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The natures of recently observed states $\eta_1(1855)$ & X(2600)

Cong-Feng Qiao

中國科學院大學

UNIVERSITY OF CHINESE ACADEMY OF SCIENCES

arXiv:2206.13133 and arXiv:2203.14014

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Contents:

- Introduction to $\eta_1(1855)$ & $X(2600)$
- Theoretical approach employed
- Results
- Concluding remarks

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- **Introduction to $\eta_1(1855)$ & X(2600)**
- Theoretical approach employed
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I. Introduction to η_1(1855) & X(2600)

- Investigation on hadronic states beyond the conventional quark model such as multiquark, hybrid and glueball, may greatly enrich our knowledge about hadron and QCD
- So far, more than thirty such states or candidates have been observed in experiment ever since the observation of X(3872)

I. Introduction to η_1(1855) & X(2600)

- It is highly expectable that a large number of new hadronic states will emerge in forthcoming years, comparable to the situation in the sixtieth of last century
- To decoding the hadronic structure of the new experimental observations is one of the intriguing and important topics in hadron physics

I. Introduction to η_1(1855) & X(2600)

- Recently, by analyzing the partial wave of the process $J/\psi \to \gamma\eta\eta'$BESIII Collaboration observed a structure at 1855 MeV, named η_1(1855), in η *and* η' invariant mass spectrum with significance of 19σ and its decay width is some 188 MeV
- The η_1(1855) possesses an exotic quantum number of J^{PC}=1^{-+}, a very interesting object

BESIII，arXiv:2202.00621 and 2202.00623

I. Introduction to η_1(1855) & X(2600)

- In the literature, η_1(1855) was interpreted as a hybrid state, analyzed in flux tube model, QCDSR and effective Lagrangian

  L. Qiu and Q. Zhao, arXiv:2202.00904;
  H.X. Chen, N. Su and S.L. Zhu, 2202.04918
  V. Shastry, C. Fischer and F. Giacosa, 2203.04327

- In fact the exotic quantum number J^{PC}=1^{-+} can also be assigned to a tetraquark

I. Introduction to η_1(1855) & X(2600)

- There had been some investigations on 1^{-+} tetraquark and molecular state, by means of QCD Sum Rules

H.X. Chen, N. Hosaka and S.L. Zhu, PRD 2008
S. Narison, PLB 2009
X.K. Dong, Y.H. Lin and B.S. Zou, arXiv: 2202.00863
F. Yang and Y. Huang, arXiv: 2203.06934

- In our work, we investigate the light isoscalar tetraquark states in configurations of $[1_c]_{\bar{s}s} \otimes [1_c]_{\bar{q}q}$ and $[1_c]_{\bar{s}q} \otimes [1_c]_{\bar{s}q}$

I. Introduction to η_1(1855) & X(2600)

- Also in very recently, by scrutinizing the radiative decays of J/ψ, BESIII Collaboration observed a structure in $\pi^+\pi^-\eta'$ invariant mass of about 2.62 GeV with significance greater than 20σ
- From its known decay products the new state most likely possesses quantum number of $J^{PC}=0^{-+}$ or $J^{PC}=2^{-+}$

BESIII，arXiv:2201.10796

I. Introduction to η_1(1855) & X(2600)

- Since the intermediate resonance f_0(1500) and final state η' are both glueball candidates, or at least have large gluonic content, it is reasonable to consider X(2600) as a glue-rich object or even a glueball
- Due to the quantum number constraint, we investigate the trigluon glueball states with quantum numbers of $J^{PC} = 0^{-+}$ and 2^{-+}

Contents:

- Introduction to $\eta_1(1855)$ & $X(2600)$
- **Theoretical approach employed**
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II. Theoretical approach employed

- The Shifman, Vainshtein, and Zakharov (SVZ) QCD sum rules (QCDSR) has some peculiar advantages in the study of hadron spectrum involving nonperturbative effect of QCD
- In order to evaluate the mass spectra of the glueballs, one has to construct appropriate currents that possesses the foremost information about the concerned hadron

II. Theoretical approach employed

- By exploiting the current, the two-point correlation function can be constructed, which has two representations: the QCD representation and the phenomenological representation.
- The QCDSR will be formally established after equating these two representations, from which the mass and decay width of hadron can be obtained

II. Theoretical approach employed

- The lowest order currents for light tetraquark states with $J^{PC}=1^{-+}$ in molecular configuration are found to be in forms:

$$
\begin{aligned}
j_\mu^A(x) &= i[\bar{s}_a(x)\gamma_5 s_a(x)][\bar{q}_b(x)\gamma^\mu\gamma_5 q_b(x)] \, , \\
j_\mu^B(x) &= i[\bar{s}_a(x)\gamma^\mu\gamma_5 s_a(x)][\bar{q}_b(x)\gamma_5 q_b(x)] \, , \\
j_\mu^C(x) &= \frac{i}{\sqrt{2}}\Big\{[\bar{s}_a(x)\gamma_5 q_a(x)][\bar{q}_b(x)\gamma^\mu\gamma_5 s_b(x)] \\
&+ [\bar{s}_a(x)\gamma^\mu\gamma_5 q_a(x)][\bar{q}_b(x)\gamma_5 s_b(x)]\Big\} \, , \\
j_\mu^D(x) &= \frac{1}{\sqrt{2}}\Big\{[\bar{s}_a(x) q_a(x)][\bar{q}_b(x)\gamma^\mu s_b(x)] \\
&- [\bar{s}_a(x)\gamma^\mu q_a(x)][\bar{q}_b(x) s_b(x)]\Big\} \, ,
\end{aligned}
$$

II. Theoretical approach employed

- For 0^{-+} trigluon glueball, the interpolating currents read:

$$
\begin{aligned}
j^{0^{-+},\,A}(x) &= g_s^3\, f^{abc}\, \tilde{G}^a_{\mu\nu}(x)\, G^b_{\nu\rho}(x)\, G^c_{\rho\mu}(x)\,, \\
j^{0^{-+},\,B}(x) &= g_s^3\, f^{abc}\, G^a_{\mu\nu}(x)\, \tilde{G}^b_{\nu\rho}(x)\, G^c_{\rho\mu}(x)\,, \\
j^{0^{-+},\,C}(x) &= g_s^3\, f^{abc}\, G^a_{\mu\nu}(x)\, G^b_{\nu\rho}(x)\, \tilde{G}^c_{\rho\mu}(x)\,, \\
j^{0^{-+},\,D}(x) &= g_s^3\, f^{abc}\, \tilde{G}^a_{\mu\nu}(x)\, \tilde{G}^b_{\nu\rho}(x)\, \tilde{G}^c_{\rho\mu}(x)\,.
\end{aligned}
$$

II. Theoretical approach employed

- For 2^{-+} trigluon glueball, the currents are:

$$
\begin{aligned}
j_{\mu\sigma}^{2^{-+},\,A}(x) &= g_s^3\, f^{abc}\, \tilde{G}^a_{\mu\nu}(x)\, G^b_{\nu\rho}(x)\, G^c_{\rho\sigma}(x)\,, \\
j_{\mu\sigma}^{2^{-+},\,B}(x) &= g_s^3\, f^{abc}\, G^a_{G^a_{\mu\nu}\,\nu}(x)\, \tilde{G}^b_{\nu\rho}(x)\, G^c_{\rho\sigma}(x)\,, \\
j_{\mu\sigma}^{2^{-+},\,C}(x) &= g_s^3\, f^{abc}\, G^a_{\mu\nu}(x)\, G^b_{\nu\rho}(x)\, \tilde{G}^c_{\rho\sigma}(x)\,. \\
j_{\mu\sigma}^{2^{-+},\,D}(x) &= g_s^3\, f^{abc}\, \tilde{G}^a_{\mu\nu}(x)\, \tilde{G}^b_{\nu\rho}(x)\, \tilde{G}^c_{\rho\sigma}(x)\,.
\end{aligned}
$$

Here, $G^a_{\mu\nu}$ represents the gluon field strength tensor

II. Theoretical approach employed

- Equipped with interpolating currents, the two-point correlation functions can be readily established, i.e.,

$$\Pi_{\mu\nu}(q^2) = i \int d^4x e^{iq\cdot x} \langle 0|T\{j_\mu(x),\ j_\nu^\dagger(0)\}|0\rangle \ ,$$

$$\Pi^k(q^2) = i \int d^4x e^{iq\cdot x} \langle 0|T\{j^k(x),\ j^k(0)\}|0\rangle \ ,$$

$$\Pi^k_{\mu\nu,\rho\sigma}(q^2) = i \int d^4x e^{iq\cdot x} \langle 0|T\{j^k_{\mu\nu}(x),\ j^k_{\rho\sigma}(0)\}|0\rangle.$$

For 1^{-+} light tetraquark state, 0^{-+} and 2^{-+} trigluon glueballs respectively

II. Theoretical approach employed

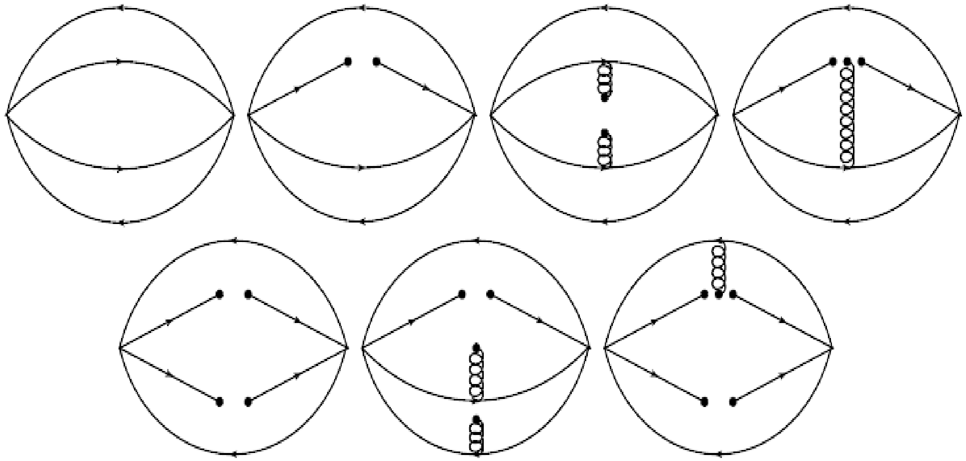

FIG. 1: The typical Feynman diagrams related to the correlation function, where the solid lines stand for the quarks and the sprial ones for gluons.

II. Theoretical approach employed

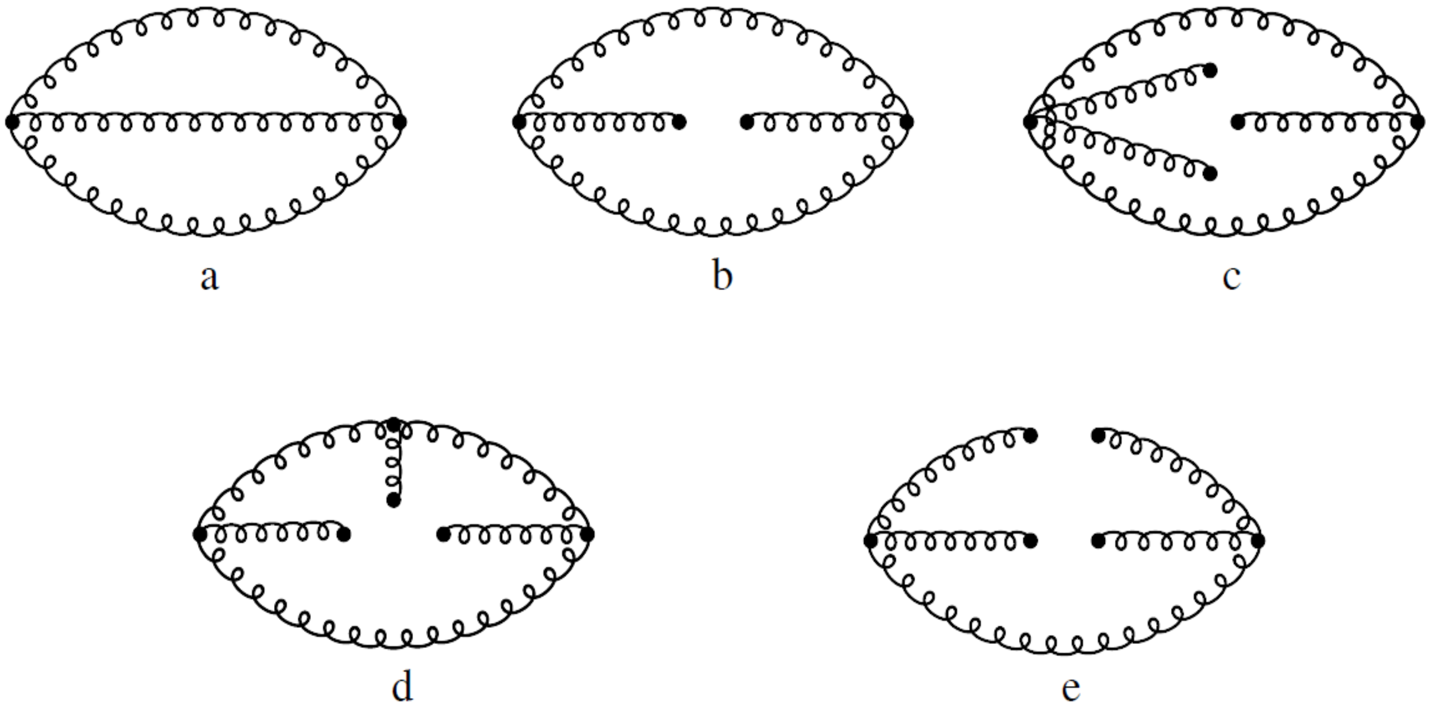

FIG. 2: The typical Feynman diagrams of trigluon glueball in the scheme of QCD sum rules. (a) Diagram for the perturbative term; (b) for the two-gluon condensate terms; (c) and (d) for the three-gluon condensate terms; (e) for the four-gluon condensate terms.

II. Theoretical approach employed

- The correlation function on the QCD representation can be obtained by the operator product expansion (OPE):

$$\Pi^{k,\,\mathrm{QCD}}_{JPC}(q^2) = \left(a_0 + a_1 \ln\frac{-q^2}{\mu^2}\right)(q^2)^4 + \left(b_0 + b_1 \ln\frac{-q^2}{\mu^2}\right)(q^2)^2\langle\alpha_s G^2\rangle + \left(c_0 + c_1 \ln\frac{-q^2}{\mu^2}\right)(q^2)\,\langle g_s^3 G^3\rangle + \left(d_0 + d_1 \ln\frac{-q^2}{\mu^2}\right)\langle\alpha_s G^2\rangle^2$$

for glueballs for instance

II. Theoretical approach employed

- On the phenomenological representation, adopting the pole plus continuum parametrization of the hadronic spectral density, the imaginary part of the correlation function can be written as

$$\frac{1}{\pi}\mathrm{Im}\,\Pi^{k,\,\mathrm{phe}}_{J^{PC}} = (\lambda^{k}_{J^{PC}})^2\,\delta\left(s-(M^{k}_{J^{PC}})^2\right)+\rho^{k,\,\mathrm{cont}}_{J^{PC}}(s)\theta(s-s_0)\ .$$

Here, $\lambda^{k}_{J^{PC}}$ is the coupling constant; $M^{k}_{J^{PC}}$ stands for the mass of J^{PC} trigluon glueball. $\rho^{k,\,\mathrm{cont}}_{J^{PC}}(s)$ represents the spectral density which contains continuous spectrum and high excited states above the continuum threshold $\sqrt{s_0}$.

II. Theoretical approach employed

➢ Using the dispersion relation

$$
\begin{aligned}
\Pi^k_{JPC}(q^2) &= \frac{1}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\Pi^k_{JPC}(s)}{s-q^2} + \Big(\Pi^k_{JPC}(0) + q^2\Pi^{k\,\prime}_{JPC}(0) \\
&+ \frac{1}{2}q^4\Pi^{k\,\prime\prime}_{JPC}(0) + \frac{1}{6}q^6\Pi^{k\,\prime\prime\prime}_{JPC}(0)\Big) \,,
\end{aligned}
$$

➢ one then gets the main function

$$
\begin{aligned}
& \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\Pi^{\mathrm{k,\,QCD}}_{\mathrm{JPC}}(s)}{s-q^2} ds \\
= & \frac{(f^k_{JPC})^2 (M^k_{JPC})^{2n}}{(M^k_{JPC})^2 - q^2} + \int_{s_0}^\infty \frac{\rho^k_{JPC}(s)\theta(s-s_0)}{s-q^2} ds \,.
\end{aligned}
$$

II. Theoretical approach employed

- Applying Borel transformation

$$\hat{B}_\tau \equiv \lim_{\substack{-q^2\to\infty, n\to\infty \\ \frac{-q^2}{n}=\frac{1}{\tau}}} \frac{(q^2)^n}{(n-1)!}\left(-\frac{d}{dq^2}\right)^n ,$$

- and the quark-hadron duality approximation

$$\frac{1}{\pi}\int_{s_0}^{\infty} e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{k,\,QCD}}_{\mathrm{JPC}}(s)ds \simeq \int_{s_0}^{\infty} \rho^k_{JPC}(s)e^{-s\tau}ds ,$$

II. Theoretical approach employed

- one then gets the moments

$$L^{k}_{JPC,\,0}(\tau, s_0) = \frac{1}{\pi}\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{k,\,QCD}}_{\mathrm{JPC}}(s)ds\,,$$
$$L^{k}_{JPC,\,1}(\tau, s_0) = \frac{1}{\pi}\int_0^{s_0} s e^{-s\tau}\mathrm{Im}\Pi^{\mathrm{k,\,QCD}}_{\mathrm{JPC}}(s)ds\,,$$

- and the mass function

$$M^{k}_{JPC}(\tau, s_0) = \sqrt{\frac{L^{k}_{JPC,\,1}(\tau, s_0)}{L^{k}_{JPC,\,0}(\tau, s_0)}}\,,$$

- Ratio to constrain τ & s_0 by the pole contribution (PC)

$$R^{k,\ \mathrm{PC}}_{J} = \frac{L^{k}_{JPC,\,0}(\tau, s_0)}{L^{k}_{JPC,\,0}(\tau, \infty)}\,.$$

II. Theoretical approach employed

- Ratio to constrain τ and s_0 by convergence of the OPE

$$R_J^{k,\,\mathrm{G}^2} = \frac{\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi_{\mathrm{JPC}}^{\mathrm{k},\,\langle\alpha_s \mathrm{G}^2\rangle}(s)ds}{\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi_{\mathrm{JPC}}^{\mathrm{k},\,\mathrm{QCD}}(s)ds}\,,$$

$$R_J^{k,\,\mathrm{G}^3} = \frac{\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi_{\mathrm{JPC}}^{\mathrm{k},\,\langle g_s \mathrm{G}^3\rangle}(s)ds}{\int_0^{s_0} e^{-s\tau}\mathrm{Im}\Pi_{\mathrm{JPC}}^{\mathrm{k},\,\mathrm{QCD}}(s)ds}\,.$$

- Input parameters

$\langle\bar{q}q\rangle = -(0.23\pm0.03)^3\ \mathrm{GeV}^3$, $\langle\bar{s}s\rangle = (0.8\pm0.1)\langle\bar{q}q\rangle$, $\langle\bar{q}g_s\sigma\cdot Gq\rangle = m_0^2\langle\bar{q}q\rangle$, $\langle\bar{s}g_s\sigma\cdot Gs\rangle = m_0^2\langle\bar{s}s\rangle$, $\langle g_s^2G^2\rangle = (0.88\pm0.25)\ \mathrm{GeV}^4$, and $m_0^2 = (0.8\pm0.2)\ \mathrm{GeV}^2$.

$$\langle\alpha_s G^2\rangle = 0.06\,\mathrm{GeV}^4\ ,\ \langle g_s G^3\rangle = (0.27\,\mathrm{GeV}^2)\langle\alpha_s G^2\rangle\ ,$$

$$\Lambda_{\overline{\mathrm{MS}}} = 300\,\mathrm{MeV}\ ,\ \alpha_s = \frac{-4\pi}{11\ln(\tau\Lambda_{\overline{\mathrm{MS}}}^2)}\ ,$$

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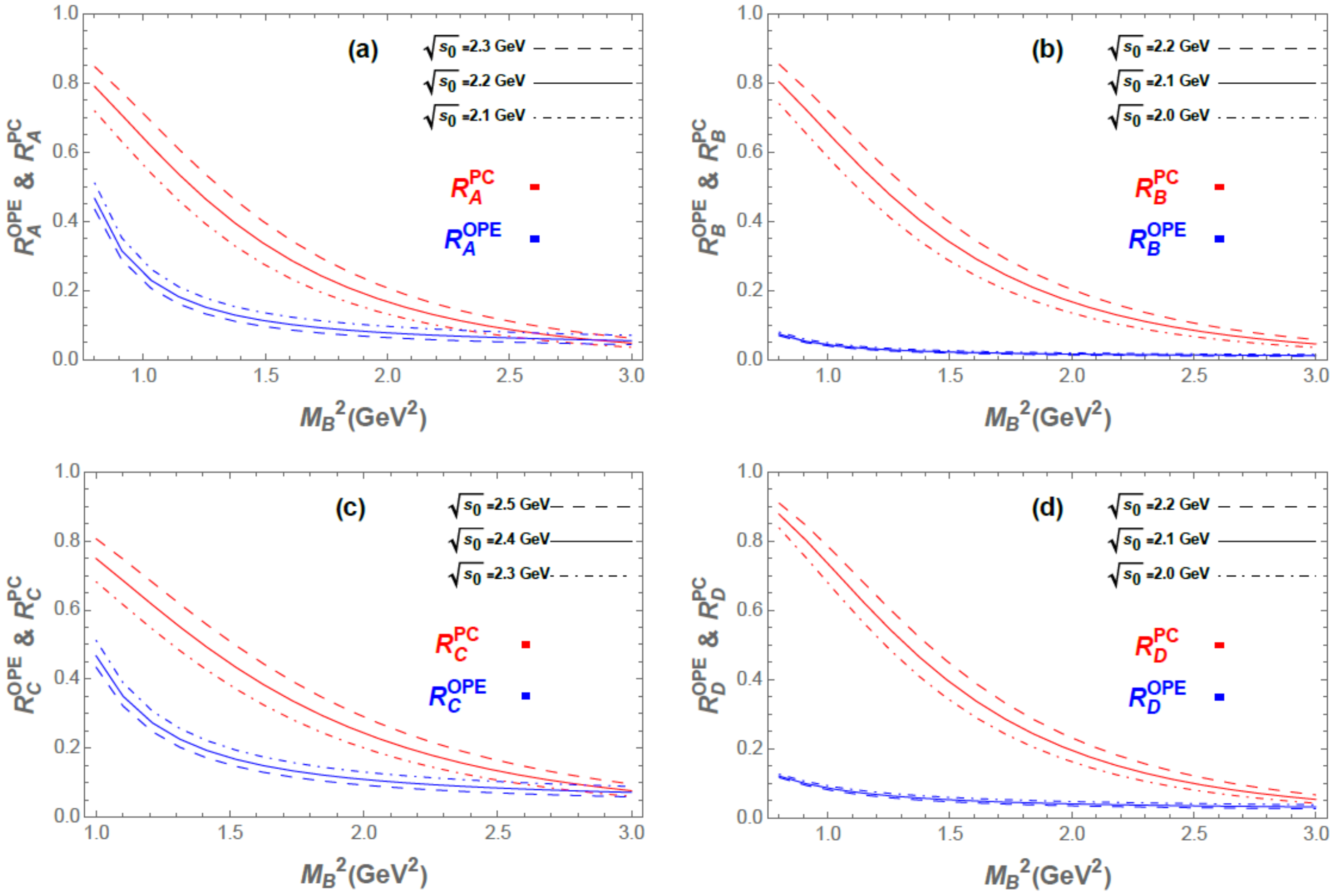
- Concluding remarks

III. Results

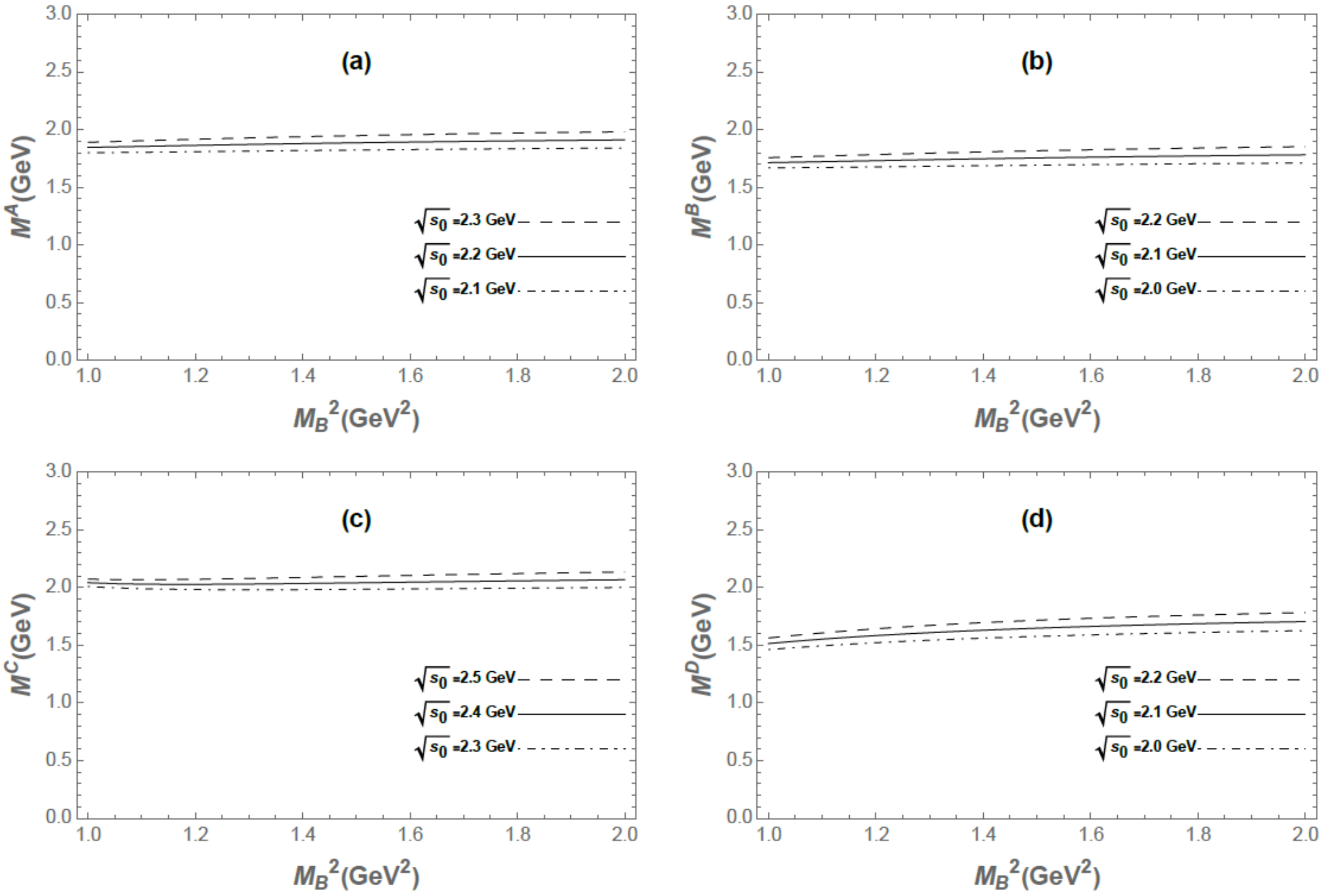
- After applying requirements for OPE convergence and pole contribution dominance, we can then find a proper value for continuum s_0 and Borel parameter M^2_B

- and the mass spectrum of aiming object

III. Results


(a)
$\sqrt{s_0}$ =2.3 GeV
$\sqrt{s_0}$ =2.2 GeV
$\sqrt{s_0}$ =2.1 GeV
R_A^{PC}
R_A^{OPE}
R_A^{OPE} & R_A^{PC}
$M_B^2(\mathrm{GeV}^2)$
(b)
$\sqrt{s_0}$ =2.2 GeV
$\sqrt{s_0}$ =2.1 GeV
$\sqrt{s_0}$ =2.0 GeV
R_B^{PC}
R_B^{OPE}
R_B^{OPE} & R_B^{PC}
$M_B^2(\mathrm{GeV}^2)$
(c)
$\sqrt{s_0}$ =2.5 GeV
$\sqrt{s_0}$ =2.4 GeV
$\sqrt{s_0}$ =2.3 GeV
R_C^{PC}
R_C^{OPE}
R_C^{OPE} & R_C^{PC}
$M_B^2(\mathrm{GeV}^2)$
(d)
$\sqrt{s_0}$ =2.2 GeV
$\sqrt{s_0}$ =2.1 GeV
$\sqrt{s_0}$ =2.0 GeV
R_D^{PC}
R_D^{OPE}
R_D^{OPE} & R_D^{PC}
$M_B^2(\mathrm{GeV}^2)$

III. Results


(a)
M^A(GeV)
M_B^2(GeV^2)
$\sqrt{s_0}$ =2.3 GeV
$\sqrt{s_0}$ =2.2 GeV
$\sqrt{s_0}$ =2.1 GeV
(b)
M^B(GeV)
M_B^2(GeV^2)
$\sqrt{s_0}$ =2.2 GeV
$\sqrt{s_0}$ =2.1 GeV
$\sqrt{s_0}$ =2.0 GeV
(c)
M^C(GeV)
M_B^2(GeV^2)
$\sqrt{s_0}$ =2.5 GeV
$\sqrt{s_0}$ =2.4 GeV
$\sqrt{s_0}$ =2.3 GeV
(d)
M^D(GeV)
M_B^2(GeV^2)
$\sqrt{s_0}$ =2.2 GeV
$\sqrt{s_0}$ =2.1 GeV
$\sqrt{s_0}$ =2.0 GeV

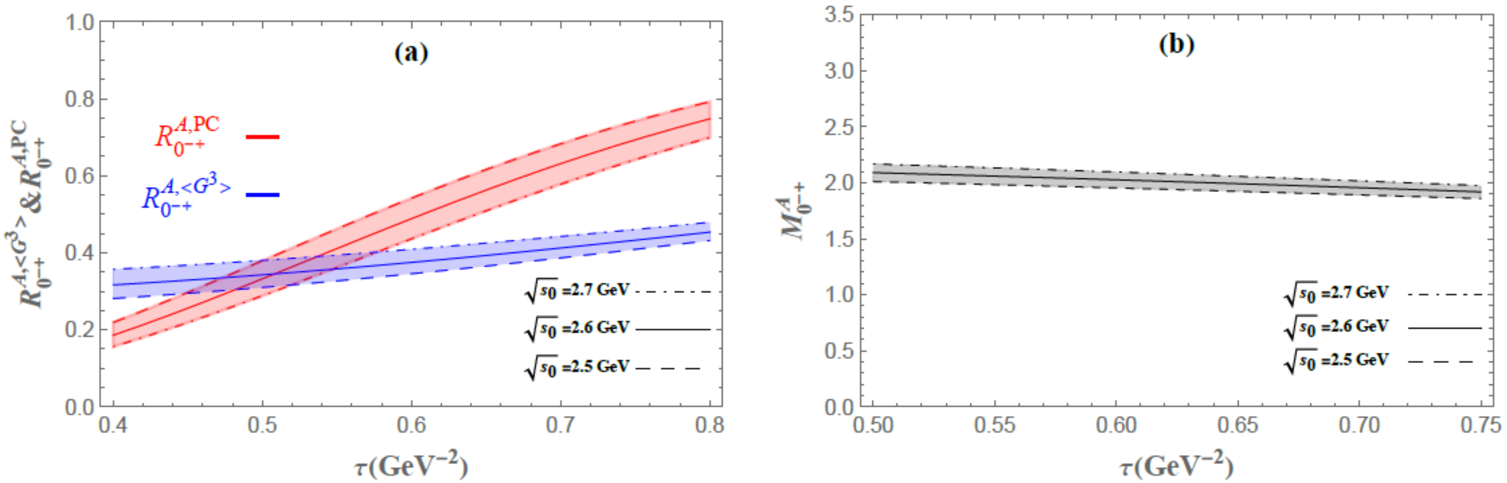
III. Results

TABLE I: The continuum thresholds, Borel parameters, and predicted masses of tetraquark states.

Configuration	Current	$\sqrt{s_0}$ (GeV)	M_B^2 (GeV^2)	M^X (GeV)
$[1_c]_{\bar{s}s} \otimes [1_c]_{\bar{q}q}$	A	2.2 ± 0.1	$1.1 - 1.6$	1.87 ± 0.08
	B	2.1 ± 0.1	$1.1 - 1.6$	1.75 ± 0.08
$[1_c]_{\bar{s}q} \otimes [1_c]_{\bar{q}s}$	C	2.4 ± 0.1	$1.3 - 1.8$	2.05 ± 0.07
	D	2.1 ± 0.1	$1.2 - 1.7$	1.63 ± 0.12
$[1_c]_{\bar{s}s} \otimes [1_c]_{\bar{s}s}$	A	2.5 ± 0.1	$1.3 - 1.9$	2.14 ± 0.07
	D	2.2 ± 0.1	$1.3 - 1.8$	1.71 ± 0.11

the tetraquark mass spectrum

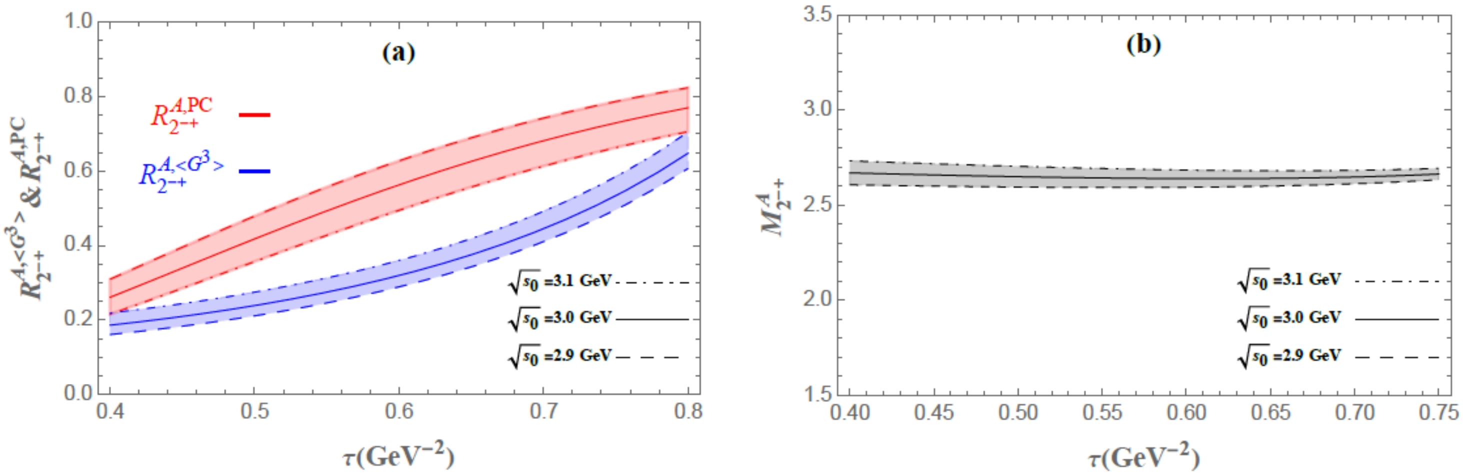
III. Results



$$M^{A,\,D}_{0^{-+}} = 2.01 \pm 0.14\,\mathrm{GeV}.$$

0^{-+} Glueball mass plots

III. Results



$$M^{A,B,D}_{2-+} = 2.66 \pm 0.06\,\mathrm{GeV}.$$

2^{-+}Glueball mass plots

Contents:

IV. Concluding remarks

- Our numerical results indicate that the $\eta_1(1855)$ is close in magnitude to the calculation of one of the feasible currents, which is to say $\eta_1(1855)$ at least has a large component in tetraquark configuration. We predict five more other tetraquark states in different configurations
- For the tetraquark state, favorable decay modes include to S-wave $\eta\ f_1(1285)$ and P-wave $\eta\ \eta'$, $\eta\ \eta(1295)$
- To discriminate from hybrid, the tetraquark in configuration $[1_c]_{\bar{s}s} \otimes [1_c]_{\bar{q}q}$ is relatively tamed to decay to $K_1(1270)\bar{K}$, while there has no hurdle for a hybrid to decay in this mode

IV. Concluding remarks

- As to X(2600), our calculation indicates that its mass is close to the 2^{-+} trigluon glueball

- To further confirm the nature of X(2600), more measurements on its various decay modes are necessary

- In general, decay modes of three mesons or baryon-antibaryon pair for two-gluon glueballs are suppressed, while trigluon glueballs exclusively decay in these channels more straightforwardly.

IV. Concluding remarks

- Various decay modes of 0^{-+} and 2^{-+} trigluon glueballs

Cases	Possible decay channels	
0^{-+} two-gluon glueball $\to$	$a_0(980)+\pi$	$2f_0(500)$
	$\{f_0(500),\, f_0(980)\}+\eta$	
0^{-+} trigluon glueball $\to$	$\{f_0(500),\, f_0(980),\, f_0(1370),\, f_0(1500)\}+\eta$	$2f_0(500),\, 2f_0(980),\, 2a_0(980)$
	$f_0(500)+f_0(980)+\eta$	$\{\omega\omega,\, \rho\rho\}+f_0(500)$
	$f_0(500)+f_0(500)+\{\eta,\, \eta'\}$	$N\bar{N}$
	$\{f_0(500),\, f_0(980)\}+a_0(980)+\pi$	$\eta\eta\eta,\, \eta\eta\eta',\, \{\eta,\, \eta'\}+\pi+\pi$
2^{-+} two-gluon glueball $\to$	$a_2(1320)+\pi$	$f_0(500)+f_1(1285)$
	$f_2(1270)+\eta$	
2^{-+} trigluon glueball $\to$	$\eta_2(1645)+f_0(500)$	$2f_1(1285),\, 2a_1(1260),\, 2h_1(1170)$
	$\{f_2(1270),\, f_2'(1525)\}+\{\eta,\, \eta'\}$	$\rho+\rho+f_0(980)$
	$a_2(1320)+f_0(500)+\pi$	$\{\omega\omega,\, \rho\rho,\, \omega+\phi\}+f_0(500)$
	$\{f_2(1270),\, f_2'(1525)\}+f_0(500)+\eta$	$h_1(1170)+\omega+\eta$
	$\{f_2(1270),\, f_2'(1525)\}+a_0(980)+\pi$	$\{h_1(1170),\, h_1(1415)\}+\rho+\pi$
	$\omega+\phi+\eta,\, \{\pi\pi,\, \omega\omega,\, \rho\rho\}+\{\eta,\, \eta'\}$	$N\bar{N},\, \Lambda\bar{\Lambda},\, \Sigma\bar{\Sigma},\, \Xi\bar{\Xi}$

THANKS