

# Investigation on the $\Omega(2012)$ from QCD sum rules

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- Background
- P-wave  $\Omega$  baryon currents
- QCD sum rule analyses
- Numerical analyses
- Summary and outlook

# New baryon candidates

- Significant progress has been achieved in **baryon spectroscopy**, with an increasing number of **new baryon candidates**

## ➤ Heavy baryons

$\Xi_c(2923)$   $\Lambda_b(5912)$

$\Xi_c(2939)$   $\Lambda_b(5920)$

$\Xi_c(2965)$   $\Xi_b(6087)$

$\Xi_b(6100)$

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## ➤ Light baryons

$\Xi(1620)$

$\Xi(1690)$

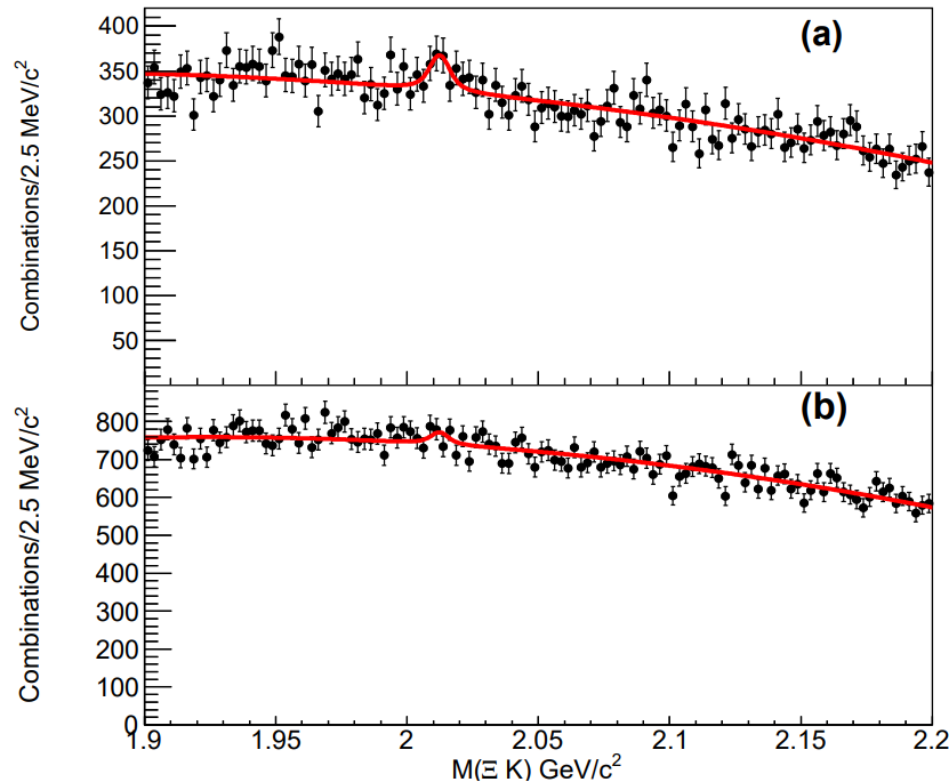
$\Xi(1820)$

$\Omega(2012)$

.....

# Discovery of the $\Omega(2012)$

- In 2018, the **excited**  $\Omega$  baryon,  $\Omega(2012)$ , was discovered **for the first time** by the Belle experiment in  $\Xi^0 K^-$  and  $\Xi^- K_S^0$  invariant mass distribution in  $\Upsilon(1S)$ ,  $\Upsilon(2S)$ ,  $\Upsilon(3S)$  decays



Belle Collaboration, Phys. Rev. Lett. 121 (5) (2018) 052003

$$M = 2012.5 \pm 0.7 \pm 0.5 \text{ MeV},$$

$$\Gamma = 6.4_{-2.0}^{+2.5} \text{ MeV}.$$

|                  |         |      |
|------------------|---------|------|
| $\Omega^-$       | $3/2^+$ | **** |
| $\Omega(2012)^-$ | $?^-$   | ***  |
| $\Omega(2250)^-$ |         | ***  |
| $\Omega(2380)^-$ |         | **   |
| $\Omega(2470)^-$ |         | **   |

# Interpretation of the $\Omega(2012)$

- In 2021, the experimental evidence has been further strengthened by  $\Omega_c \rightarrow \pi^+ \Omega(2012) \rightarrow \pi^+ (\bar{K}\Xi)^-$  decay

Belle Collaboration, Phys. Rev. D 104 (5) (2021) 052005

- The mass of  $\Omega(2012)$  is about 340 MeV higher than the ground state  $\Omega(1672)$  and is just 16.6 MeV below the  $K^-$  and  $\Xi^*(1530)$  mass threshold

the first P-wave excitation of the ground  $\Omega$  baryon

a hadronic molecular

# P-wave excitation of $\Omega$ baryon state

- The conventional **quark model** may naively explain the  $\Omega(2012)$  to be the first P-wave excitation of the ground  $\Omega$  baryon

|        |   |   |
|--------|---|---|
| before | { | K. T. Chao, N. Isgur and G. Karl, Phys. Rev. D 23, 155 (1981)                           |
|        |   | S. Capstick, N. Isgur, Phys. Rev. D 34 (1986) 9, 2809-2835                              |
|        |   | L. Y. Xiao and X. H. Zhong, Phys. Rev. D 98, 034004 (2018)                              |
| after  | { | Z.Y. Wang, L.C. Gui, Q.F. Lu, L.Y. Xiao, X.-H. Zhong, Phys. Rev. D98 (11) (2018) 114023 |
|        |   | A. J. Arifi, D. Suenaga, A. Hosaka, Y. Oh, Phys. Rev. D 105 (9) (2022) 094006           |
|        |   | .....   |

- One important feature of this quark model picture is that there should be **spin-orbit partners** of both  $J^P = 1/2^-$  and  $J^P = 3/2^-$

# P-wave excitation of $\Omega$ baryon state

- The spin  $3/2^-$  state may decay to  $\bar{K}\Xi$  via D-wave, or more likely decay to  $\bar{K}\Xi(1530)$  via S-wave, but such a decay will be suppressed due to a small phase space factor  
(small decay width)
- The spin  $1/2^-$  state will be easier to decay to  $\bar{K}\Xi$  via S-wave with no phase space suppression, or decay to  $\bar{K}\Xi(1530)$  via D-wave  
(large decay width)
- The spin-parity of  $\Omega(2012)$  is likely to be  $J^P = 3/2^-$

# Molecule state

- A molecular picture of  $\bar{K}\Xi(1530)$  has been proposed and extensively discussed in **the one-boson-exchange model** and **the chiral unitary approach**

M. P. Valderrama, Phys. Rev. D 98, 054009 (2018)

Y. H. Lin and B. S. Zou, Phys. Rev. D 98,056013 (2018)

Y. Huang, M. Z. Liu, J. X. Lu, J. J. Xie and L. S. Geng, Phys. Rev. D 98,076012 (2018)

R. Pavao and E. Oset, Eur. Phys. J. C 78, 857 (2018)

T. Gutsche, V. E. Lyubovitskij, J. Phys. G 48 (2) (2020) 025001

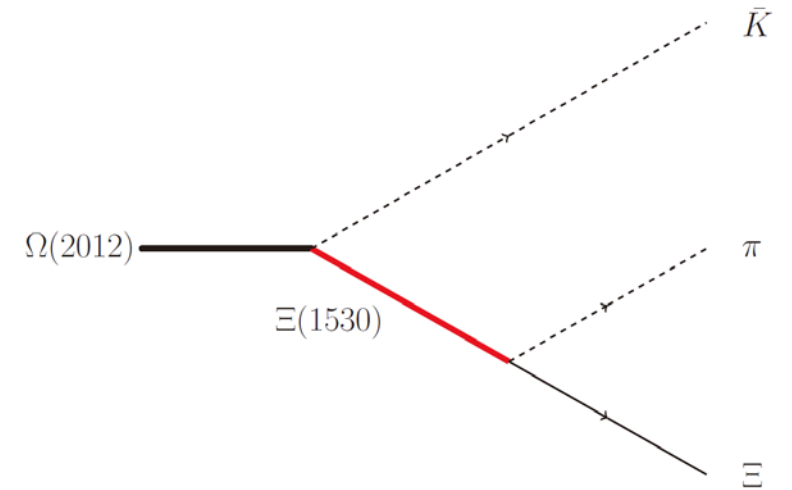
X. Liu, H. Huang, J. Ping, D. Chen, Phys. Rev. C 103 (2) (2021) 025202.

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- Unlike the quark model picture, **the spin  $1/2^-$  state is not easily generated** in the molecular picture

# Molecule state

- The molecular state may lead to a large contribution to the **three-body decay** of  $\Omega(2012) \rightarrow \bar{K}\Xi(1530) \rightarrow \bar{K}\Xi\pi$



- In **experiments**, it was first reported that such a three-body decay was not observed in 2019, but in 2022 the measurement was revisited and the possibility of the three-body decay was discussed  
*Belle collaboration, arXiv: 2207.03090*

$$\mathcal{R}_{\Xi\bar{K}}^{\Xi\pi\bar{K}} \equiv \frac{\mathcal{B}[\Omega(2012) \rightarrow \Xi(1530)\bar{K} \rightarrow \Xi\pi\bar{K}]}{\mathcal{B}[\Omega(2012) \rightarrow \Xi\bar{K}]} < 11.9\%$$

*S. Jia, et al, Phys. Rev. D 100 (3) (2019) 032006*

# Mixing of two picture

- A **hybrid picture** of three-quark and molecular structures was proposed in a coupled-channel approach. It was found that both the three-quark core and  $\bar{K}\Xi(1530)$  channel are essential for the description of  $\Omega(2012)$  resonance

Q.-F. Lu, H. Nagahiro, A. Hosaka, *Phys.Rev. D* 107 (1) (2023) 014025

- It would be fair to say that the structure of  $\Omega(2012)$  is not yet well understood, and this has motivated us to study further properties of this state in another theoretical approach based on QCD sum rules

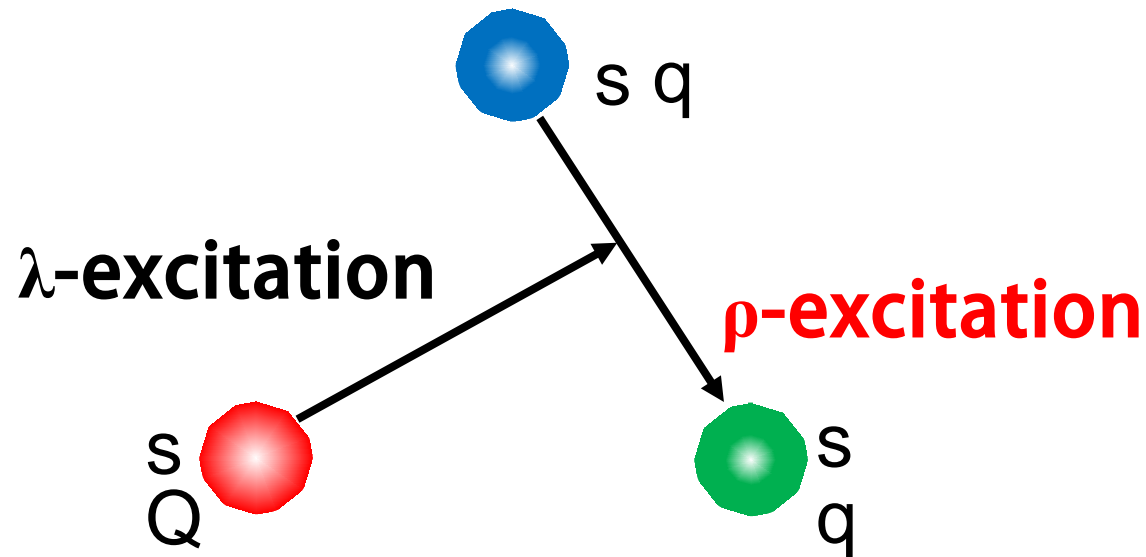


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# P-wave $ss$ diquark

- We construct the P-wave  $\Omega$  baryon currents **with a covariant derivative**, assuming that the parity of  $\Omega(2012)$  is negative
- It is not easy to differentiate the  $\rho$  mode and  $\lambda$  mode orbital excitations. We find it **much easier** to construct the currents of the  $\rho$  mode, which contains the **P-wave  $ss$  diquark field**



# P-wave ss diquark

- For the P-wave ss diquark within a derivative, **three types** of diquarks remain non-vanishing:

$$\epsilon^{abc} s_a^T C \gamma_5 \overleftrightarrow{D}_\mu s_b \quad \epsilon^{abc} s_a^T C \overleftrightarrow{D}_\mu s_b \quad \epsilon^{abc} s_a^T C \gamma_\mu \gamma_5 \overleftrightarrow{D}_\mu s_b$$

- This diquark has a **suitable internal P -wave structure**. In fact,  $s_a^T C \gamma_5 s_b$  has quantum number  $J^P = 0^+$  which is S-wave, with total spin  $S_{12} = 0$ . After applying a derivative,  $s_a^T C \gamma_5 \overleftrightarrow{D}_\mu s_b$  becomes a **pure P -wave diquark**

# P-wave ss diquark

- For the P-wave ss diquark within a derivative, **three types** of diquarks remain non-vanishing:

$$\epsilon^{abc} s_a^T C \gamma_5 \overleftrightarrow{D}_\mu s_b \quad \epsilon^{abc} s_a^T C \overleftrightarrow{D}_\mu s_b \quad \epsilon^{abc} s_a^T C \gamma_\mu \gamma_5 \overleftrightarrow{D}_\mu s_b$$

- Phenomenologically, the P-wave ss diquark with the color representation  $\bar{3}_c$  can only have the total spin  $S_{12} = 0$ . Thus it has antisymmetric color, symmetric flavor, antisymmetric orbital structure, and antisymmetric spin, so its total structure is antisymmetric, satisfying **the Pauli principle**

# P-wave ss diquark

- For the P-wave ss diquark within a derivative, **three types** of diquarks remain non-vanishing:

$$\epsilon^{abc} s_a^T C \gamma_5 \overleftrightarrow{D}_\mu s_b$$

$$\epsilon^{abc} s_a^T C \overleftrightarrow{D}_\mu s_b$$

$$\epsilon^{abc} s_a^T C \gamma_\mu \gamma_5 \overleftrightarrow{D}_\mu s_b$$

- In contrast, the other two diquarks, before applying a derivative,  $s_a^T C s_b$  and  $s_a^T C \gamma_\mu \gamma_5 s_b$  themselves have already P -wave nature ( $J^P = 0^-$  and  $J^P = 1^-$ ). When a derivative is applied, they obtain **complicated structure unlike the P-wave field**

# P-wave $\Omega$ baryon currents

- Combining the diquark with the third quark field of spin  $1/2$ , we can write the currents for the P-wave  $\Omega$  baryon with the total angular momentum  $J_{tot} = 1/2, 3/2$

$$\begin{aligned}
 J &= s_1 \otimes s_2 \otimes s_3 \otimes l_\rho \otimes l_\lambda \\
 &\rightarrow [s_1 \otimes s_2 \otimes l_\rho] \otimes s_3 && s_1 / s_2 / s_3 = 1/2 \\
 &= s_{12} \otimes l_\rho \otimes s_3 && s_{12} = 0, j_{12} = 1 \\
 &= j_{12} \otimes s_3 && l_\rho = 1 \\
 &= 1/2 \oplus 3/2.
 \end{aligned}$$

- Their corresponding P-wave  $\Omega$  baryon currents are

$$\begin{aligned}
 J &= -2\epsilon^{abc} [(D^\mu s_a^T) C \gamma_5 s_b] \gamma_\mu s_c, \\
 J_\mu &= -2\epsilon^{abc} [(D^\nu s_a^T) C \gamma_5 s_b] (g_{\mu\nu} - \frac{1}{4} \gamma_\mu \gamma_\nu) s_c
 \end{aligned}$$



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# QCD sum rule analyses

- The current matrix element  $J_\mu = -2\epsilon^{abc} [(D^\nu s_a^T)C\gamma_5 s_b] (g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma_\nu)s_c$

$$\langle 0|J_\mu|\Omega; 3/2^-\rangle = f_- u_\mu(q)$$

$$\langle 0|J_\mu|\Omega; 3/2^+\rangle = f_+ \gamma_5 u_\mu(q)$$

- We study the correlation function with the following Lorentz structure

$$\begin{aligned}\Pi_{\mu\nu}(q^2) &= i \int d^4x e^{iqx} \langle 0|\mathbf{T}[J_\mu(x)J_\nu^\dagger(0)]|0\rangle \\ &= \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\Pi(q^2) + \dots\end{aligned}$$

# QCD sum rule analyses

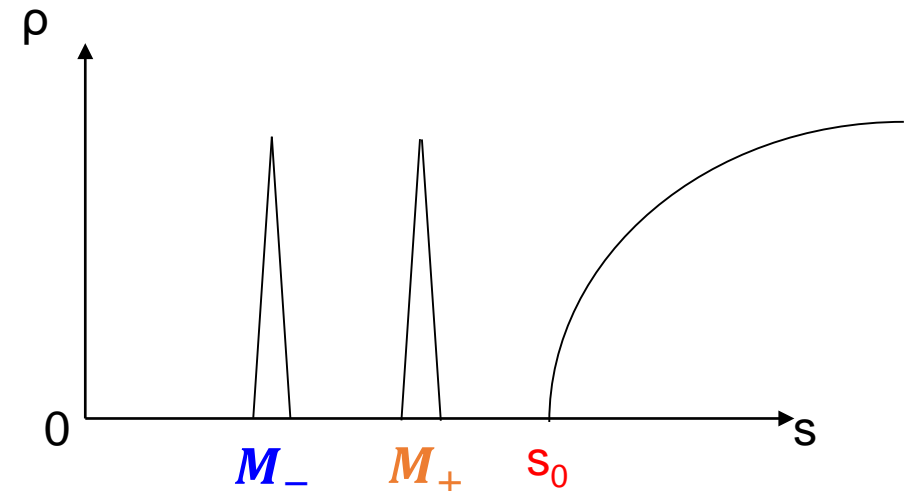
- $\Pi(q^2)$  can be expressed as a **dispersion relation**

$$\Pi(q^2) = \int_{s_<}^{\infty} \frac{\rho(s)}{s - q^2 - i\varepsilon} ds \quad \rho(s) \equiv \text{Im}\Pi(s)/\pi$$

$$s_< = 9m_s^2$$

- **At the hadron level**, we obtain the spectral density by inserting the complete set of intermediate hadronic states

$$\begin{aligned} \rho^{\text{phen}}(s) &\equiv \sum_n \delta(s - M_n^2) \langle 0 | J_\mu | n \rangle \langle n | J_\nu^\dagger | 0 \rangle \\ &= \frac{f_-^2 (\not{q} + M_-) \delta(s - M_-^2)}{\quad} \\ &\quad + \frac{f_+^2 (\not{q} - M_+) \delta(s - M_+^2)}{\quad} \\ &\quad + \theta(s - s_0) \rho^{\text{cont}}(s), \end{aligned}$$



# QCD sum rule analyses

- At **the hadron level**, the correlation function can be given as

$$\begin{aligned}\Pi^{\text{phen}}(q^2) &= f_-^2 \frac{\not{q} + M_-}{M_-^2 - q^2 - i\epsilon} + f_+^2 \frac{\not{q} - M_+}{M_+^2 - q^2 - i\epsilon} \\ &= \Pi_1^{\text{phen}}(q^2)\not{q} + \Pi_0^{\text{phen}}(q^2),\end{aligned}$$



$$\begin{aligned}\rho_1^{\text{phen}}(s) &= f_-^2 \delta(s - M_-^2) + f_+^2 \delta(s - M_+^2), \\ \rho_0^{\text{phen}}(s) &= f_-^2 M_- \delta(s - M_-^2) - f_+^2 M_+ \delta(s - M_+^2)\end{aligned}$$

- Finally we can get **the spectral densities for negative and positive parity states** as

$$\rho_{\mp}^{\text{phen}}(s) = \sqrt{s} \rho_1^{\text{phen}}(s) \pm \rho_0^{\text{phen}}(s)$$

# QCD sum rule analyses

- At **the quark-gluon level**, we insert explicit forms of currents into the correlation function, **contract out** pairs of quark field which are quark propagator of QCD
- We get the **master formula** expressed in terms of quark propagator

$$\Pi_{\mu\nu}(q) = i \int d^4x e^{iq \cdot x} \langle 0 | \mathcal{T} \{ J_\mu(x) J_\nu^\dagger(0) \} | 0 \rangle$$



$$\begin{aligned} \Pi_{\mu\nu}^{\text{OPE}}(q) = & \epsilon_{abc} \epsilon_{a'b'c'} \int d^4x e^{iqx} \langle 0 | \left\{ S_s^{ca'}(x) \gamma_\nu \tilde{S}_s^{ab'}(x) \gamma_\mu S_s^{bc'}(x) - S_s^{ca'}(x) \gamma_\nu \tilde{S}_s^{bb'}(x) \gamma_\mu S_s^{ac'}(x) \right. \\ & - S_s^{cb'}(x) \gamma_\nu \tilde{S}_s^{aa'}(x) \gamma_\mu S_s^{bc'}(x) + S_s^{cb'}(x) \gamma_\nu \tilde{S}_s^{ba'}(x) \gamma_\mu S_s^{ac'}(x) - S_s^{cc'}(x) \text{Tr} \left[ S_s^{ba'}(x) \gamma_\nu \tilde{S}_s^{ab'}(x) \gamma_\mu \right] \\ & \left. + S_s^{cc'}(x) \text{Tr} \left[ S_s^{bb'}(x) \gamma_\nu \tilde{S}_s^{aa'}(x) \gamma_\mu \right] \right\} | 0 \rangle, \end{aligned}$$

# QCD sum rule analyses

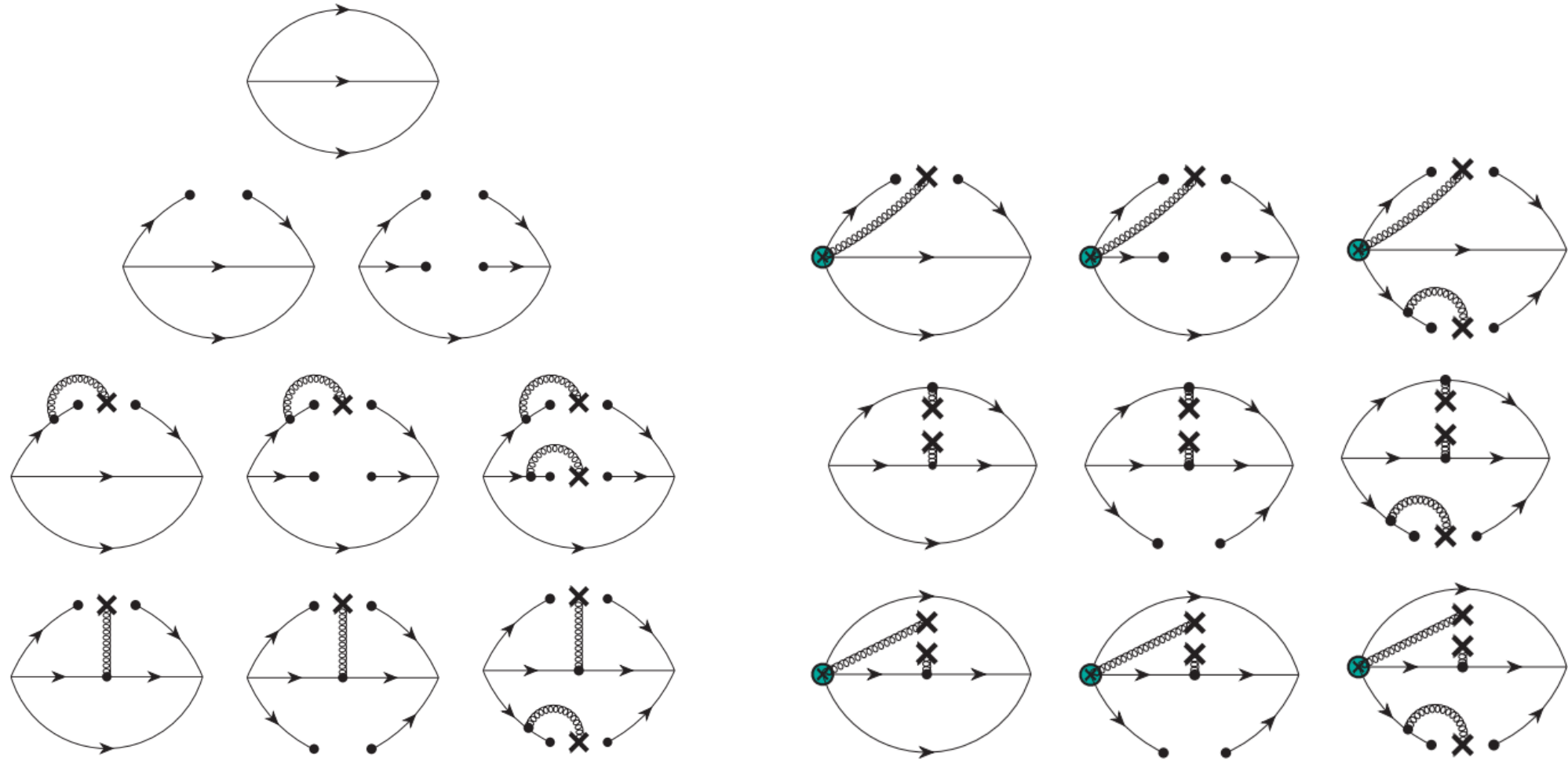
- Insert propagator to master formula, we can get the correlation function in the form of the operator product expansion(OPE)

$$\begin{aligned}
 & \rho_1^{\text{OPE}}(s) \quad \text{Dimension 10} \\
 = & \frac{5s^3}{36864\pi^4} - \frac{167m_s^2s^2}{40960\pi^4} \\
 & - \left( \frac{5\langle g_s^2 GG \rangle}{49152\pi^4} - \frac{11m_s\langle \bar{s}s \rangle}{1024\pi^2} \right) s \\
 & + \left( -\frac{263m_s\langle g_s\bar{s}\sigma Gs \rangle}{73728\pi^2} + \frac{155m_s^2\langle g_s^2 GG \rangle}{196608\pi^4} \right) \\
 & - \left( \frac{89\langle \bar{s}s \rangle\langle g_s\bar{s}\sigma Gs \rangle}{3072} - \frac{29m_s\langle g_s^2 GG \rangle\langle \bar{s}s \rangle}{147456\pi^2} \right. \\
 & \left. - \frac{m_s^2\langle \bar{s}s \rangle^2}{128} \right) \delta(s) + \left( -\frac{361\langle g_s\bar{s}\sigma Gs \rangle^2}{36864} \right. \\
 & \left. - \frac{\langle g_s^2 GG \rangle\langle \bar{s}s \rangle^2}{1152} + \frac{m_s\langle g_s^2 GG \rangle\langle g_s\bar{s}\sigma Gs \rangle}{3072\pi^2} \right. \\
 & \left. + \frac{7m_s^2\langle \bar{s}s \rangle\langle g_s\bar{s}\sigma Gs \rangle}{192} \right) \delta'(s),
 \end{aligned}$$

$$\begin{aligned}
 & \rho_0^{\text{OPE}}(s) \quad \text{Dimension 11} \\
 = & \frac{23m_s s^3}{32768\pi^4} - \frac{\langle \bar{s}s \rangle s^2}{96\pi^2} - \left( \frac{317m_s\langle g_s^2 GG \rangle}{589824\pi^4} \right. \\
 & \left. - \frac{103m_s^2\langle \bar{s}s \rangle}{1536\pi^2} + \frac{37\langle g_s\bar{s}\sigma Gs \rangle}{3072\pi^2} \right) s \\
 & + \left( \frac{441m_s^2\langle g_s\bar{s}\sigma Gs \rangle}{8192\pi^2} + \frac{25\langle \bar{s}s \rangle\langle g_s^2 GG \rangle}{24576\pi^2} \right) \\
 & - \left( \frac{13m_s^2\langle g_s^2 GG \rangle\langle \bar{s}s \rangle}{4096\pi^2} - \frac{13\langle g_s^2 GG \rangle\langle g_s\bar{s}\sigma Gs \rangle}{294912\pi^2} \right. \\
 & \left. + \frac{5m_s\langle \bar{s}s \rangle\langle g_s\bar{s}\sigma Gs \rangle}{64} \right) \delta(s) + \left( -\frac{5m_s\langle g_s\bar{s}\sigma Gs \rangle^2}{2084} \right. \\
 & \left. - \frac{3m_s\langle g_s^2 GG \rangle\langle \bar{s}s \rangle^2}{1024} + \frac{9m_s^2\langle g_s^2 GG \rangle\langle g_s\bar{s}\sigma Gs \rangle}{8192\pi^2} \right) \delta'(s)
 \end{aligned} \tag{1}$$

# QCD sum rule analyses

- Feynman diagrams in the present study



# QCD sum rule analyses

- By **equating** the spectral densities at the hadron and quark-gluon levels, and by performing the Borel transformation, we derive the sum rule equation as

$$\begin{aligned}\Pi_{\mp}(s_0, M_B) &= 2M_{\mp} f_{\mp}^2 e^{-M_{\mp}^2/M_B^2} \\ &= \int_{s_<}^{s_0} (\sqrt{s} \rho_1^{\text{OPE}}(s) \pm \rho_0^{\text{OPE}}(s)) e^{-s/M_B^2} ds.\end{aligned}$$

- $\Pi_-$  is the equation for **the negative parity state**,  $\Pi_+$  is the equation **for the positive parity state**

# QCD sum rule analyses

- The masses and coupling constants are obtained as

$$\begin{aligned} & M_{\mp}^2(s_0, M_B) \\ = & \frac{\int_{s <}^{s_0} (\sqrt{s} \rho_1^{\text{OPE}}(s) \pm \rho_0^{\text{OPE}}(s)) s e^{-s/M_B^2} ds}{\int_{s <}^{s_0} (\sqrt{s} \rho_1^{\text{OPE}}(s) \pm \rho_0^{\text{OPE}}(s)) e^{-s/M_B^2} ds} \\ & f_{\mp}^2(s_0, M_B) \\ = & \frac{\int_{s <}^{s_0} (\sqrt{s} \rho_1^{\text{OPE}}(s) \pm \rho_0^{\text{OPE}}(s)) e^{-s/M_B^2} ds \times e^{M_{\mp}^2/M_B^2}}{2M_{\mp}} \end{aligned}$$



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# Numerical analyses

- Take  $J^p = 3/2^-$  state as an concrete example

- Mass formula

$$M_{\mp}^2(s_0, M_B) = \frac{\int_{s <}^{s_0} (\sqrt{s} \rho_1^{\text{OPE}}(s) \pm \rho_0^{\text{OPE}}(s)) s e^{-s/M_B^2} ds}{\int_{s <}^{s_0} (\sqrt{s} \rho_1^{\text{OPE}}(s) \pm \rho_0^{\text{OPE}}(s)) e^{-s/M_B^2} ds}$$

- Two parameters:  $s_0, M_B$

- Criteria:

1. The convergence of OPE
2. The pole contribution
3. The mass dependence on these two parameters

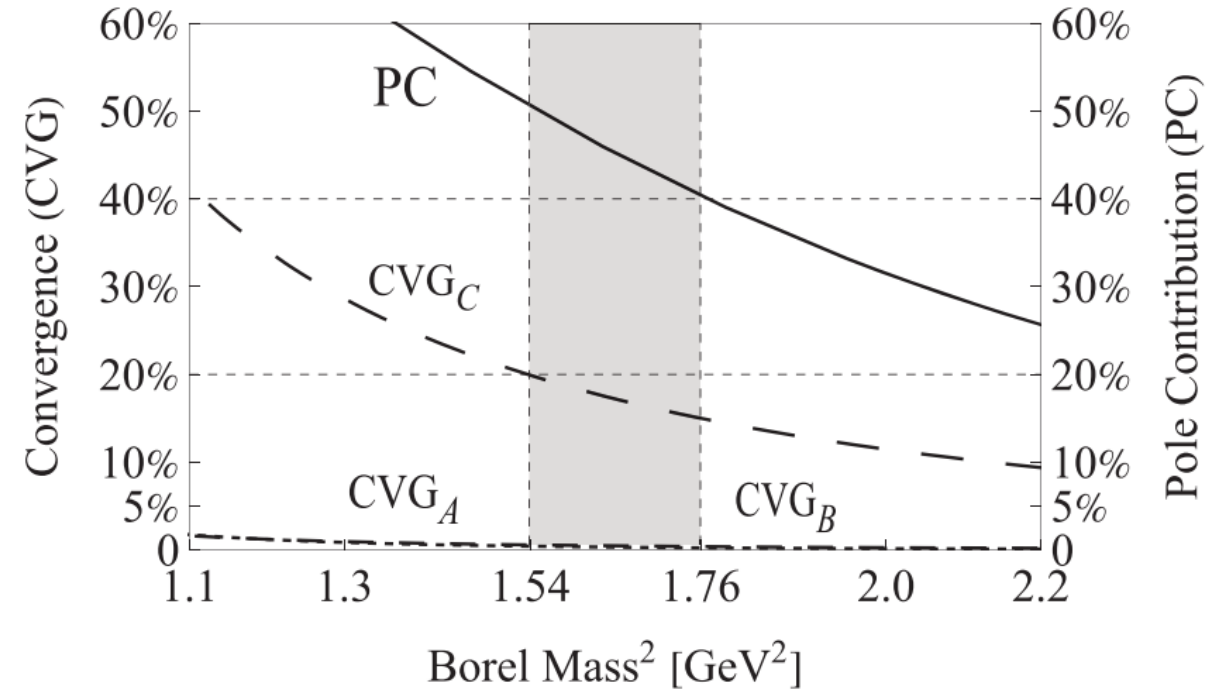
# Numerical analyses

## ● The convergence of OPE

$$\text{CVG}_A \equiv \left| \frac{\Pi_-^{\text{D}=11+10+9+8}(\infty, M_B^2)}{\Pi_-(\infty, M_B^2)} \right| \leq 5\%,$$

$$\text{CVG}_B \equiv \left| \frac{\Pi_-^{\text{D}=7+6}(\infty, M_B^2)}{\Pi_-(\infty, M_B^2)} \right| \leq 10\%,$$

$$\text{CVG}_C \equiv \left| \frac{\Pi_-^{\text{D}=5+4}(\infty, M_B^2)}{\Pi_-(\infty, M_B^2)} \right| \leq 20\%,$$



## ● The pole contribution

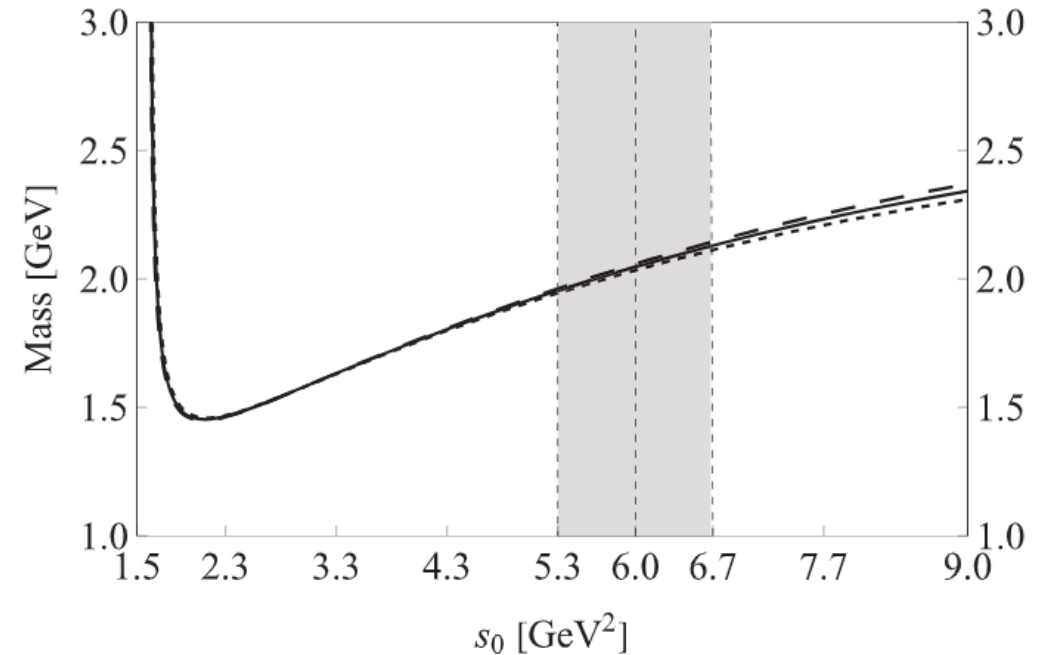
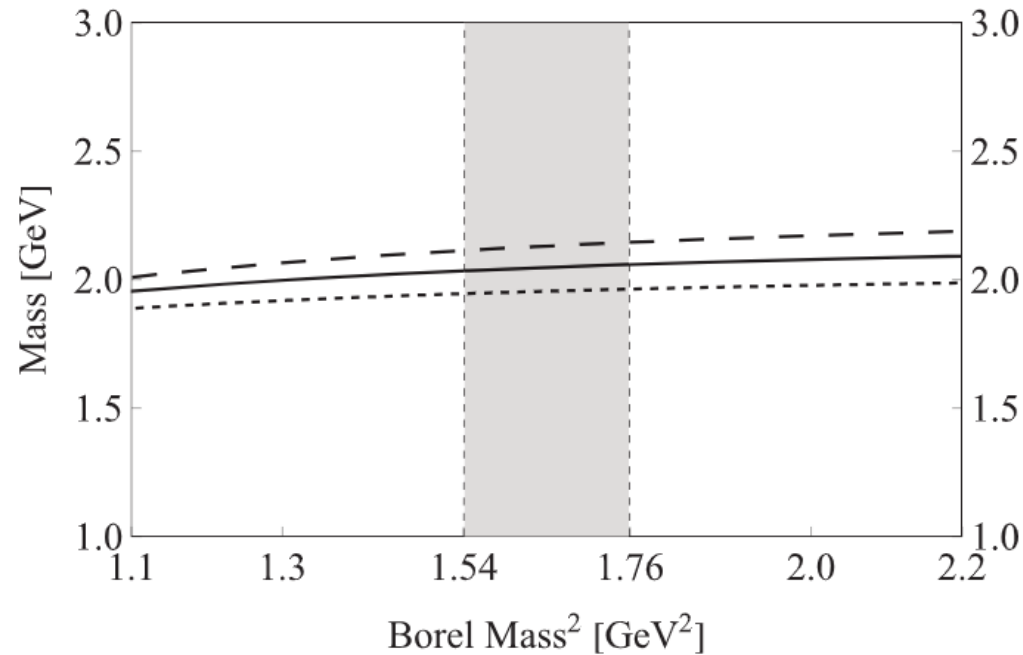
$$\text{PC} \equiv \left| \frac{\Pi_-(s_0, M_B^2)}{\Pi_-(\infty, M_B^2)} \right| \geq 40\%.$$

$$1.54 \text{ GeV}^2 \leq M_B^2 \leq 1.76 \leftrightarrow s_0 = 6.0 \text{ GeV}^2$$

$$s_0^{\min} = 5.3 \text{ GeV}^2$$

# Numerical analyses

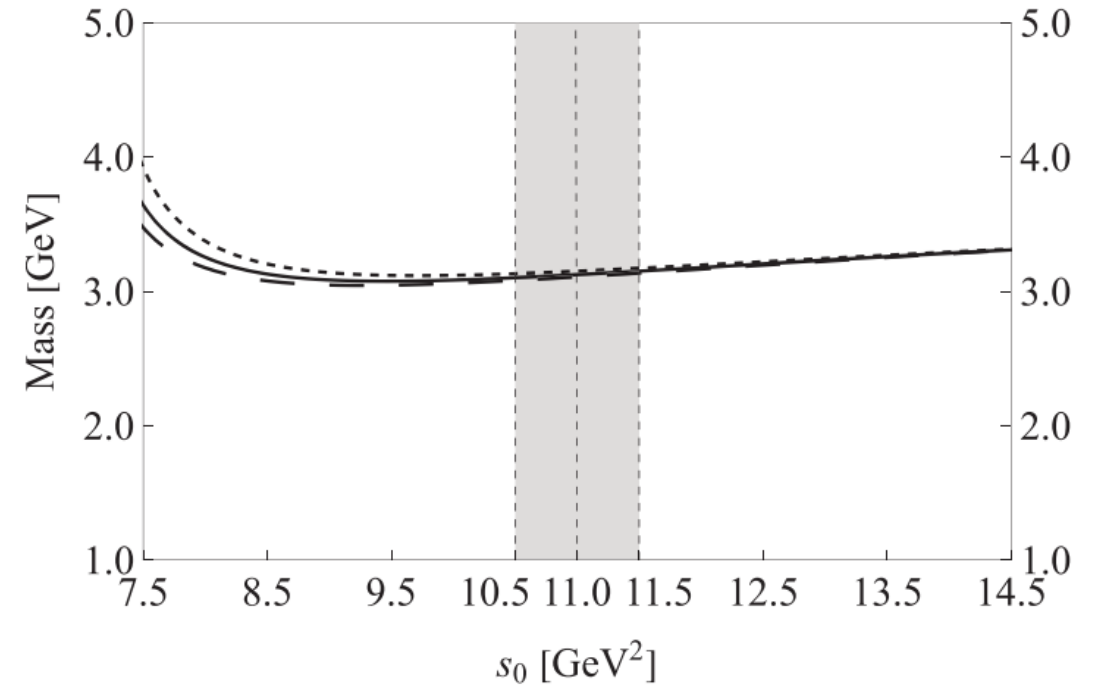
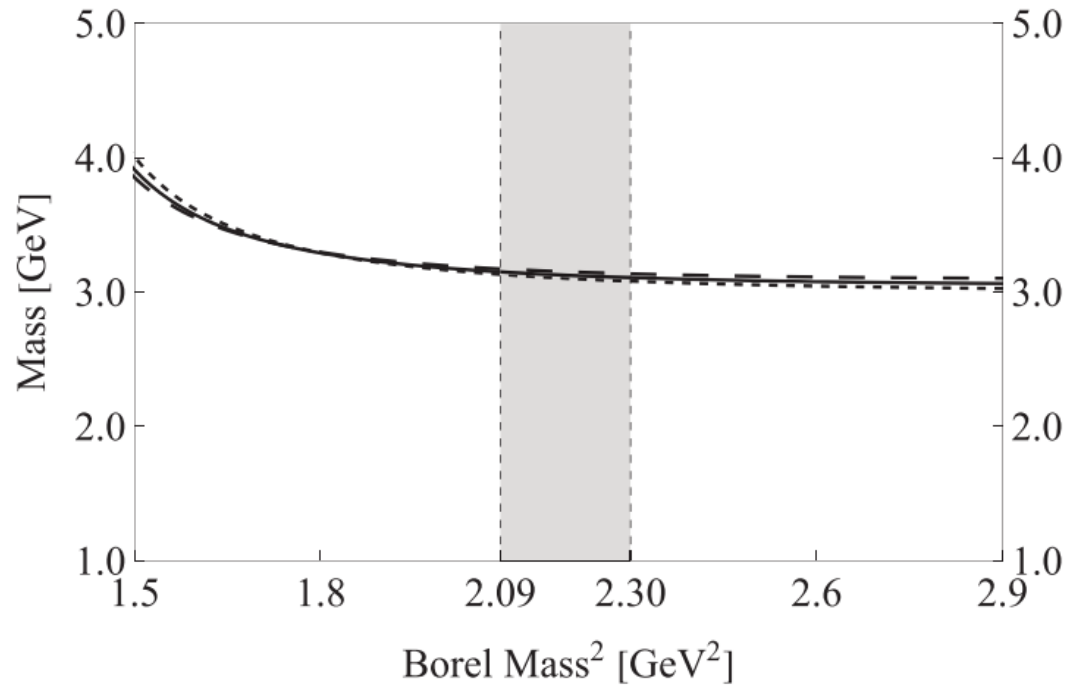
- The mass of  $3/2^-$  state as a function of these two parameters



$$M_{3/2^-} = 2.05^{+0.09}_{-0.10} \text{ GeV}, \quad s_0^{\min} = 5.3 \text{ GeV}^2$$
$$f_{3/2^-} = 0.037^{+0.007}_{-0.007} \text{ GeV}^3$$

# Numerical analyses

- The mass dependence on the two parameters for the  $J^p = 3/2^+$  state



$$M_{3/2^+} = 3.13^{+0.27}_{-0.18} \text{ GeV},$$

$$f_{3/2^+} = 0.074^{+0.015}_{-0.009} \text{ GeV}^3$$

# Numerical analyses

- The masses and the couple constants for the  $J^p = 3/2$  state using the current with a derivative

$$J_\mu = -2\epsilon^{abc} [(D^\nu s_a^T)C\gamma_5 s_b] (g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma_\nu)s_c$$

TABLE I: Masses and coupling constants extracted from the currents  $J$  in Eq. (5),  $J_\mu$  in Eq. (6), and  $J'_\mu$  in Eq.( 30).

| Current  | state                    | $s_0^{\min}$ [GeV <sup>2</sup> ] | Working Regions             |                           | Pole [%] | Mass [GeV]             | Couple constant [GeV <sup>3</sup> ] |
|----------|--------------------------|----------------------------------|-----------------------------|---------------------------|----------|------------------------|-------------------------------------|
|          |                          |                                  | $M_B^2$ [GeV <sup>2</sup> ] | $s_0$ [GeV <sup>2</sup> ] |          |                        |                                     |
| $J$      | $ \Omega; 1/2^+\rangle$  | 9.7                              | 1.91-2.40                   | 11.0                      | 40-55    | $3.05^{+0.21}_{-0.15}$ | $0.168^{+0.045}_{-0.040}$           |
|          | $ \Omega; 1/2^-\rangle$  | 5.5                              | 1.58-1.73                   | 6.0                       | 40-47    | $2.07^{+0.07}_{-0.07}$ | $0.079^{+0.011}_{-0.011}$           |
| $J_\mu$  | $ \Omega; 3/2^+\rangle$  | 10.5                             | 2.09-2.30                   | 11.0                      | 40-46    | $3.13^{+0.27}_{-0.18}$ | $0.074^{+0.015}_{-0.009}$           |
|          | $ \Omega; 3/2^-\rangle$  | 5.3                              | 1.54-1.76                   | 6.0                       | 40-51    | $2.05^{+0.09}_{-0.10}$ | $0.037^{+0.007}_{-0.007}$           |
| $J'_\mu$ | $ \Omega'; 3/2^+\rangle$ | 3.3                              | 1.48-1.77                   | 4.0                       | 40-52    | $1.59^{+0.10}_{-0.12}$ | $0.033^{+0.006}_{-0.006}$           |
|          | $ \Omega'; 3/2^-\rangle$ | 11.5                             | 3.30-3.93                   | 13.0                      | 40-51    | $3.15^{+0.16}_{-0.17}$ | $0.092^{+0.018}_{-0.018}$           |

# Numerical analyses

- The masses and the couple constants for the  $J^p = 1/2$  state using the current with a derivative

$$J = -2\epsilon^{abc} [(D^\mu s_a^T) C \gamma_5 s_b] \gamma_\mu s_c$$

TABLE I: Masses and coupling constants extracted from the currents  $J$  in Eq. (5),  $J_\mu$  in Eq. (6), and  $J'_\mu$  in Eq.( 30).

| Current  | state                    | $s_0^{\min}$ [GeV <sup>2</sup> ] | Working Regions             |                           | Pole [%] | Mass [GeV]             | Couple constant [GeV <sup>3</sup> ] |
|----------|--------------------------|----------------------------------|-----------------------------|---------------------------|----------|------------------------|-------------------------------------|
|          |                          |                                  | $M_B^2$ [GeV <sup>2</sup> ] | $s_0$ [GeV <sup>2</sup> ] |          |                        |                                     |
| $J$      | $ \Omega; 1/2^+\rangle$  | 9.7                              | 1.91-2.40                   | 11.0                      | 40-55    | $3.05^{+0.21}_{-0.15}$ | $0.168^{+0.045}_{-0.040}$           |
|          | $ \Omega; 1/2^-\rangle$  | 5.5                              | 1.58-1.73                   | 6.0                       | 40-47    | $2.07^{+0.07}_{-0.07}$ | $0.079^{+0.011}_{-0.011}$           |
| $J_\mu$  | $ \Omega; 3/2^+\rangle$  | 10.5                             | 2.09-2.30                   | 11.0                      | 40-46    | $3.13^{+0.27}_{-0.18}$ | $0.074^{+0.015}_{-0.009}$           |
|          | $ \Omega; 3/2^-\rangle$  | 5.3                              | 1.54-1.76                   | 6.0                       | 40-51    | $2.05^{+0.09}_{-0.10}$ | $0.037^{+0.007}_{-0.007}$           |
| $J'_\mu$ | $ \Omega'; 3/2^+\rangle$ | 3.3                              | 1.48-1.77                   | 4.0                       | 40-52    | $1.59^{+0.10}_{-0.12}$ | $0.033^{+0.006}_{-0.006}$           |
|          | $ \Omega'; 3/2^-\rangle$ | 11.5                             | 3.30-3.93                   | 13.0                      | 40-51    | $3.15^{+0.16}_{-0.17}$ | $0.092^{+0.018}_{-0.018}$           |

# Numerical analyses

- The masses and the couple constants for the  $J^p = 3/2$  state using the current without a derivative

$$J'_\mu = -\sqrt{3}\epsilon^{abc}s_a^T C\gamma_\mu s_b s_c$$

TABLE I: Masses and coupling constants extracted from the currents  $J$  in Eq. (5),  $J_\mu$  in Eq. (6), and  $J'_\mu$  in Eq.( 30).

| Current  | state                    | $s_0^{\min}$ [GeV <sup>2</sup> ] | Working Regions             |                           | Pole [%] | Mass [GeV]             | Couple constant [GeV <sup>3</sup> ] |
|----------|--------------------------|----------------------------------|-----------------------------|---------------------------|----------|------------------------|-------------------------------------|
|          |                          |                                  | $M_B^2$ [GeV <sup>2</sup> ] | $s_0$ [GeV <sup>2</sup> ] |          |                        |                                     |
| $J$      | $ \Omega; 1/2^+\rangle$  | 9.7                              | 1.91-2.40                   | 11.0                      | 40-55    | $3.05^{+0.21}_{-0.15}$ | $0.168^{+0.045}_{-0.040}$           |
|          | $ \Omega; 1/2^-\rangle$  | 5.5                              | 1.58-1.73                   | 6.0                       | 40-47    | $2.07^{+0.07}_{-0.07}$ | $0.079^{+0.011}_{-0.011}$           |
| $J_\mu$  | $ \Omega; 3/2^+\rangle$  | 10.5                             | 2.09-2.30                   | 11.0                      | 40-46    | $3.13^{+0.27}_{-0.18}$ | $0.074^{+0.015}_{-0.009}$           |
|          | $ \Omega; 3/2^-\rangle$  | 5.3                              | 1.54-1.76                   | 6.0                       | 40-51    | $2.05^{+0.09}_{-0.10}$ | $0.037^{+0.007}_{-0.007}$           |
| $J'_\mu$ | $ \Omega'; 3/2^+\rangle$ | 3.3                              | 1.48-1.77                   | 4.0                       | 40-52    | $1.59^{+0.10}_{-0.12}$ | $0.033^{+0.006}_{-0.006}$           |
|          | $ \Omega'; 3/2^-\rangle$ | 11.5                             | 3.30-3.93                   | 13.0                      | 40-51    | $3.15^{+0.16}_{-0.17}$ | $0.092^{+0.018}_{-0.018}$           |



# Contents

- Background
- P-wave  $\Omega$  baryon currents
- QCD sum rule analyses
- Numerical analyses
- Summary and outlook

# Summary and outlook

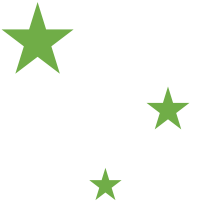
- We have constructed the P-wave  $\Omega$  baryon currents with a derivative of spin 1/2 and 3/2 by performing the spin projections
- We have analyzed the parity projected QCD sum rules to separate the positive parity state and negative parity state
- Our QCD sum rule results predict both  $1/2^-$  and  $3/2^-$  states in the mass region of  $\Omega(2012)$ . Considering the narrow width, the  $\Omega(2012)$  is likely to be a P-wave excited  $\Omega$  baryon with spin  $3/2^-$ . The  $1/2^-$  state is expected to have a wider width, which could be tested by future experiments.
- We will study its decay properties to further determine its spin

# Summary and outlook

- Until now, only the mass and width of the  $\Omega(2012)$  resonance were measured, more experimental information is needed
- Furthermore, both on theoretical and experimental sides, the studies on the **radiative decay** of  $\Omega(2012)$  are also important, and they will be helpful to further understand the nature of the  $\Omega(2012)$  state

Thanks for your attention !

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**Backup slides**

# Backup

$$\mathcal{R}_{\Xi\bar{K}}^{\Xi\pi\bar{K}} = 0.97:$$

Belle collaboration, arXiv: 2207.03090

# Backup

## Propagator of the strange quark

$$\begin{aligned} & iS_s^{ab}(x-y) \tag{28} \\ &= \langle 0 | \mathbf{T} [s^a(x) \bar{s}^b(y)] | 0 \rangle \\ &= \frac{i\delta^{ab}}{2\pi^2(x-y)^4} (\hat{x} - \hat{y}) - \frac{\delta^{ab}}{12} \langle \bar{s}s \rangle \\ &+ \frac{i}{32\pi^2(x-y)^2} \frac{\lambda_{ab}^n}{2} g_s G_{\mu\nu}^n (\sigma^{\mu\nu}(\hat{x} - \hat{y}) + (\hat{x} - \hat{y})\sigma^{\mu\nu}) \\ &- \frac{1}{4\pi^2(x-y)^4} \frac{\lambda_{ab}^n}{2} g_s G_{\mu\nu}^n x^\mu y^\nu (\hat{x} - \hat{y}) \\ &+ \frac{\delta^{ab}(x-y)^2}{192} \langle g_s \bar{s}\sigma G s \rangle - \frac{m_s \delta^{ab}}{4\pi^2(x-y)^2} \\ &+ \frac{i\delta^{ab}}{48} m_s \langle \bar{s}s \rangle (\hat{x} - \hat{y}) + \frac{i\delta^{ab}}{8\pi^2(x-y)^2} m_s^2 (\hat{x} - \hat{y}). \end{aligned}$$