masses [MeV]													
$\frac{1}{\sqrt{2}}\ \bar{u}u \pm \bar{d}d$	88	4.5	3.0	1.0	0	0	–	1.0	1.0	0	19	24.4	1246(94)
$\bar{u}s$	43	4.3	6.0	1.0	3.4	3	13.4	1.1	4.2	0	23	25.3	1276(58)
$\bar{s}s$	51	2.8	3.5	0.9	7.6	4.2	21.8	1.3	4.9	0	16.5	17.7	1288(62)
(Couplings $/m_q$) $\times 10^3$													
$\frac{1}{\sqrt{2}}\ \bar{u}u \pm \bar{d}d$	38	1.1	3.0	1.5	0	0	–	2.8	0.0	0.0	1.6	0.7	274(38)
$\bar{u}s$	23	1.2	2.4	2.1	0.5	0.2	0.8	3	0.15	0	1.2	0.5	264(24)
$\bar{s}s$	17	0.95	2.4	1.9	1.5	0.4	2.5	3.4	0.2	0	1.3	0.5	256(18)

- ⊙ These LSR results should be compared to the Breit-Wigner or On-Shell mass [29,30] (not to be confused with complex pole [31-35])
- ⊙ The lightest $\bar{q}q$ mesons are in the range : $M_{\bar{q}q'} = 1040 \sim 1353$ MeV $(q, q' \equiv u, d, s)$.
  - Consistent with the **On-shell** masses of the observed mesons : $\sigma/f_0(500)$, $f_0/a_0(980)$ and $K_0^*(1430)$.
  - The estimated hadronic width [3] does not favour a pure $\bar{q}q$ interpretation of broader $\sigma/f_0(500)$.
- ⊙ $M_{\bar{u}s} = 1276$ MeV comparable with $K_0^*(1350)$.
  The predicted total width from vertex sum rules [3] narrower than the data.
- ⊙ $\bar{s}s$ mass too low compared to $f_0(1710)$.
  The predicted K^+K^- width $\sim 1/2$ the data.

$\sigma/f_0(500)$ as dipion molecule







$$M_{\pi^+\pi^-} = 1017(159)\ \text{MeV}$$
$$f_{\pi^+\pi^-} = 1657(277)\ \text{keV}$$

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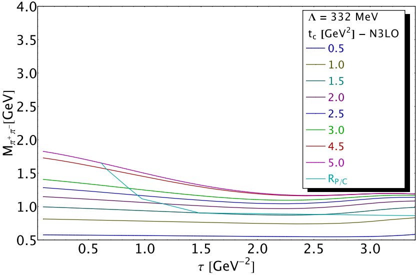

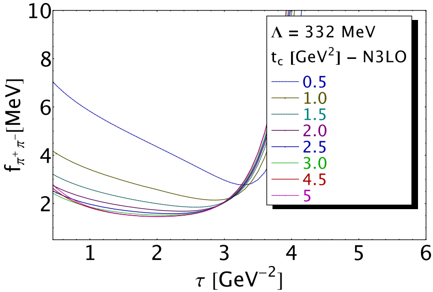

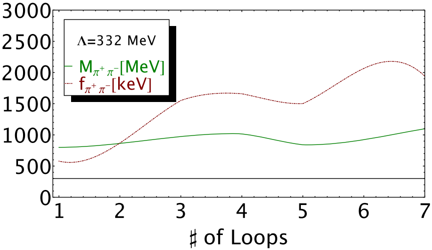

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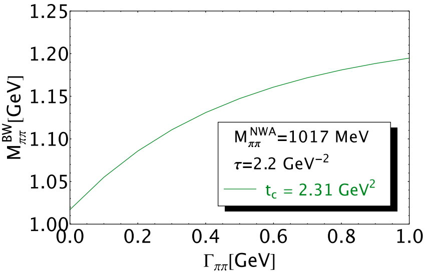


Results

Currents	t_c	τ	$\Delta\Lambda$	PT	m_q	$\bar{q}q$	κ	G^2	$\bar{q}Gq$	G^3	$\bar{q}q^2$	OPE	Value
Molecules													
Masses [MeV]													
$\pi^+\pi^-$	144	5	25	31	–	11	–	0.1	0.4	0	11.5	52	1017(159)
K^+K^-	165	3.9	18.4	115	1.1	11.8	7.4	0.6	3.5	0	13	67	1056(214)
$K^+\pi^-$	54	4.5	19.4	105	0.2	10.4	2.5	0.1	1.8	0	13.1	59	1035(134)
$\eta\pi$	57	4.5	15.8	109	0.2	10.8	2.8	0.2	1.8	0	10	60	1040(139)
Couplings [keV]													
$\pi^+\pi^-$	193	24.5	2.0	81	–	87	–	2.5	2.0	0.	70	140	1657(277)
K^+K^-	161	15	0.8	32	4	70	39	2.8	24	0	78	158	1380(255)
$K^+\pi^-$	170	16.5	0.2	56	3	73	16	1.9	13	0	78	176	1504(275)
$\eta\pi$	165	19	0.1	53	2	71	9.6	2.6	13.7	0	77	142	1462(249)

▷ Comments

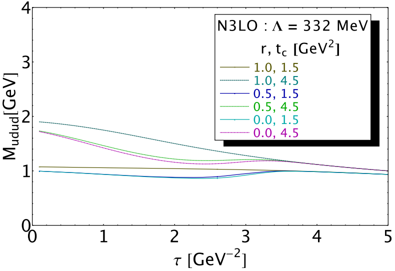
◎ Masses of molecule about $(230 - 250)$ MeV lower than $\bar{q}q$.

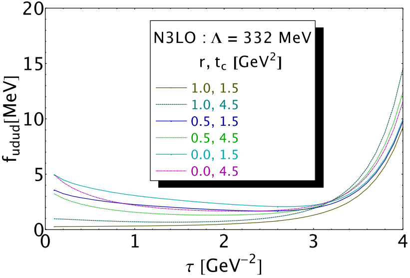
- Including width corrections, they tend to meet around 1.1 GeV.

◎ Masses of $\bar{q}q$ and molecule coincide with the mass of lowest scalar digluonium : $(M_{\sigma_B}^{glue} = 1041(111)$ MeV [4]).

$\Rightarrow$ Cannot yet establish the nature of σ : $\bar{q}q$, $\pi\pi$, gluonium ? ?.

S⊕PS four-quark





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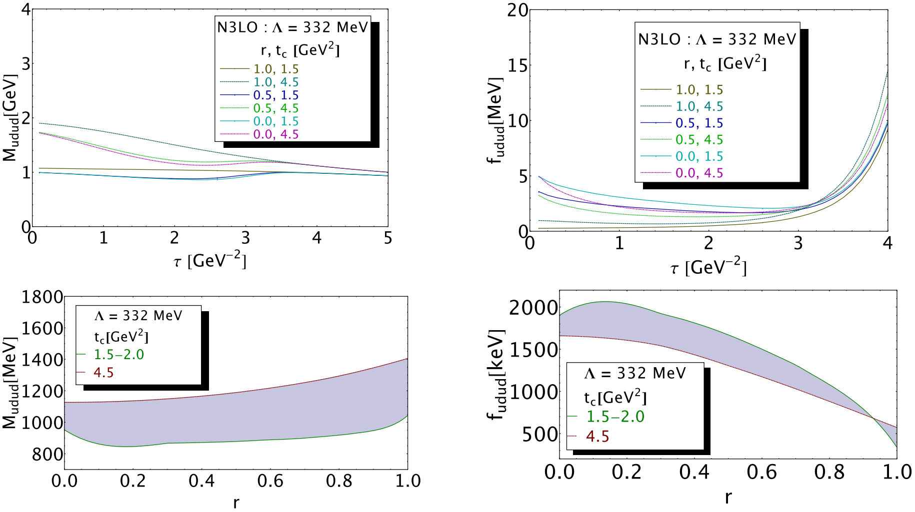


$$
\begin{aligned}
r = 0: &\quad M_{\bar u\bar d ud} = 1040(119)\ \text{MeV}, &\quad f_{\bar u\bar d ud} = 1780(540)\ \text{keV}\\
= 1/\sqrt{2}: &\quad = 1078(186)\ \text{MeV}, &\quad = 1160(357)\ \text{keV}\\
= 1: &\quad = 1225(184)\ \text{MeV}, &\quad = 458(288)\ \text{keV}
\end{aligned}
$$

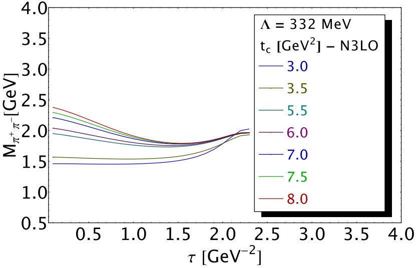
S$\oplus$PS four-quark

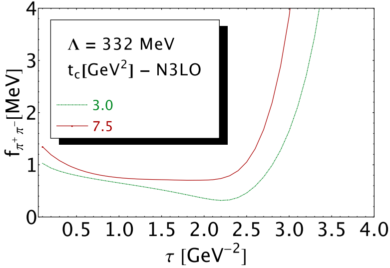
Currents	t_c	τ	$\Delta\Lambda$	PT	m_q	$\bar{q}q$	κ	G^2	$\bar{q}Gq$	G^3	$\bar{q}q^2$	OPE	Value
$\mathcal{O}^{S/P}_{\bar{u}\bar{d}ud}$													
Masses [MeV]													
1	181	13	3	28	0	0	–	1	0.2	0	0	–	1225(184)
$1/\sqrt{2}$	177	6.1	22	31	0	10.2	–	0.2	0.4	0	10.3	32	1078(184)
0	87	5.3	25	31	0	10.5	–	0	0	0	11.5	60	1040(114)
Couplings [keV]													
1	124	14	12	5	0	0	–	2.6	0	0	0	–	449(126)
$1/\sqrt{2}$	140	12	6.5	26	0	52	–	2.5	1.5	0	53	54	1160(170)
0	122	29	7.5	119	0	94	–	1.5	1.5	0	95	152	1780(266)
$\mathcal{O}^{S/P}_{\bar{u}\bar{s}ud}$													
Masses [MeV]													
1	155	26	2	3.5	0.7	1.2	1	0.8	1.7	0	0	1.9	1215(157)
$1/\sqrt{2}$	134	7	25	31	0.1	11	2.6	0.2	0.4	0	26	33	997(147)
0	56	8	21	62	0.5	7	2.7	0.2	0.3	0	23	84	957(123)
Couplings [keV]													
1	129	13	8.5	5	4	1.7	1.7	4	2.3	0	0.4	8.5	422(131)
$1/\sqrt{2}$	203	28	5.5	118	1	90	20.5	1.5	5.5	0	189	214	1835(382)
0	169	60	7.5	355	1.5	140	33.5	1	9.5	0	293	377	2902(638)
$\mathcal{O}^{S/P}_{\bar{u}\bar{s}ds}$													
Masses [MeV]													
1	164	25	1	2.5	0.5	2.0	2.0	0.15	2.2	0	0	2	1214(166)
$1/\sqrt{2}$	153	5	25	30	0.2	11.5	6.5	0.4	2.1	0	24.3	56	1012(170)
0	60	8	27	37	0	9.5	5.6	0.15	1.1	0	22.4	79	971(112)
Couplings [keV]													
1	150	16	12	11	3.5	3.5	5	4	4.5	0	0	16	413(153)
$1/\sqrt{2}$	192	25	4	98	2.5	81	45.5	1.5	10	0	178	171	1683(342)
0	129	56	11	297	2.5	113	79	0.7	38	0	278	241	2637(514)

V⊕A four-quark

Currents	t_c	τ	$\Delta\Lambda$	PT	m_q	$\bar{q}q$	κ	G^2	$\bar{q}Gq$	G^3	$\bar{q}q^2$	OPE	Value
$\mathcal{O}^{V/A}_{\bar{u}\bar{d}ud}$													
Masses [MeV]													
1	100	3.5	16	97	0	4.5	–	1.8	0	0	0	9.3	831(141)
$1/\sqrt{2}$	141	14	15.8	130	0	0.8	–	0.9	2.5	0	0	3.6	941(193)
Couplings [keV]													
1	45	16.5	4	269	0	19.5	–	16.5	2.9	0	2.1	45.5	983(278)
$1/\sqrt{2}$	40	1.5	0	20	0	1.6	–	48	5.6	0	0	7	601(66)
$\mathcal{O}^{V/A}_{\bar{u}\bar{s}ud}$													
Masses [MeV]													
1	99	18	16	97	0.1	4.5	1	1.8	0	0	0	9.3	834(141)
$1/\sqrt{2}$	129	14.5	15.8	130	0.1	0.8	10	0.9	2.5	0	0	3.6	978(185)
Couplings [keV]													
1	108	16.5	4	269	0.1	19.5	1.7	16.5	2.9	0	2.1	45.5	1042(295)
$1/\sqrt{2}$	11	2.2	0	20	0.1	1.6	19.5	48	5.6	0	0	7	408(57)
$\mathcal{O}^{V/A}_{\bar{u}\bar{s}ds}$													
Masses [MeV]													
1	97	23	16	97	0.1	4.5	3	1.8	0	0	0	9.3	840(140)
$1/\sqrt{2}$	57	21	15.8	130	0.1	0.8	10	0.9	2.5	0	0	3.6	1282(145)
Couplings [keV]													
1	63	21.5	4	269	0.3	19.5	4	16.5	2.9	0	2.1	45.5	829(282)
$1/\sqrt{2}$	40	13	0	20	0.15	1.6	16	48	5.6	0	0	7	354(69)

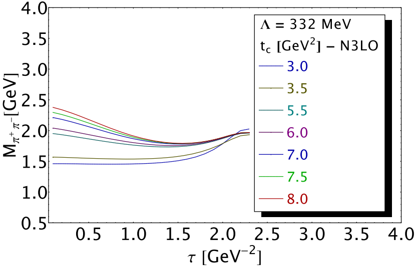
First radial excitations

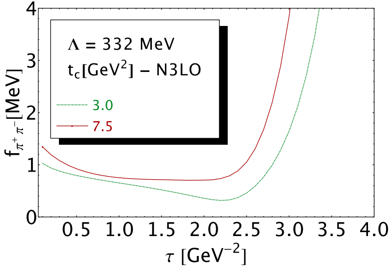




$$M^{(1)}_{\pi^+\pi^-} = 1621(514)\,\text{MeV}, \quad f^{(1)}_{\pi^+\pi^-} = 665(338)\,\text{keV}$$

First radial excitations





$$M^{(1)}_{\pi^+\pi^-} = 1621(514)\,\text{MeV}, \quad f^{(1)}_{\pi^+\pi^-} = 665(338)\,\text{keV}$$

▷ Ordinary $\bar{u}d$

$$M^{(1)}_{\bar{u}d} = 1378(186)\,\text{MeV}, \quad f^{(1)}_{\bar{u}d} = 212(38)\,\text{keV}$$

▷ S⊕PS : $\bar{u}\bar{d}ud$

$r = 0:\quad M^{S/P(1)}_{\bar{u}\bar{d}ud} = 1588(511)\,\text{MeV}, \quad f^{S/P(1)}_{\bar{u}\bar{d}ud} = 777(189)\,\text{keV}$

$= 1/\sqrt{2}:\quad = 1920(317)\,\text{MeV}, \quad = 498(97)\,\text{keV}$

$= 1:\quad = 1670(168)\,\text{MeV}, \quad = 381(128)\,\text{keV}$

▷ V⊕A : $\bar{u}\bar{d}ud$

$r = 1:\quad M^{V/A(1)}_{\bar{u}\bar{d}ud} = 931(193)\,\text{MeV}, \quad f^{V/A(1)}_{\bar{u}\bar{d}ud} = 1137(334)\,\text{keV}$

$= 1/\sqrt{2}:\quad = 1489(380)\,\text{MeV}, \quad = 287(99)\,\text{keV}$

Summary

◎ $SU(3)$ breakings effect :

- Few tens of MeV.
- S/P four-quark state : decrease the masses.

◎ $\sigma/f_0(500)$

- The $R_{P/C}$ condition favours the estimation $M_\sigma \sim 1$ GeV
- The constraint on $\sigma\bar{K}K$ coupling does not favour $\bar{u}\bar{d}ud$ and $\pi^+\pi^-$ nature of σ.

◎ $f_0(980)$

- σ and $f_0(980)$ seems to emerge from meson-gluonium mixing :

$(M_{\bar{q}q}, \Gamma_{\pi\pi}) = (1229, 120)$ MeV and $(M_{glue}, \Gamma_{glue}) = (1070, 890)$ MeV [4].

- K^+K^- : $1056(214)$ MeV and mean prediction of four-quark :$1045(112)$ MeV

compatible with $f_0(980)$.

◎ $a_0(980)$

- $\eta\pi$: $1040(139)$ MeV and mean prediction of four-quark are compatible.

◎ $f_0(1370)$

- 1^{st} radial excitation of $\bar{q}q$:$1378(186)$ MeV. Can mix with :

- scalar gluonium $M_{\sigma'} = 1110(117)$ MeV [4] to give the large $\pi\pi$ width (can be the dragon [6,36])

- 1^{st} radial excitation of four-quark state :$1409(112)$ MeV.

◎ $a_0(1450)$: 1^{st} radial excitation of the $\bar{u}\bar{s}ud$.

Summary

⊚ $f_0(1500)$: Expected to be gluonium state from : $M_{G'} = 1563(141)$ MeV and $U(1)-$like decays ($\eta'\eta,\ \eta\eta$).

- From our results : four-quark 1^{st} radial excitation$\sim 1409(112)$ MeV (which may mix with the gluonium state).

⊚ $f_0(1710)$

Likely the 1^{st} radial excitation of the four-quark/molecule (next radial excitation of gluonium higher : $M_{G_2} = 2992(221)$ MeV)

⊚ $K_0^*(700)$: On-shell mass :$845(17)$ MeV

- $\bar{u}s$ is relatively to high :$1276(58)$ MeV.
- $K\pi$:$1035(134)$ MeV and four-quark are compatible with On-shell mass.

⊚ $K_0^*(1430)$

Better fit the $\bar{u}s$ and/or its radial excitation expected to be ~ 1400 MeV.

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Thank you !