0^+ XTZ states from QCD spectral sum rules [1]

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QCD22 25th HIGH-ENERGY PHYSICS INTERNATIONAL CONFERENCE IN QUANTUM CHROMODYNAMICS

MONTPELLIER 4-7 july 2022

1. R. Albuqueruqe, S. Narison and D. Rabetiarivony, Nucl. Phys. A 1023 (2022) 122451; Phys. Rev. D 103 (2021) 074015; Phys. Rev. D 105 (2022) 114035

Two-point function

Evaluation of two point function $\rightsquigarrow$ Hadron parameters

$$\Pi^{\mu\nu}_{\mathcal{H}}(q^2) = i\int d^4x\; e^{-iqx}\langle 0|\mathcal{T}\,\mathcal{O}^{\mu}_{\mathcal{H}}(x)\left(\mathcal{O}^{\nu}_{\mathcal{H}}(0)\right)^{\dagger}|0\rangle$$

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Currents ($q \equiv u, d, s$)

$J^P = 0^+$

$$\mathcal{O}^{0^+}_{S_c} = \epsilon_{ijk}\,\epsilon_{mnk}\Big[\left(q_i^T\,C\gamma_5 c_j\right)\left(\bar{q}_m\,\gamma_5 C\bar{c}_n^T\right) + k\left(q_i^T\,Cc_j\right)\left(\bar{q}_m C\bar{c}_n^T\right)\Big]$$

$$\mathcal{O}^{0^+}_{T_{ud}} = \frac{1}{\sqrt{2}}\epsilon_{ijk}\,\epsilon_{mnk}\left(c_i^T\,C\gamma^\mu\,c_j\right)\Big[\left(\bar{u}_m\,\gamma_\mu C\,\bar{d}_n^T\right) + \left(\bar{d}_m\,\gamma_\mu C\,\bar{u}_n^T\right)\Big]$$

$$\mathcal{O}^{0^+}_{T_{qs}} = \epsilon_{ijk}\,\epsilon_{mnk}\left(c_i\,C\gamma^\mu c_j^T\right)\left(\bar{q}_m\,\gamma_\mu C\bar{s}_n^T\right)$$

X(3872)

$$\mathcal{O}^{1^+}_{X_c} = \epsilon_{ijk}\,\epsilon_{mnk}\Big[\left(q_i^T\,C\gamma_5\,c_j\right)\left(\bar{c}_m\,\gamma^\mu C\,\bar{q}_n^T\right) + \left(q_i^T\,C\gamma^\mu\,c_j\right)\left(\bar{c}_m\,\gamma_5 C\,\bar{q}_n^T\right)\Big]$$

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♣ QCD side : quark and gluon fields, OPE $\Rightarrow$ Dispertion relation
♣ Phenomenological side : Hadron parameters $\Rightarrow$ Dispertion relation

♣ Quark Hadron duality principle
QCD side $\simeq$ PHEN side

♣ Inverse Laplace Transform

Mass Extraction

Finite Energy Inverse Laplace Transform Sum Rule :

$$\mathcal{L}_n^c|_{\mathcal{H}}(\tau,\mu) = \int_{(2M_c+m_q+m_s)^2}^{t_c} dt\ t^n\ e^{-t\tau}\frac{1}{\pi}\text{Im}\ \Pi_{\mathcal{H}}(t,\mu)$$

Ansatz :

$$\frac{1}{\pi}\mathrm{Im}\,\Pi_{\mathcal{H}}(t) = f_{\mathcal{H}}^2 M_{\mathcal{H}}^8\,\delta(t-M_{\mathcal{H}}^2) + \frac{1}{\pi}\mathrm{Im}\,\Pi_{\mathcal{H}}^{\mathrm{QCD}}(t)\,\theta(t-t_c)$$

$$M_{\mathcal{H}}^2 = \mathcal{R}_{\mathcal{H}}^c(\tau_0) = \frac{\mathcal{L}_1^c|_{\mathcal{H}}}{\mathcal{L}_0^c|_{\mathcal{H}}};$$

$$r_{\mathcal{H}'/\mathcal{H}}(\tau_0) \equiv \sqrt{\frac{\mathcal{R}_{\mathcal{H}'}^c}{\mathcal{R}_{\mathcal{H}}^c}} = \frac{M_{\mathcal{H}'}}{M_{\mathcal{H}}}$$

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⊛ **OPE convergence** obtained for condensates up to $d \leq 6$

⊛ To prevent the **violation of factorization** the inclusion of higher dimension condensates is not suggested

NLO corrections

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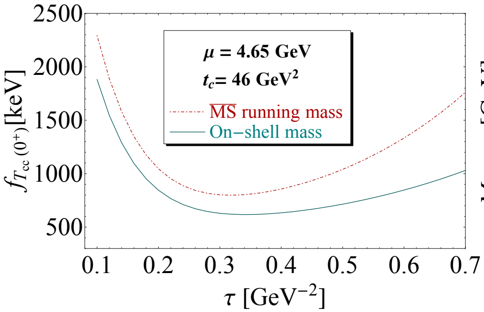

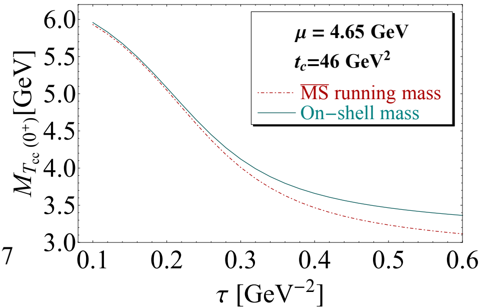

$$\Delta f \simeq 80\,\text{keV} \;\&\; \Delta M \simeq 25\,\text{MeV}$$

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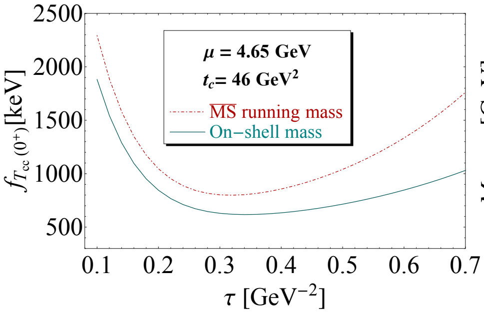

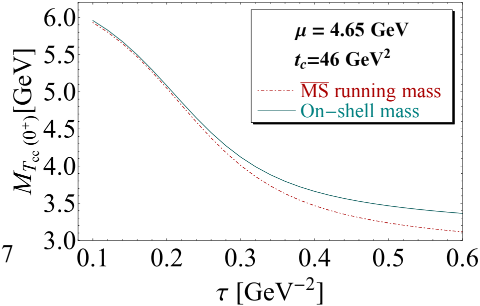

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♣ Inclusion of **NLO PT corrections** : Solve this ambiguity and justify the use of MS running quark mass.

◇ NLO corrections are small ⇒ Lucky choice of MS running mass at LO

Stability criteria

♣ (τ, t_c, μ) free external parameters $\Rightarrow$ minimum sensitivity of (M_H, f_H) vs (τ, t_c, μ)

τ stability

- Optimal result extracted at the **minimum or inflexion** point (Harmonic oscillator of QM & J/ψ LSR)[ref test Zc and ref therein]
- More precise than the hand-waving $10 \sim 20\%$ Borel window criteria
- Lowest ground state dominance & OPE convergence satisfied at τ extremum

t_c stability

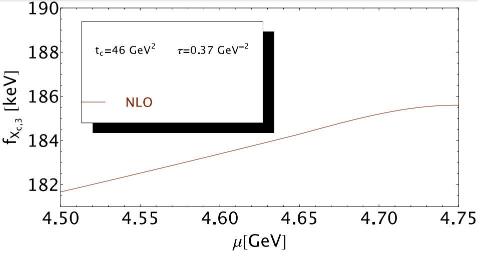
- From the beginning of τ stability $>$ the beginning of t_c stability
- Large range of t_c value (choice of t_c inside this range confirmed by FESR : [PRD 105 (2022) 114035])

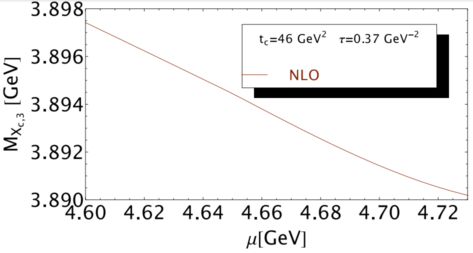
μ stability

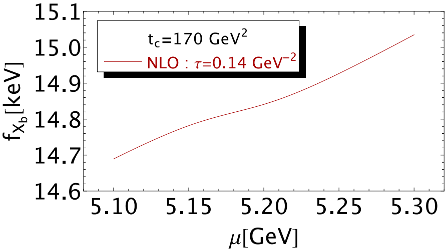
- To fix the arbitrary subtraction constant in HO PT evaluation of Wilson coeff. & QCD input renormalized parameters
- Almost universal for $[QQ\overline{qq}]$ & $[Qq\overline{Qq}]$: $\mu_c \simeq 4.65$ GeV $\|$ $\mu_b \simeq 5.20$ GeV

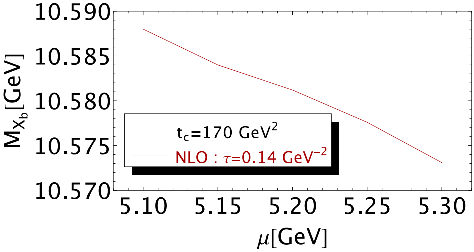
[IJMPA 31 (2016) 1650196 ; IJMPA 33 (2018) 1850082 & NPA/PRD in title page]

μ analysis



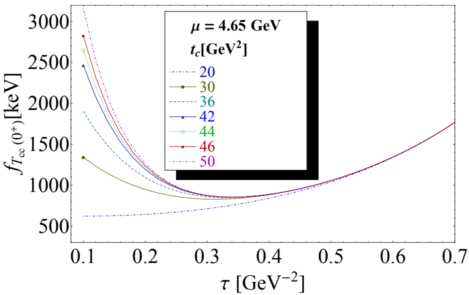


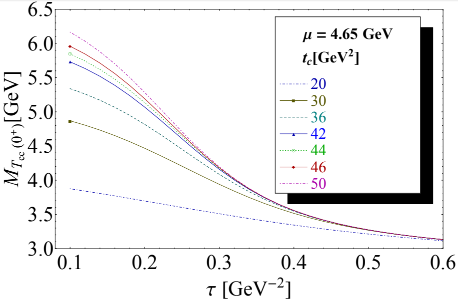




$$\Rightarrow \mu_c \simeq 4.65\,\text{GeV} \;\&\; \mu_b \simeq 5.20\,\text{GeV}$$

$0^+ \; T_{cc\bar{u}\bar{d}}$



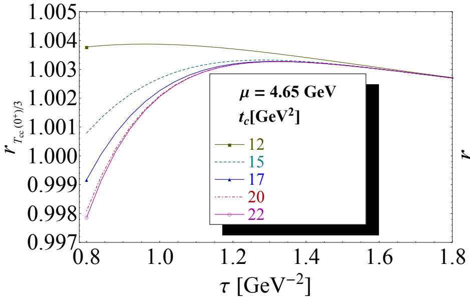


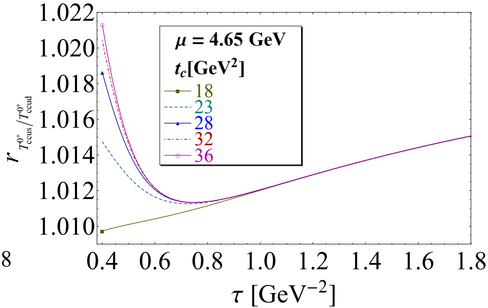
♣ Stability region

- ▷ Beginning of τ-stability for : $(\tau,\, t_c) = (0.31,\, 30)$ ($\text{GeV}^{-2},\ \text{GeV}^2$)
- ▷ t_c-stability reached for : $(\tau,\, t_c) = (0.34,\, 46)$ ($\text{GeV}^{-2},\ \text{GeV}^2$)

$$\Rightarrow f_{T_{cc}}(0^+) = 841(83)\ \text{keV}\ ; \quad M_{T_{cc}}(0^+) = 3882(129)\ \text{MeV}$$

DRSR





♣ Stability region :

$(\tau,\, t_c) = (1.28,\, 15) \sim (1.32,\, 20)$

⊙ $r_{T^{0+}_{cc}/X_c} = 1.0033(10)$

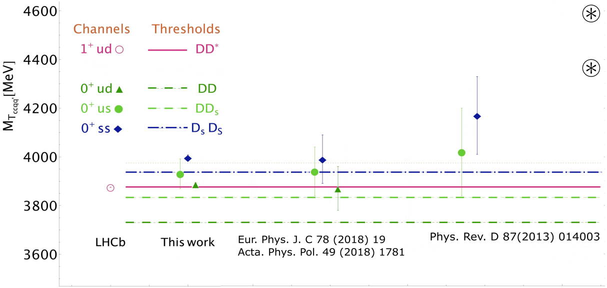
$\Rightarrow M_{T_{cc}}(0^+) = 3885(4)$ MeV

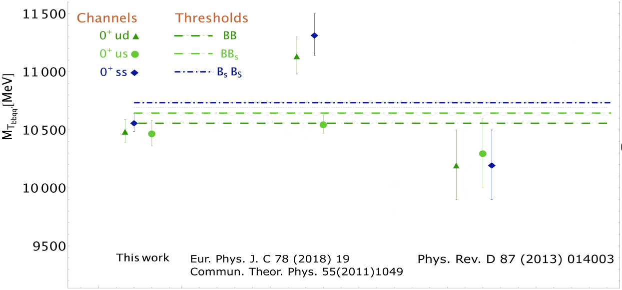
♣ Stability region :

$(\tau,\, t_c) = (0.72,\, 23) \sim (0.74,\, 32)$

⊙ $r_{T_{cc\bar{u}\bar{s}}/T_{cc}(0^+)} = 1.0113(12)$

$\Rightarrow M_{T_{cc\bar{s}\bar{u}}}(0^+) = 3927(6)$ MeV



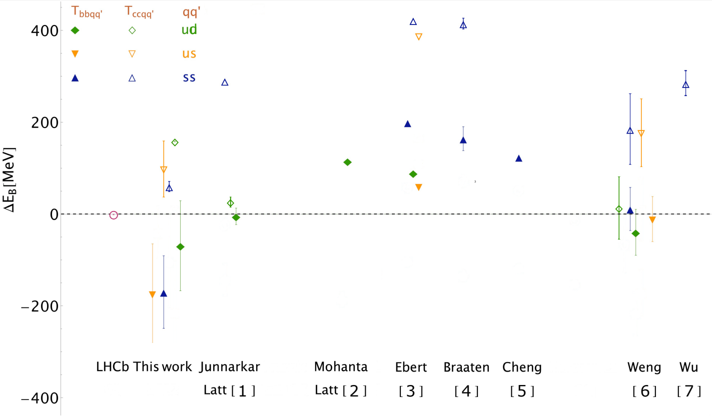


⊛ Quoted results : SR ⊕ DRSR

⊛ - T_{cc}-like above threshold
- T_{bb}-like below threshold

⊛ Sources of discrepancies :
- Different inputs (obsolete and inaccurate values in other literature)
- Missing diagrams in the QCD expressions of the propagators
- Different Optimization criteria

⊛ Within the errors there are almost good agreement among the ≠ predictions



[1] Phys. Rev. D99(2019)034507
[2] Phys. Rev. D102(2020)094516
[3] Phys. Rev. D76(2007)114015
[4] Phys. Rev. D103(2021)016001
[5] Chin. Phys. C45(2021)043102
[6] arXiv :2108.07242[hep-ph]
[7] arXiv :2112.05967[hep-ph]

- ⊛ Our predictions are grouped around the physical threshold.
- ⊛ Mass shifts due to SU3 breaking effect positive but tiny
- ⊛ Experimental check of the peculiar feature of $T_{cc\bar{s}\bar{s}}$ and $T_{bb\bar{s}\bar{s}}$ is needed.