The study of P-wave D_s mesons in coupled channel framework

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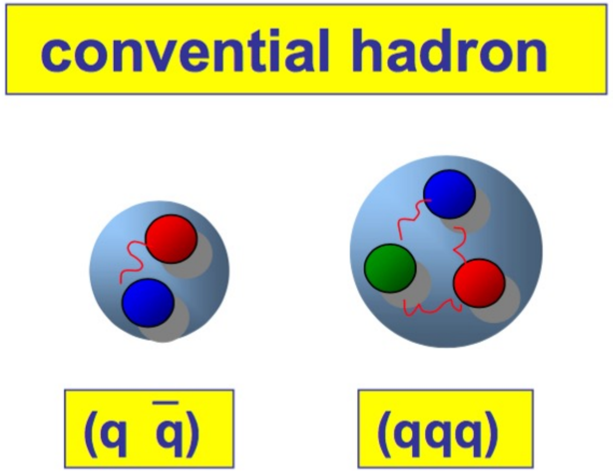
Together with Zhi Yang (UESTC), Jia-Jun Wu (UCAS), Makoto Oka (JAEA) and Shi-Lin Zhu (PKU)

Based on Phys. Rev. Lett. 128. 112001, 2007.07320
25th International Conference, Montpellier, 6th July 2022

Classical Quark model

- Mesons ($\bar{q}q$) and Baryons (qqq) in a simple picture



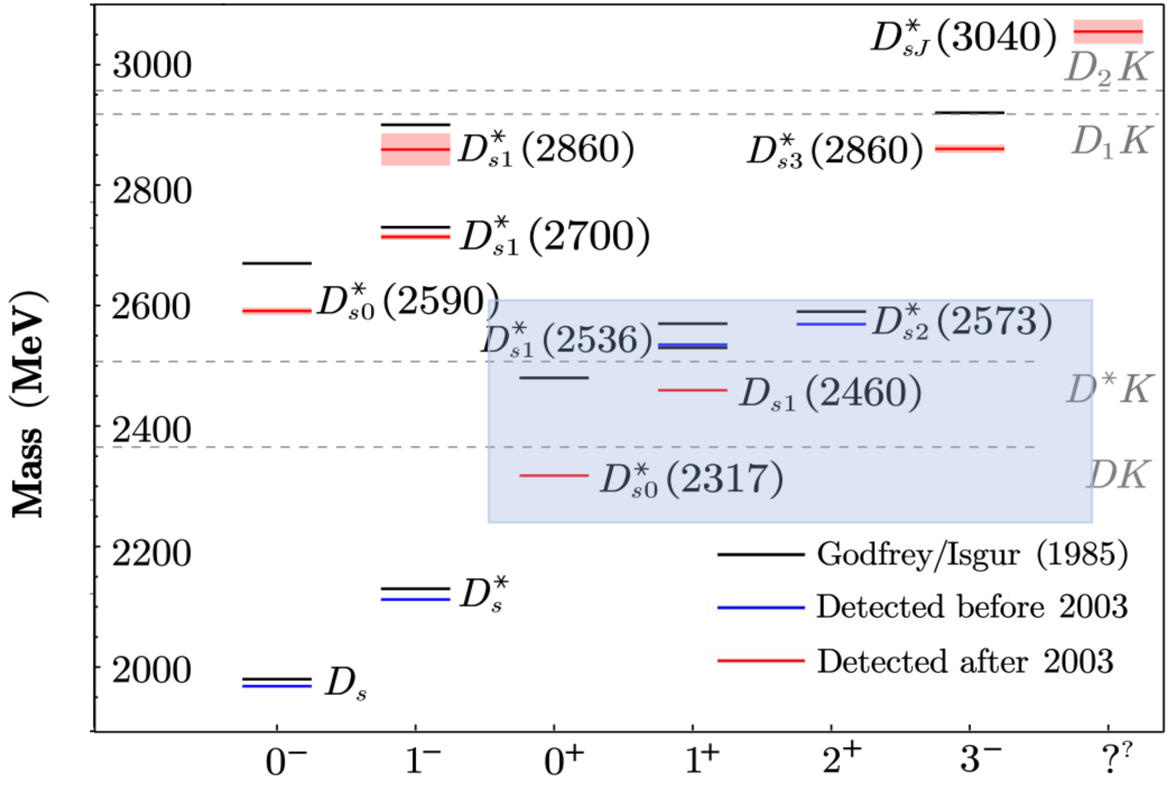
- The quark model provided predictions for the both ground and excited hadrons.
- Failed badly in some excited mesons and baryons.
- At 2003, the observation of $D_{s0}^{*}(2317)$ and X(3872) : great challenge to the quark model.

Puzzles of P-wave D_s mesons

- Four P-wave excited $c\bar{s}$ mesons in QM:

$$S_{\bar{c}s} = 0,\ J^P = 1^+$$
$$S_{\bar{c}s} = 1,\ J^P = 0^+, 1^+, 2^+$$

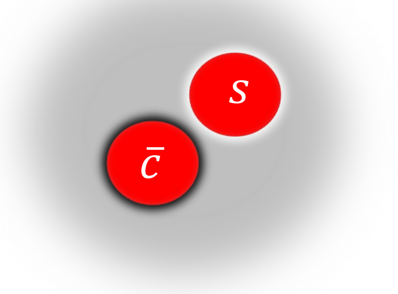
- $D_{s0}^*(2317)$ & $D_{s1}(2460)$: $m_{exp} < m_{the}$?
- $D_{s1}^*(2536)$ & $D_{s2}^*(2573)$: $m_{exp} \sim m_{the}$.
- Close to the $D^{(*)}K$ threshold.



Lu *et al*,. arXiv:2004.08716

$D_{s0}^{*}(2317)$& $D_{s1}(2460)$

- Various theoretical models: molecule, tetraquark, quenched quark model …



$\bar{c}s$ meson

S. Godfrey et al,. Phys. Rev. D 32, 189 (1985)
Y.-B. Dai et al,. Phys. Rev. D 68, 114011 (2003)
D. S. Hwang et al,. Phys. Lett. B 601, 137 (2004)
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H.-Y. Cheng et al,. Eur. Phys. J. C 77, 668 (2017)
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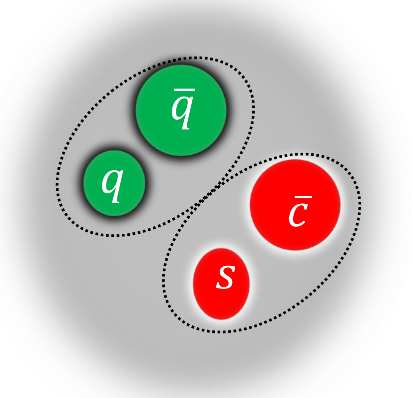

Compact $\bar{c}\bar{q}sq$

H.-Y. Cheng et al,., Phys. Lett. B 566, 193 (2003)
Y.-Q. Chen et al,. Phys. Rev. Lett. 93, 232001(2004)
V. Dmitrasinovic, Phys. Rev. Lett. 94, 162002 (2005)
H. Kim et al,. Phys. Rev. D 72, 074012 (2005)
J.-R. Zhang, Phys. Lett. B 789, 432 (2019)
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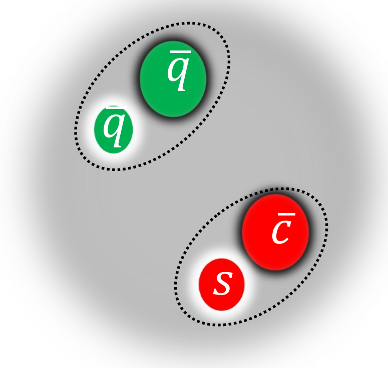
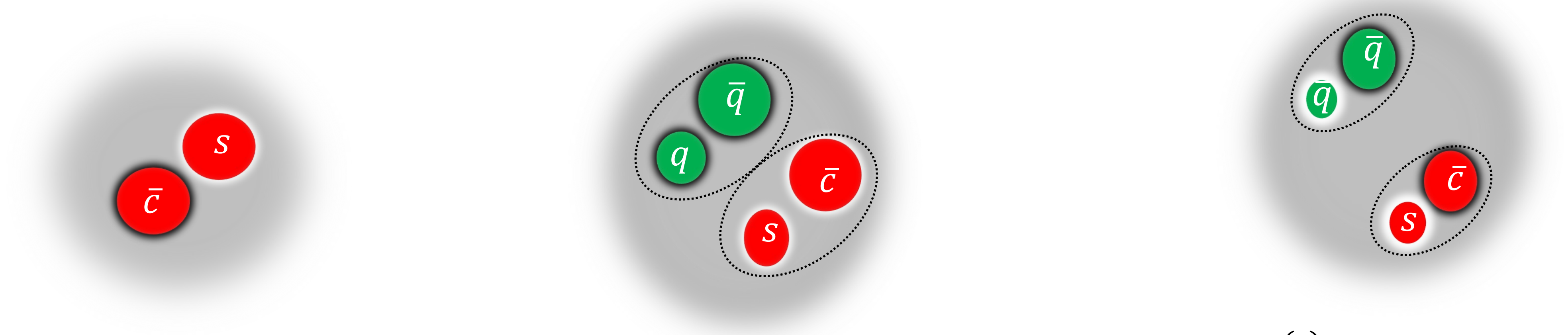


$D^{(*)}K$ molecule

E. E. Kolomeitsev et al,. Phys. Lett. B 582, 39 (2004)
A. P. Szczepaniak, Phys. Lett. B 567, 23 (2003)
J. Hofmann et al,. Nucl. Phys. A 733, 142 (2004)
E. van Beveren et al,. Phys. Rev. Lett. 91, 012003 (2003)
T. Barnes et al,. Phys. Rev. D68, 054006 (2003)
D. Gamermann et al,. Phys. Rev. D 76,074016 (2007)
F.-K. Guo et al,. Phys. Lett. B 647, 133 (2007)
J. M. Flynn et al,. , Phys. Rev. D 75, 074024 (2007)
A. Faessler et al,. Phys. Rev. D 76, 014005 (2007)
F.-K. Guo et al,. Eur. Phys. J. A 40, 171 (2009)
Z.-X. Xie et al,. Phys. Rev. D 81, 036014 (2010)
……

$D_{s0}^{*}(2317)$& $D_{s1}(2460)$

- Various theoretical models: molecule, tetraquark, quenched quark model …



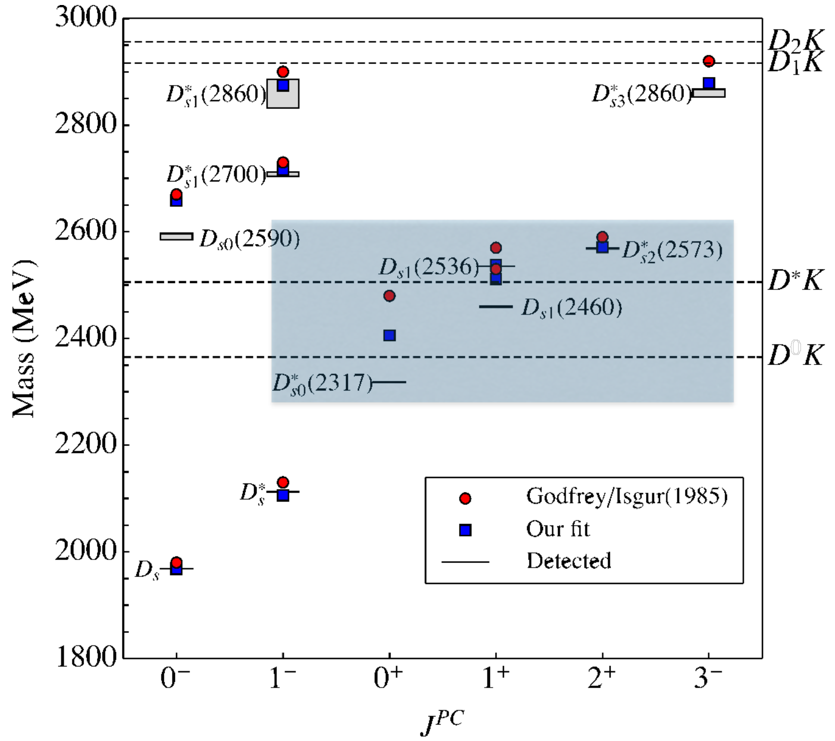
$\bar{c}s$ Meson | Compact $\bar{c}\bar{q}sq$ | $D^{(*)}K$ molecule

- Quark model successfully described other D_S mesons: $D_{s1}(2536)$ & $D_{s2}^{*}(2573)$.

Quark model ($\bar{c}s$) + Coupled-channel effects ($D^{(*)}K$) to study the four P-wave D_S states.

- The lack of $D^{(*)}K \to D^{(*)}K$ experimental data: Large uncertainties of parameters in theoretical models
- Lattice QCD result helps to determine the undetermined parameters.-- Hamiltonian Effective Field Theory (HEFT)

Quark model: bare $\bar{c}s$ state



	$\bar{c}s$ cores		channel	
	$B(\lvert^{2S+1}L_J\rangle)$	B(mass)	α	L
$D_{s0}^*(2317)$	$\lvert^3P_0\rangle$	2405.9	DK	S
$D_{s1}^*(2460)$	$0.68\lvert^1P_1\rangle - 0.74\lvert^3P_1\rangle$ $= -0.99\phi_s + 0.13\phi_d$	2511.5	D^*K	S, D
$D_{s1}^*(2536)$	$-0.74\lvert^1P_1\rangle - 0.68\lvert^3P_1\rangle$ $= -0.13\phi_s - 0.99\phi_d$	2537.8	D^*K	S, D
$D_{s2}^*(2573)$	$\lvert^3P_2\rangle$	2571.2	DK, D^*K	D

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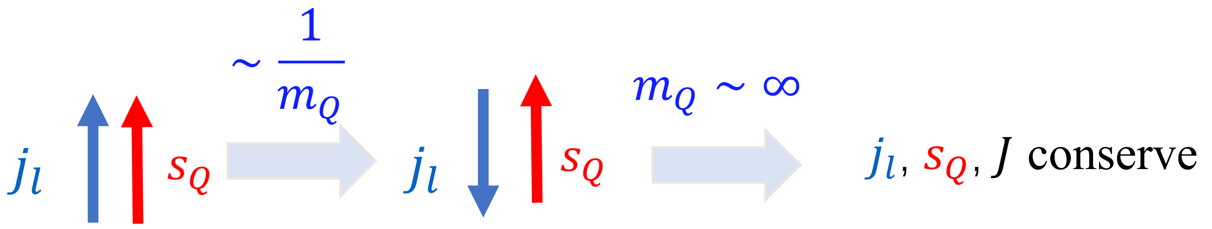
- The predicted lowest $0^+/1^+$ bare $\bar{c}s$ mesons
  -located above the $D_{s0}^*(2317)\&D_{s1}(2460)$ states.

- Good heavy quark symmetry (HQS)
  - Heavy quark spin symmetry

$$J = s_Q \otimes s_q \otimes L = S(s_Q \otimes s_q) \otimes L = s_Q \otimes j_l(s_q \otimes L)$$



M. Neubert, Phys. Rept. 245 (1994) 259-396

0^+ $\lvert \frac{1}{2}_l \otimes \frac{1}{2}_H \rangle_0$

1^+ $\phi_s = \lvert \frac{1}{2}_l \otimes \frac{1}{2}_H \rangle_1$

→ S-wave $D^{(*)}K$

1^+ $\phi_d = \lvert \frac{3}{2}_l \otimes \frac{1}{2}_H \rangle_1$

2^+ $\lvert \frac{3}{2}_l \otimes \frac{1}{2}_H \rangle_2$

→ D-wave $D^{(*)}K$

Formalism

$$H = H_0 + H_I,$$

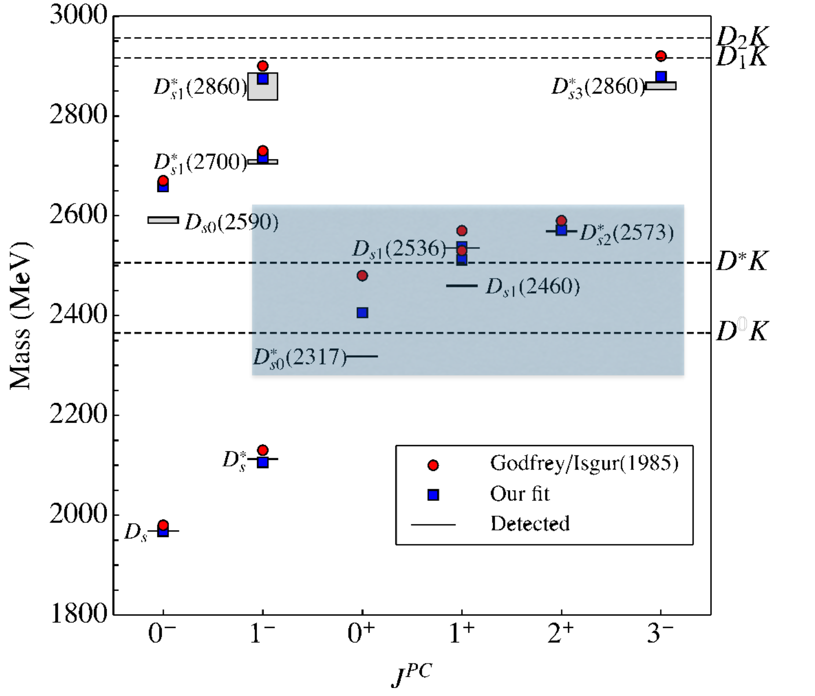
$$H_0 = \sum_{i=1,n} \underline{|B_i\rangle} m_i \langle B_i| + \sum_{\alpha} \underline{|\alpha(k_\alpha)\rangle} \left[\sqrt{m_{\alpha 1}^2 + k_\alpha^2} + \sqrt{m_{\alpha 2}^2 + k_\alpha^2} \right] \langle \alpha(k_\alpha)|$$

bare $\bar{c}s$ meson   two-meson $D^{(*)}K$ channel

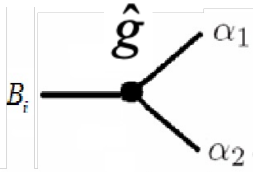
$$H_I = \hat{g} + \hat{v}$$

$$\hat{g} = \sum_{\alpha} \sum_{i=1,n} \left[|\alpha(k_\alpha)\rangle g_{i,\alpha}^{+} \langle B_i| + |B_i\rangle g_{i,\alpha} \langle \alpha(k_\alpha)| \right]$$

$$\hat{v} = \sum_{\alpha,\beta} |\alpha(k_\alpha)\rangle v_{\alpha,\beta} \langle \beta(k_\beta)|$$



bare state core -> channel：



$^3P_0$ model

at quark level

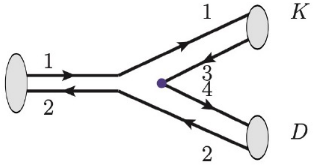
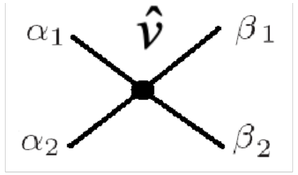
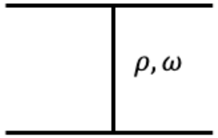


Figure 2: The diagram contribute to the process $D_s^*(2317) \to DK$.

$\hat{g}$, $\hat{v}$ determined by lattice QCD

channel -> channel：





Hamiltonian Effective Field Theory (HEFT)

- In the finite volume, the momentum is discretized as

$$k_n = 2\pi\sqrt{n}/L, \quad n = n_x^2 + n_y^2 + n_z^2, \quad n = 0, 1, 2, \ldots$$

Continuous → Discrete

Continuous				
$\int d\vec{k}$	and	$\vert\alpha(\vec{k}_\alpha)\rangle$	and	$\langle\beta(\vec{k}_\beta)\vert\alpha(\vec{k}_\alpha)\rangle = \delta_{\alpha\beta}\delta(\vec{k}_\alpha - \vec{k}_\beta)$
↓		↓		↓
$\sum_i \left(2\pi/L\right)^3$	and	$\left(2\pi/L\right)^{-3/2}\vert\vec{k}_i, -\vec{k}_i\rangle_\alpha$	and	${}_\beta\langle\vec{k}_j, -\vec{k}_j\vert\vec{k}_i, -\vec{k}_i\rangle_\alpha = \delta_{\alpha\beta}\delta_{ij}$

J.M.M. Hall, et al. Phys. Rev. D 87, 094510.
J.J. Wu, et al. Phys. Rev. C 90, 055206.
Z. W. Liu, et al. Phys. Rev. Lett.116,082004

$$H_0 = \sum_{i=1,n} |B_i\rangle m_i \langle B_i| + \sum_{\alpha,i} |\vec{k}_i, -\vec{k}_i\rangle_\alpha \left[\sqrt{m_{\alpha_B}^2 + k_\alpha^2} + \sqrt{m_{\alpha_M}^2 + k_\alpha^2}\right]_\alpha \langle\vec{k}_i, -\vec{k}_i|$$

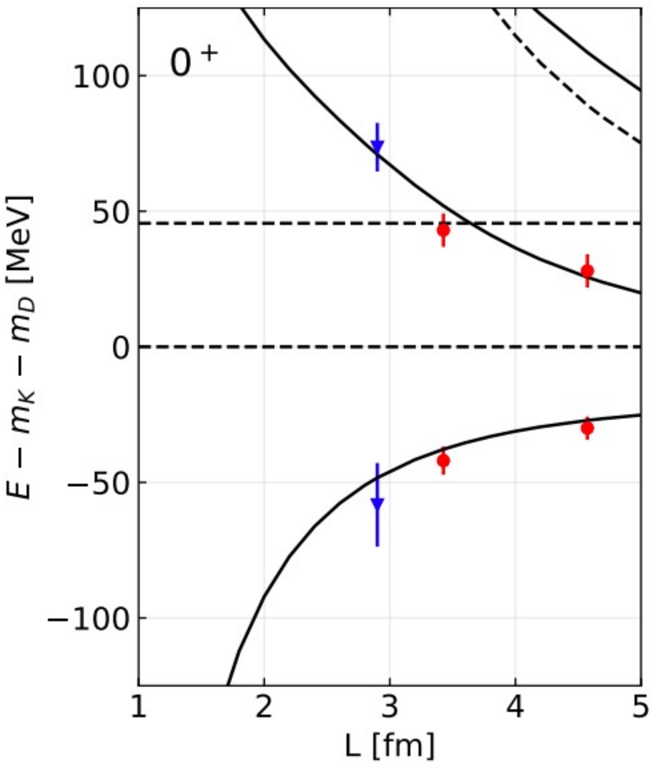
$$H_I = \sum_j \left(2\pi/L\right)^{3/2} \sum_\alpha \sum_{i=1,n} \left[|\vec{k}_j, -\vec{k}_j\rangle_\alpha\, g_{i,\alpha}^+ \langle B_i| + |B_i\rangle g_{i,\alpha}\, {}_\alpha\langle\vec{k}_j, -\vec{k}_j|\right]$$

$$+ \sum_{i,j} \left(2\pi/L\right)^3 \sum_{\alpha,\beta} |\vec{k}_i, -\vec{k}_i\rangle_\alpha\, v_{\alpha,\beta}\, {}_\beta\langle\vec{k}_j, -\vec{k}_j|$$

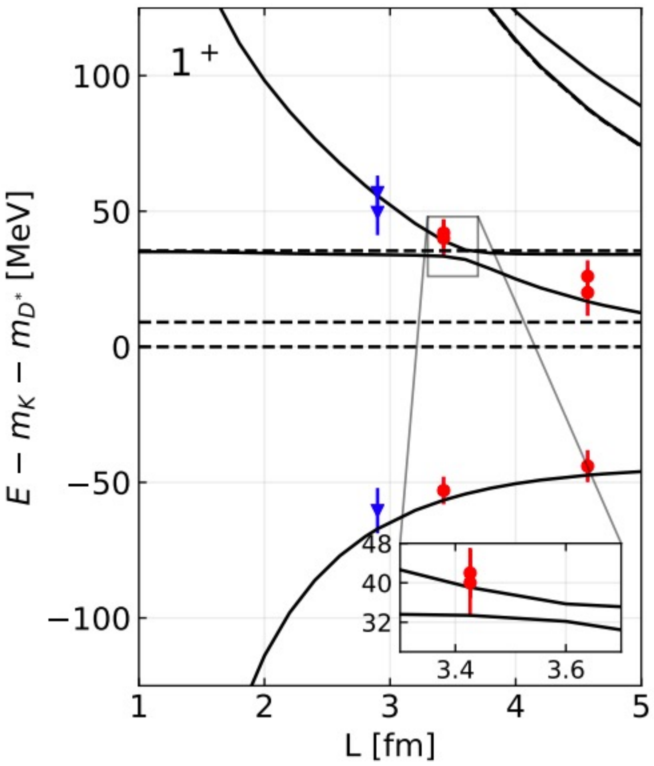
$$(H_0 + H_I)|\Psi\rangle = \underline{E}|\Psi\rangle$$

$\hat{g}$, $\hat{v}$ ⟵ Energy levels in lattice QCD

Fitting the Lattice data



$D_{s0}(2317)$



$D_{s1}(2460)$ & $D_{s1}(2536)$

- The data from Lattice QCD with $m_\pi = 150$ MeV [1] & $m_\pi = 156$ MeV [2].

- No chiral extrapolation.

2. We find a fit for the lattice result with χ^2/dof =0.95 with fixed $\Lambda = 1.0$ GeV.

[1] G. Bali, et al. (RQCD Collaboration) Phys. Rev. D 96, 074501 (2017).
[2] C. Lang, et al. Phys. Rev. D 90, 034510 (2014) .

Parameters

- Parameters

Parameters	g_c ($g_{DDV} * g_{KKV}$)	Λ' ($^3P_0$)[GeV]	γ($^3P_0$)
Best fit	$4.2^{+2.2}_{-3.1}$	$0.323^{+0.033}_{-0.031}$	$10.3^{+1.1}_{-1.0}$
Ref. [1]		0.84	6.5
Ref. [2]	-	-	6.9
Ref. [3]	18.2/9.8	-	-
Ref. [4]	8.4	-	-

[1] P. Ortega, et al. Phys. Rev. D 94 , 074037 (2016).
[2] S. Godfrey, et al. Phys .Rev. D 93 (2016) 3, 034035.
[3] C. W. Shen, et al. Phys. Rev. D 100, 056006 (2019).
[4] Z.W. Lin, et al. Phys. Rev. C 61, 024904 (2000).

- Pole mass: solving the scattering T-matrix in infinite limit,

$$T_{\alpha,\beta}(k,k';E) = \mathcal{V}_{\alpha,\beta}(k,k';E) + \sum_{\alpha'} \int q^2 dq \frac{\mathcal{V}_{\alpha,\alpha'}(k,q;E) T_{\alpha,\beta}(q,k';E)}{E - E_{\alpha'}(q) + i\epsilon}$$

Results

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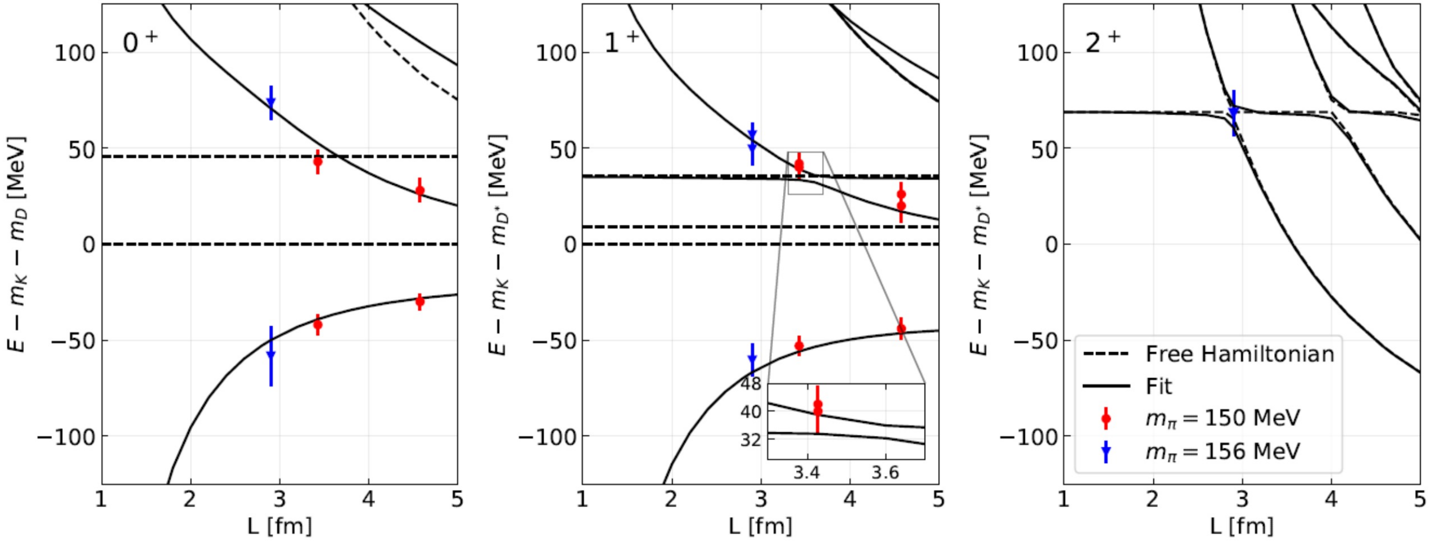
- Probability: wave function with length L=4.57 fm.

	$P(c\bar{s})[\%]$	ours	exp
$D^*_{s0}(2317)$	$32.0^{+5.2}_{-3.9}$	$2338.9^{+2.1}_{-2.7}$	2317.8 ± 0.5
$D^*_{s1}(2460)$	$52.4^{+5.1}_{-3.8}$	$2459.4^{+2.9}_{-3.0}$	2459.5 ± 0.6
$D^*_{s1}(2536)$	$98.2^{+0.1}_{-0.2}$	$2536.6^{+0.3}_{-0.5}$	2535.11 ± 0.06
$D^*_{s2}(2573)$	$95.9^{+1.0}_{-1.5}$	$2570.2^{+0.4}_{-0.8}$	2569.1 ± 0.8



- Pole position by solving T-matrix at infinite volume.

Prediction I: $D_{s2}(2573)$

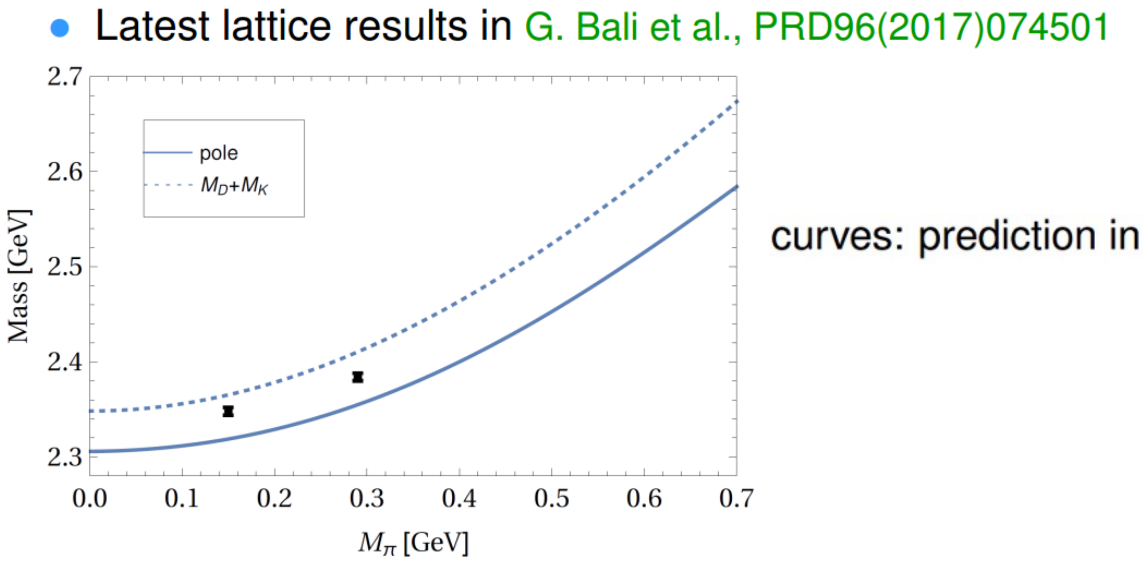
- Different mass splitting patterns:

$D^*_{s0}(2317)$ and $D_{s1}(2460)$- coupled with S-wave $D^{(*)}K$ → Sizable mass shift & mixing

$D_{s1}(2536)$ and $D_{s2}(2573)$- coupled with D-wave $D^{(*)}K$ → Small mass shift & tiny mixing

Prediction II: m_π- dependence

- DK molecule: Tends to become larger with larger m_π.



curves: prediction in Du et al., EPJC77(2017)728

- Bare state ($c\bar{s}$): Tends to become stable with larger m_π.

"…for the lower lying pseudoscalar and vector D_s meson masses which decrease by 3 MeV (from 1980(1) MeV at m_π = 290 MeV to 1977(1) at m_π = 150 MeV) and 7 MeV (from 2101(1) MeV to 2094(1) MeV), respectively, hinting that the 0+ and 1+ states may have a more complicated internal structure."

G. Bali et al., PRD96(2017)074501

Prediction II: m_π- dependence

- Our prediction: the mass of $D_{s0}^*(2317)$ finally tends to become stable with increasing m_π.

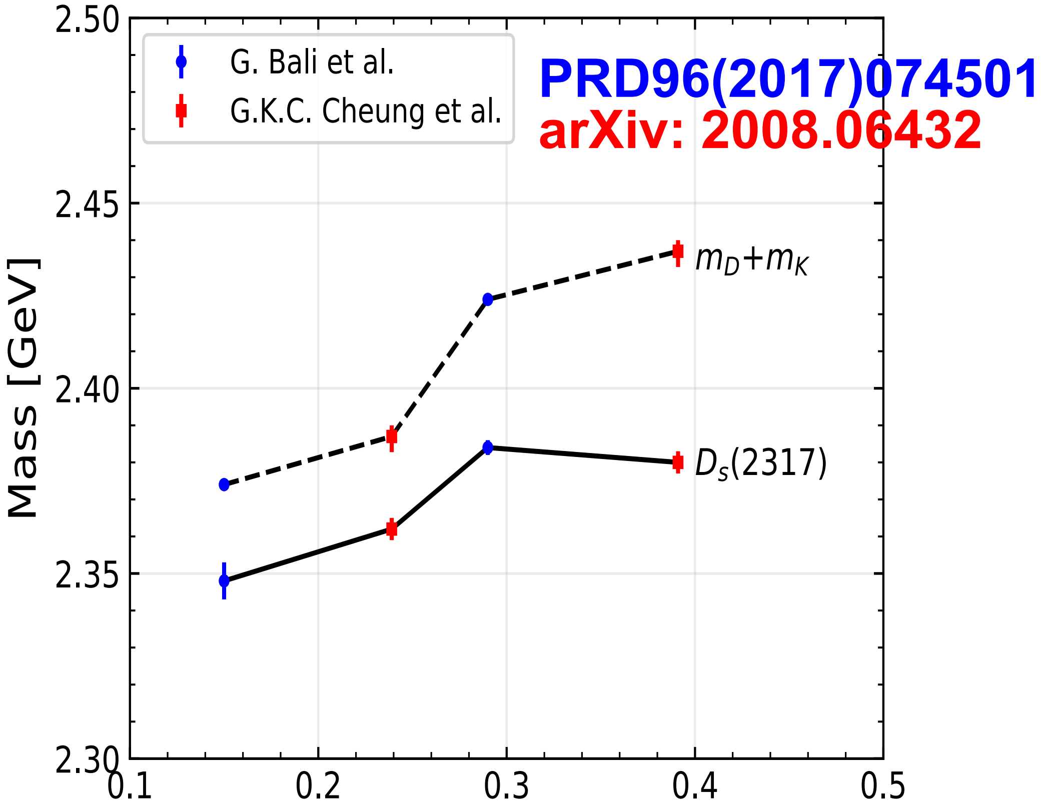
- m_π ↗ , m_{DK} ↗ , $m_{\bar{c}s}$ → stable

- $m_{DK} < m_{\bar{c}s}$

- $D_{s0}^*(2317)$: mainly dominated by DK, increasing

- $m_{DK} >> m_{\bar{c}s}$:

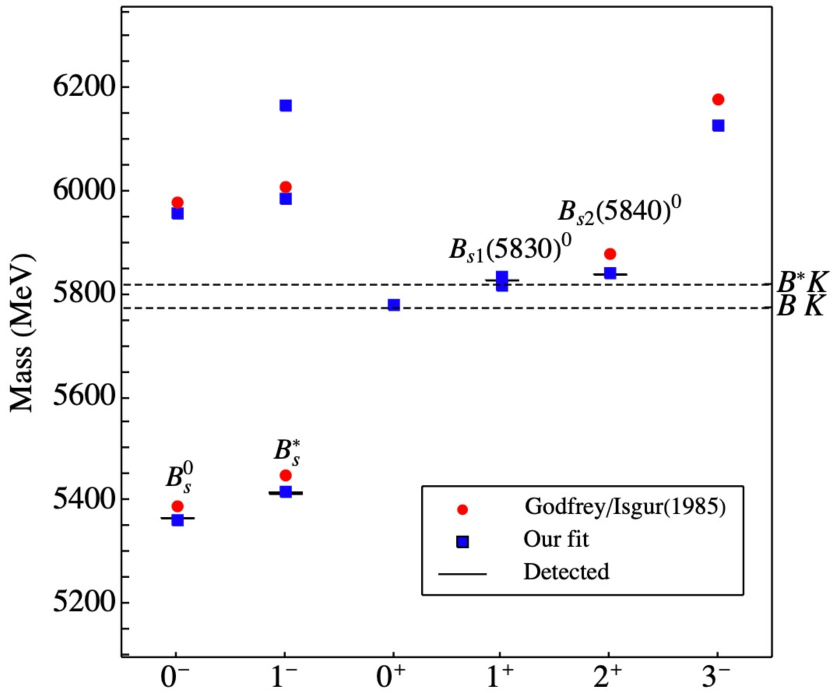
- $D_{s0}^*(2317)$ is mainly $\bar{c}s$. $M_{D_{s0}^*(2317)}$ tends to be stable.



G. Bali et al., PRD96(2017)074501

Gavin K. C. Cheung et al., arXiv: 2008.06432

B_{s0}^{*} & B_{s1}^{*} with HQS



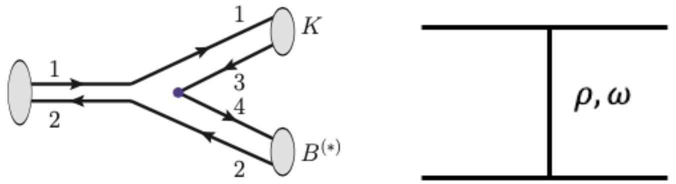
	$b\,(\vert{}^{2S+1}L_J\rangle)$	b (mass)	α	L
B_{s0}^*	$\vert{}^3P_0\rangle$	5780.9	BK	S
B_{s1}^*	$-0.74\,\vert{}^1P_1\rangle + 0.67\,\vert{}^3P_1\rangle$	5818.5	B^*K	$S,\ D$
	$= 0.98\,\phi_s - 0.22\,\phi_d$			
$B_{s1}^{*\prime}$	$0.67\,\vert{}^1P_1\rangle + 0.74\,\vert{}^3P_1\rangle$	5835.6	B^*K	$S,\ D$
	$= 0.22\,\phi_s + 0.98\,\phi_d$			

$$H = H_0 + H_I,$$

$$H_0 = \sum_{i=1,n} |B_i\rangle m_i \langle B_i| + \sum_{\alpha} |\alpha(k_\alpha)\rangle \left[\sqrt{m_{\alpha 1}^2 + k_\alpha^2} + \sqrt{m_{\alpha 2}^2 + k_\alpha^2}\right] \langle \alpha(k_\alpha)|$$

bare $\bar{b}s$ meson   two-meson channel

$$H_I = \hat{g} + \hat{v}$$



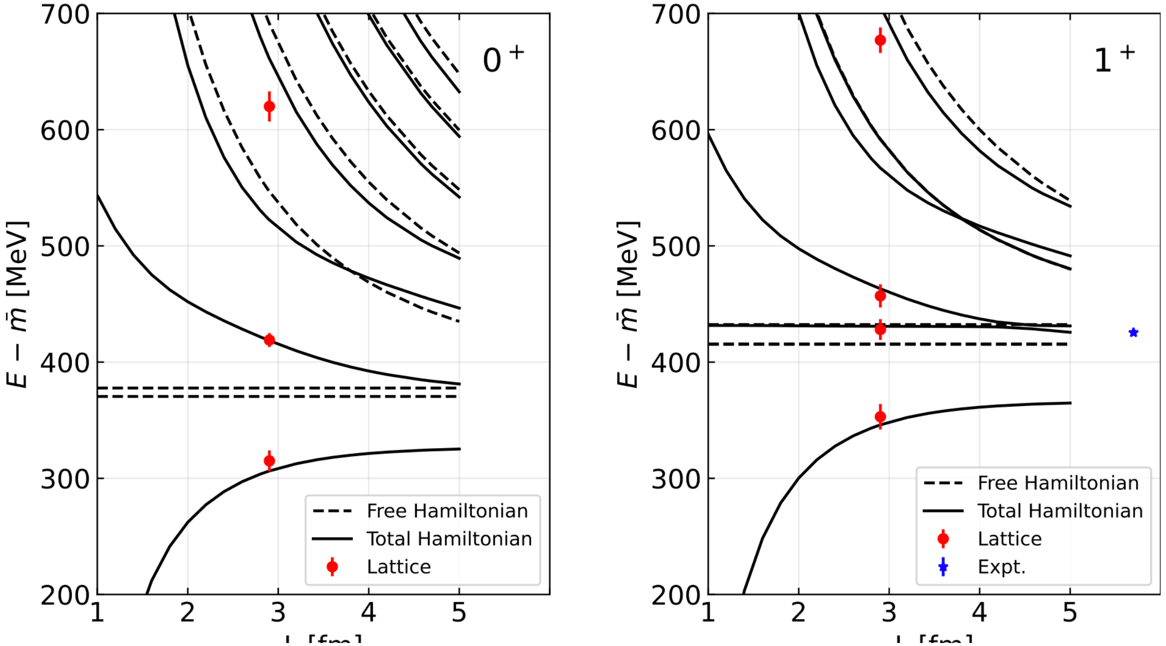
- Heavy quark flavor symmetry:
  Same parameters determined in the D_s sectors.

$$g_c = 4.2^{+2.2}_{-3.1}\,, \quad \Lambda' = 0.323^{+0.033}_{-0.031}\ \text{GeV}\,, \quad \gamma = 10.3^{+1.1}_{-1.0}\,.$$

B_{s0}^{*}& B_{s1}^{*}: Predictions VS lattice results

- The predicted P-wave B_s energy levels with HQS versus Lattice QCD results with $m_\pi = 156.$

C. B. Lang et al., Phys. Lett. B 750, 17 (2015)



	$P(b\bar{s})[\%]$	Mass [MeV]	LQCD
B_{s0}^{*}	$54.7^{+5.2}_{-4.1}$	$5730.2^{+2.4}_{-1.5}$	$5713 \pm 11 \pm 19$
B_{s1}^{*}	$56.7^{+4.6}_{-3.7}$	$5769.6^{+2.4}_{-1.6}$	$5750 \pm 17 \pm 19$

Summary

• Quark model + coupled channel effect +HEFT & Lattice QCD: near threshold P-wave D_ss.

	$P(c\bar{s})[\%]$	$P(D^{(*)}K)[\%]$	ours	exp
$D_{s0}^*(2317)$	$32.0^{+5.2}_{-3.9}$	68.0	$2338.9^{+2.1}_{-2.7}$	2317.8 ± 0.5
$D_{s1}^*(2460)$	$52.4^{+5.1}_{-3.8}$	47.6	$2459.4^{+2.9}_{-3.0}$	2459.5 ± 0.6
$D_{s1}^*(2536)$	$98.2^{+0.1}_{-0.2}$	1.8	$2536.6^{+0.3}_{-0.5}$	2535.11 ± 0.06
$D_{s2}^*(2573)$	$95.9^{+1.0}_{-1.5}$	4.1	$2570.2^{+0.4}_{-0.8}$	2569.1 ± 0.8

1. Different mass splitting patterns:

$D_{s0}^*(2317)$ and $D_{s1}(2460)$- coupled with S-wave $D^{(*)}K$ → Sizable mass shift & mixing

$D_{s1}(2536)$ and $D_{s2}(2573)$- coupled with D-wave $D^{(*)}K$ → Small mass shift & tiny mixing

• Extension to B_ss $(0^+, 1^+)$ with HQS: Predictions of the energy levels and masses consistent with lattice QCD.

Thank you for your attention!

Back up side

$D_{s0}^*(2317)\& \ D_{s1}^*(2460)$

- **Quenched and unquenched quark model**
  1. S. Godfrey and N. Isgur, **Phys. Rev. D 32, 189 (1985)**
  2. Y.-B. Dai, C.-S. Huang, C. Liu, and S.-L. Zhu, **Phys. Rev. D 68, 114011 (2003)**
  3. D. S. Hwang and D.-W. Kim, **Phys. Lett. B 601, 137 (2004)**
  4. Y. A. Simonov and J. A. Tjon, **Phys. Rev. D 70, 114013 (2004)**
  5. H.-Y. Cheng and F.-S. Yu, **Phys. Rev. D 89, 114017 (2014)**
  6. Q.-T. Song, D.-Y. Chen, X. Liu, and T. Matsuki, **Phys. Rev. D 91, 054031 (2015)**
  7. H.-Y. Cheng and F.-S. Yu, **Eur. Phys. J. C 77, 668 (2017)**
  8. S.-Q. Luo, B. Chen, X. Liu, and T. Matsuki, **Phys. Rev. D 103, 074027 (2021)**
  9. Z.-Y. Zhou and Z. Xiao, **Eur. Phys. J. C 81, 551 (2021)**

  ……
- **Tetraquark**
  1. H.-Y. Cheng and W.-S. Hou, **Phys. Lett. B 566, 193 (2003)**
  2. Y.-Q. Chen and X.-Q. Li, **Phys. Rev. Lett. 93, 232001(2004)**
  3. V. Dmitrasinovic, **Phys. Rev. Lett. 94, 162002 (2005)**
  4. H. Kim and Y. Oh, **Phys. Rev. D 72, 074012 (2005)**
  5. J.-R. Zhang, **Phys. Lett. B 789, 432 (2019)**

  ……
- **Hadronic molecule**
  1. E. E. Kolomeitsev and M. F. M. Lutz, **Phys. Lett. B 582, 39 (2004)**
  2. A. P. Szczepaniak, **Phys. Lett. B 567, 23 (2003)**
  3. J. Hofmann and M. F. M. Lutz, **Nucl. Phys. A 733, 142 (2004)**
  4. E. van Beveren and G. Rupp, **Phys. Rev. Lett. 91, 012003 (2003)**
  5. T. Barnes, F. E. Close, and H. J. Lipkin, **Phys. Rev. D68, 054006 (2003)**
  6. D. Gamermann, E. Oset, D. Strottman, and M. J. Vi- cente Vacas, **Phys. Rev. D 76,074016 (2007)**
  7. F.-K. Guo, P.-N. Shen, and H.-C. Chiang, **Phys. Lett. B 647, 133 (2007)**
  8. F.-K. Guo, P.-N. Shen, H.-C. Chiang, R.-G. Ping, and B.-S. Zou, **Phys. Lett. B 641, 278 (2006)**
  9. J. M. Flynn and J. Nieves, **Phys. Rev. D 75, 074024 (2007)**
  10. A. Faessler, T. Gutsche, V. E. Lyubovitskij, and Y.-L. Ma, **Phys. Rev. D 76, 014005 (2007)**
  11. F.-K. Guo, C. Hanhart, and U.-G. Meissner, **Eur. Phys. J. A 40, 171 (2009)**
  12. Z.-X. Xie, G.-Q. Feng, and X.-H. Guo, **Phys. Rev. D 81, 036014 (2010)**

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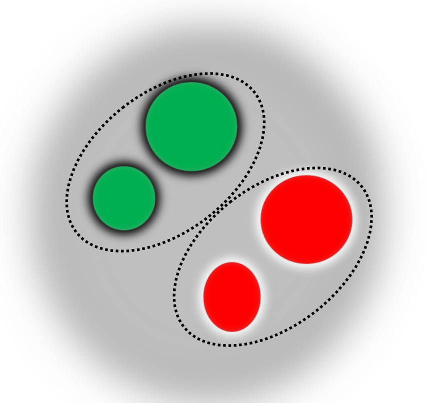
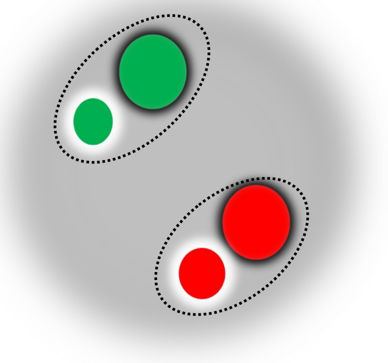
Meson

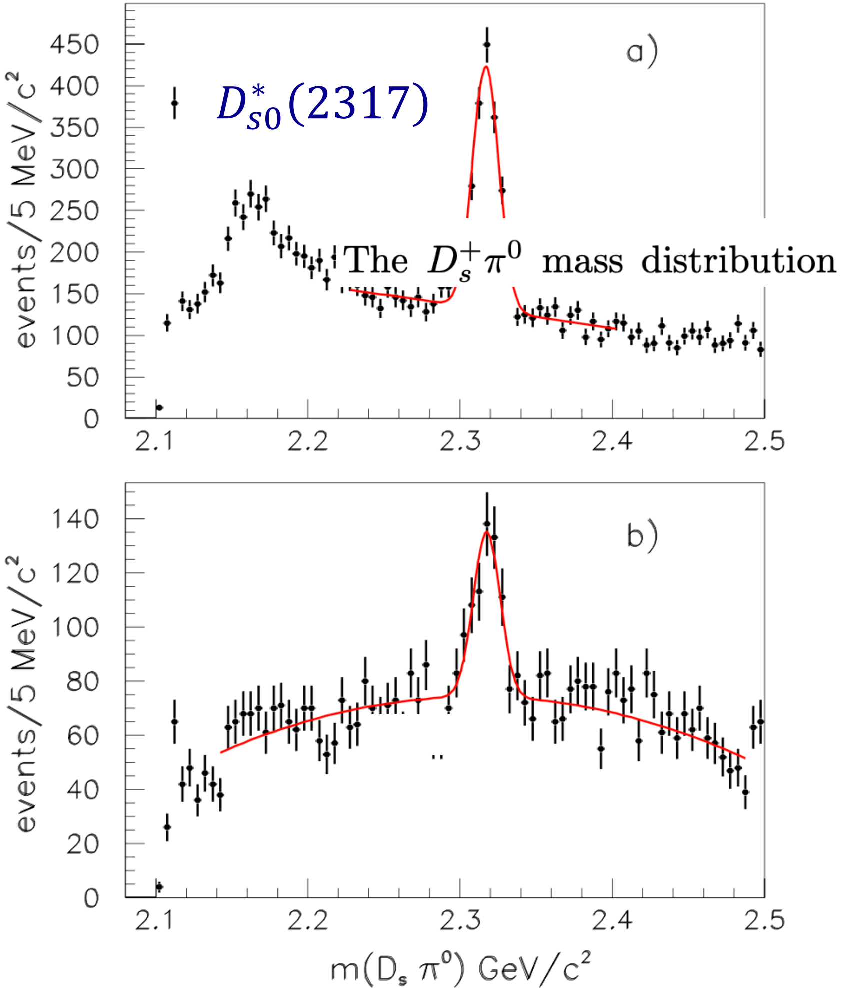


Compact multiquark



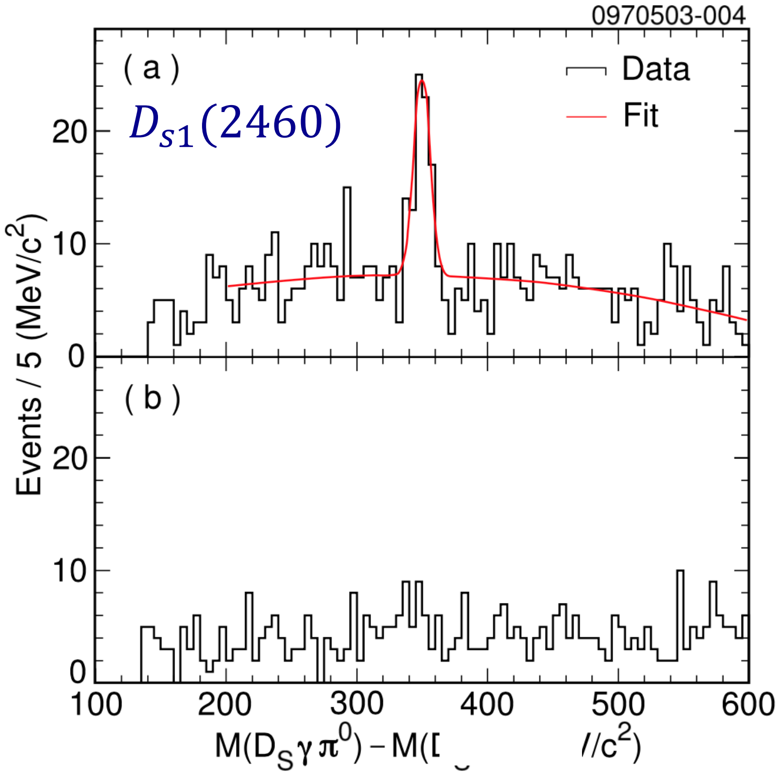
Hadronic molecule

$D_{s0}^*(2317)$ & $D_{s1}(2460)$



The $D_s\pi^0$ mass distribution

BaBar, PRL90 (2003) 242001



CLEO, PRD68 (2003) 032002

$D_{s0}^*(2317)$: M= 2317.8 ± 0.5 MeV, $\Gamma < 3.8$ MeV.
$I(J^P) = 0(0^+)$.

$D_{s1}(2460)$: M=2459.5 ± 0.6 MeV, $\Gamma < 3.5$ MeV.
$I(J^P) = 0(1^+)$.

Hamiltonian effective field theory (HEFT)

- Coupled channel effect: 2→2 scattering process,

$$D^{(*)}K \rightarrow D^{(*)}K$$

- The scattering amplitude cannot be extracted from experiments and need lattice QCD data.

- The result is helpful in the relevant analysis of experimental processes, e.g.,

$$B_s/B \rightarrow D^{(*)}D^{(*)}K \text{ or } D^{(*)}KK$$

Our fit VS GI model

GI-model		Our fit	
Mass (MeV)		Mass (MeV)	
$m_q = 220$		$m_q = 294$	
$m_s = 419$		$m_s = 497$	
$m_c = 1628$		$m_c = 1720$	
$m_b = 4977$		$m_b = 5065$	
Potential		Potential	
$b = 0.18$ GeV^2		$b = 0.18$ GeV^2	
$\alpha_s^{\text{critical}} = 0.6$		$\alpha_s^{\text{critical}} = 0.6$	
$\Lambda = 200$ MeV		$\Lambda = 200$ MeV	
$c = -253$ MeV		$c = -426$ MeV	
Relativistic effects		Relativistic effects	
smearing	$\sigma_0 = 1.80$ GeV	smearing	$\sigma_0 = 1.45$ GeV
	$s = 1.55$		$s = 1.55$
$m \leftrightarrow E$ ambiguity	$\epsilon_c = -0.168$	$m \leftrightarrow E$ ambiguity	$\epsilon_c = -0.194$
	$\epsilon_t = +0.025$		$\epsilon_t = -0.016$
	$\epsilon_{so(V)} = -0.035$		$\epsilon_{so(V)} = -0.277$
	$\epsilon_{so(s)} = +0.055$		$\epsilon_{so(s)} = -0.289$

Our fit VS GI model

0.1 Our parameter:

$$H = \begin{pmatrix} 2525.7 & 13.1 \\ 13.1 & 2523.6 \end{pmatrix}$$

$$E_{2460} = 2511.5\,\text{MeV}, |\psi_{2460}\rangle = -0.771241|^1P_1, 1^+\rangle + 0.636544|^3P_1, 1^+$$
$$E_{2537} = 2537.8\,\text{MeV}, |\psi_{2537}\rangle = -0.636544|^1P_1, 1^+\rangle - 0.771241|^3P_1, 1^+$$

$$\langle ^1P_1, 1^+|D^*K\rangle = {}^1P_1, 1^+|D^*K\rangle_S + \langle ^1P_1, 1^+|D^*K\rangle_D$$

$$|^1P_1, 1^+\rangle = -\frac{1}{\sqrt{3}}|s_l = \frac{1}{2}, s_Q = \frac{1}{2}\rangle + \sqrt{\frac{2}{3}}|s_l = \frac{3}{2}, s_Q = \frac{1}{2}\rangle$$

$$|^3P_1, 1^+\rangle = \sqrt{\frac{2}{3}}|s_l = \frac{1}{2}, s_Q = \frac{1}{2}\rangle + \frac{1}{\sqrt{3}}|s_l = \frac{3}{2}, s_Q = \frac{1}{2}\rangle$$

Then, one has

$$|\psi_{2460}\rangle = -0.991737|s_l = \frac{1}{2}, s_Q = \frac{1}{2}\rangle + 0.128284|s_l = \frac{3}{2}, s_Q = \frac{1}{2}\rangle$$

$$|\psi_{2537}\rangle = -0.128284|s_l = \frac{1}{2}, s_Q = \frac{1}{2}\rangle - 0.991737|s_l = \frac{3}{2}, s_Q = \frac{1}{2}\rangle$$

0.2 GI model

$$H = \begin{pmatrix} 2558.9 & 2.3 \\ 2.3 & 2550.3 \end{pmatrix}$$

$$E_{2460} = 2549.74\,\text{MeV}, |\psi_{2460}\rangle = -0.970732|^1P_1, 1^+\rangle + 0.240165|^3P_1, 1^+\rangle$$
$$E_{2537} = 2559.46\,\text{MeV}, |\psi_{2537}\rangle = -0.240165|^1P_1, 1^+\rangle - 0.970732|^3P_1, 1^+\rangle$$

$$|\psi_{2460}\rangle = 0.756546|s_l = \frac{1}{2}, s_Q = \frac{1}{2}\rangle - 0.65394|s_l = \frac{3}{2}, s_Q = \frac{1}{2}\rangle$$

$$|\psi_{2537}\rangle = -0.65394|s_l = \frac{1}{2}, s_Q = \frac{1}{2}\rangle - 0.756546|s_l = \frac{3}{2}, s_Q = \frac{1}{2}\rangle$$

T −matrix

In the infinite volume, the scattering T-matrix reads

$$T_{\alpha,\,\beta}(k,k';E)=\mathcal{V}_{\alpha,\,\beta}(k,k';E)+\sum_{\alpha'}\int q^2dq\frac{\mathcal{V}_{\alpha,\,\alpha'}(k,q;E)T_{\alpha,\,\beta}(q,k';E)}{E-E_{\alpha'}(q)+i\epsilon}$$

where the effective potential reads

$$\mathcal{V}_{\alpha,\,\beta}(k,k';E)=\sum_{B}\frac{g_{\alpha\,B}(k)g^{*}_{\beta\,B}(k')}{E-m_B}+V^{L}_{\alpha,\beta}(k,k').$$

$$T_{\alpha,\beta}(k,k',E)=t^{\alpha,\beta}_{\rm bg}(k,k',E)+t^{\alpha,\beta}_{res}(k,k',E)$$

$$t^{\alpha,\beta}_{res}(k,k',E)=\frac{\Gamma_{\alpha,B_i}(k,E)\bar{\Gamma}_{B_j,\beta}(k',E)}{(E-m_B)\delta_{B_iB_j}-\Sigma_{B_iB_j}(E)}$$

$$\bar{\Gamma}_{B_i,\alpha}(k',E)=g_{B_i,\alpha}(k',E)+\int dqq^2g_{B_i,\beta}(q,E)G_{\beta}(q)t^{\beta,\alpha}_{bg}(q,k'),$$

$$\Sigma_{B_i,B_j}(E)=\int dk''k''^2g_{B_i,\alpha}(k,E)G_{\alpha}(k'')\bar{\Gamma}_{\alpha,B_j}(k',E),$$

$$t^{\beta,\alpha}_{bg}(q,k'')=V^{L}_{\beta,\alpha}(q,k'')+\int dq'q'^2V^{L}_{\beta,\gamma}(q,q')G_{\gamma}(q')t^{\gamma,\alpha}_{bg}(q',k),$$

$$\det[(E-m_B)\delta_{B_i,B_j}-\Sigma_{B_i,B_j}(E)]=0$$ → Pole position

Comparison with other works

state	L=4.57 fm	Pole mass at $L \to \infty$	
	$P(c\bar{s})[\%]$	ours	exp
$D^*_{s0}(2317)$	$32.0^{+5.2}_{-3.9}$	$2338.9^{+2.1}_{-2.7}$	2317.8 ± 0.5
$D^*_{s1}(2460)$	$52.4^{+5.1}_{-3.8}$	$2459.4^{+2.9}_{-3.0}$	2459.5 ± 0.6
$D^*_{s1}(2536)$	$98.2^{+0.1}_{-0.2}$	$2536.6^{+0.3}_{-0.5}$	2535.11 ± 0.06
$D^*_{s2}(2573)$	$95.9^{+1.0}_{-1.5}$	$2570.2^{+0.4}_{-0.8}$	2569.1 ± 0.8

Yang's talk

A. M. Torres, E. Oset, S. Prelovsek, and A. Ramos
JHEP 05, 153 (2015)

$$P(KD) = 72 \pm 13 \pm 5\ \%\,, \text{ for the } D^*_{s0}(2317)$$

$$P(KD^*) = 57 \pm 21 \pm 6\ \%\,, \text{ for the } D_{s1}(2460)$$

Prediction II: m_π- dependence

	0^+ channel		
	$m_\pi = 290$ MeV	$m_\pi = 150$ MeV	Expt.
a_0 [fm]	$-1.13(0.04)(+0.05)$	$-1.49(0.13)(-0.30)$	
r_0 [fm]	$0.08(0.03)(+0.08)$	$0.20(0.09)(+0.31)$	
$\|p_B\|$ [MeV]	$180(6)(0)$	$142(11)(-9)$	
Δm [MeV]	$40(3)(0)$	$26(4)(-3)$	$42.6(0.7)(2.0)$
m_{D_s} [MeV]	$2384(2)(-1)$	$2348(4)(+6)$	$2317.7(0.6)(2.0)$
g [GeV]	$11.9(0.3)(+0.5)$	$11.0(0.6)(+1.2)$	

G. Bali et al., PRD96(2017)074501

		$m_\pi = 239$ MeV	$m_\pi = 391$ MeV
$a_t\sqrt{s_{\rm pole}}$	=	0.3886(5)	0.4200(5)
$a_t\|c\|$	=	0.234(9)	0.305(20)
$\sqrt{s_{\rm pole}}$	=	2362(3) MeV	2380(3) MeV
$\|c\|$	=	1420(50) MeV	1730(110) MeV.

$$\Delta E = m_D + m_K - \sqrt{s_{\rm pole}} = 57(3)\ \text{MeV for } m_\pi = 391\ \text{MeV}$$

$$\Delta E = 25(3)\ \text{MeV for } m_\pi = 239\ \text{MeV}$$

Gavin K. C. Cheung et al., arXiv: 2008.06432