Anomalous currents in hadronic fluids

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Other references: K.Landsteiner, EM, F.Pena-Benitez, PRL107('11); Lect. Notes Phys. 81 ('13); EM, M.Valle, JHEP1411 ('14).

Issues

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   - Equilibrium Partition Function
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   - The Wess-Zumino-Witten partition function

Issues

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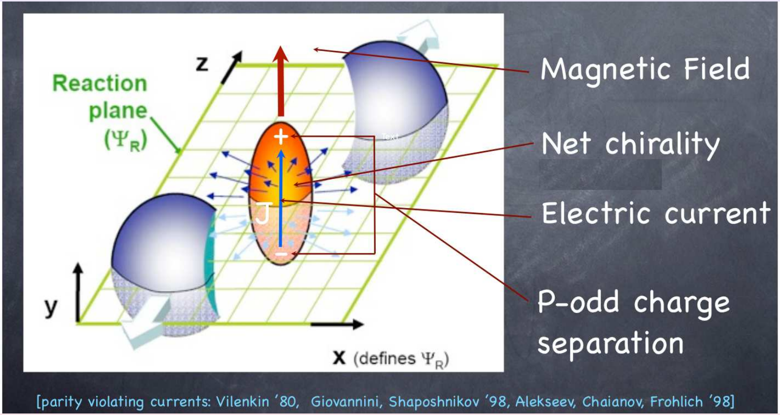
The Chiral Magnetic Effect (CME)

[Kharzeev, McLerran, Warringa '07]



Strong Magnetic field induces a $\mathcal{P}$-odd charge separation $\Longrightarrow$

$\Longrightarrow$ Electric current: $\vec{J} = \sigma^{\mathcal{B}} \vec{B}$.

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Hydrodynamics of Relativistic Fluids

[Son,Surowka], [Eling,Neiman,Oz], [Erdmenger et al.], [Banerjee et al.], [Loganayagam], [Kharzeev, Yee], [Sadovyev et al.], [Landsteiner, EM, Pena-Benitez], . . .

$$\langle T^{\mu\nu}\rangle = \underbrace{(\varepsilon+\mathcal{P})u^\mu u^\nu + \mathcal{P}g^{\mu\nu}}_{\text{Ideal Hydro}} + \underbrace{\langle T^{\mu\nu}\rangle_{\text{diss \& anom}}}_{\text{Dissipative \& Anomalous}} ,$$

$$\langle J^\mu\rangle = \underbrace{nu^\mu}_{\text{Ideal Hydro}} + \underbrace{\langle J^\mu\rangle_{\text{diss \& anom}}}_{\text{Dissipative \& Anomalous}} .$$

- Landau frame: $\langle T^{0i}\rangle \sim u^i$

$$\langle T^{\mu\nu}\rangle_{\text{diss \& anom}} = -\eta P^{\mu\alpha}P^{\nu\beta}\left(D_\alpha u_\beta + D_\beta u_\alpha - \frac{2}{3}g_{\alpha\beta}D^\lambda u_\lambda\right) - \zeta P^{\mu\nu}D^\alpha u_\alpha + \cdots$$

$$\langle J^\mu\rangle_{\text{diss \& anom}} = -\sigma T P^{\mu\nu}D_\nu\left(\frac{\mu}{T}\right) + \sigma E^\mu + \underbrace{\sigma^{\mathcal{B}}B^\mu}_{\text{CME}} + \underbrace{\sigma^{\mathcal{V}}\omega^\mu}_{\text{CVE}} + \cdots$$

where $P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$, and
vorticity: $\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\lambda}u_\nu D_\rho u_\lambda$ ➞ Chiral Vortical Effect (CVE).

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Equilibrium Partition Function

[Banerjee et al '12], [Jensen et al '13], [Bhattacharyya '14], [EM, Valle '14]

- Relativistic Invariant Quantum Field Theory on the manifold

$$ds^2 = -e^{2\sigma(\vec{x})}(dt + a_i(\vec{x})dx^i)^2 + g_{ij}(\vec{x})dx^i dx^j$$

  and time independent background $U(1)$ gauge connection:

$$\mathcal{A} = \mathcal{A}_0(\vec{x})dx^0 + \mathcal{A}_i(\vec{x})dx^i \,.$$

- Partition function of the system:

$$Z = \mathrm{Tr}\, e^{-\frac{H-\mu_0 Q}{T_0}}$$

  → Dependence of Z on σ, g_{ij}, a_i and $\mathcal{A}_\mu$?

- Most general partition function consistent with:
  - 3-dim diffeomorphism invariance.
  - Kaluza-Klein (KK) invariance: $t \to t + \phi(\vec{x})\,,\ \vec{x} \to \vec{x}$.
  - $U(1)$ *time-independent* gauge invariance (up to an anomaly).

Equilibrium Partition Function

- Stress Tensor and $U(1)$ current $\longrightarrow$ under t-indep variations

$$\delta \log Z = \frac{1}{T_0}\int d^3x\sqrt{g_3}\left(-\frac{1}{2}T_{\mu\nu}\delta g^{\mu\nu} + J^\mu \delta\mathcal{A}_\mu\right)$$

$\longrightarrow$ $T_{\mu\nu} = -2T_0\frac{\delta \log Z}{\delta g^{\mu\nu}}\,, \qquad J^\mu = T_0\frac{\delta\log Z}{\delta\mathcal{A}_\mu}.$

- In particular, for $\log Z = \mathcal{W}(e^\sigma, A_0, a_i, A_i, g^{ij}, T_0, \mu_0)$, where

$$A_0 = \mathcal{A}_0\,, \qquad A_i = \mathcal{A}_i - a_i\mathcal{A}_0\,,$$

are KK invariant quantities, one gets the consistent currents

$$\boxed{\begin{aligned} \langle J^i\rangle_{\rm cons} &= \frac{T_0}{\sqrt{-G}}\frac{\delta\mathcal{W}}{\delta A_i}\,, & \langle J_0\rangle_{\rm cons} &= -\frac{T_0 e^{2\sigma}}{\sqrt{-G}}\frac{\delta\mathcal{W}}{\delta A_0}\,, \\ \langle T_0{}^i\rangle &= \frac{T_0}{\sqrt{-G}}\left(\frac{\delta\mathcal{W}}{\delta a_i} - A_0\frac{\delta\mathcal{W}}{\delta A_i}\right)\,, & \langle T_{00}\rangle &= -\frac{T_0 e^{2\sigma}}{\sqrt{-G}}\frac{\delta\mathcal{W}}{\delta\sigma}\,. \end{aligned}}$$

- $\longrightarrow$ $\mathcal{W}$ is a generating functional for the hydrodynamic constitutive relations.

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Equilibrium Partition Function at first order

- Partition function at 1st order in derivative expansion **[Banerjee et al '12; EM, M.Valle '14]**:

$$\boxed{\mathcal{W}^{(1)}=\int d^3x\sqrt{g_3}\left[\alpha_1(\sigma,A_0)\epsilon^{ijk}A_iA_{jk}+\alpha_2(\sigma,A_0)\epsilon^{ijk}A_if_{jk}+\alpha_3(\sigma,A_0)\epsilon^{ijk}a_if_{jk}\right]}$$

where $A_{ij}=\partial_iA_j-\partial_jA_i\,,\ f_{ij}=\partial_ia_j-\partial_ja_i$.

- Ideal gas of Dirac fermions $\longrightarrow$ from a computation of $\langle {T_0}^i\rangle$ and $\langle J^i\rangle$, one gets

$$\alpha_1(\sigma,A_0)=-\frac{C}{6T_0}A_0\,,\quad \alpha_2(\sigma,A_0)=-\frac{1}{2}\left(\frac{C}{6T_0}A_0^2-C_2T_0\right)\,,\quad \alpha_3(\sigma,A_0)=0\,.$$

where: $\begin{cases} C=\frac{1}{4\pi^2} & \textit{(chiral anomaly)}\text{: [Son, Surowka '09], [Erdmenger et al '09], ...} \\ C_2=\frac{1}{24} & \textit{(gauge-gravitational anomaly)}\text{: [Landsteiner, EM, Pena-Benitez '11]}\end{cases}$

- Coefficients related to chiral magnetic and vortical conductivities:

$$\sigma^{\mathcal{B}}=C\mu\,,\qquad \sigma^{\mathcal{V}}=\frac{1}{2}C\mu^2+C_2T^2\,,\qquad [\mu=e^{-\sigma}A_0,\ T=e^{-\sigma}T_0]\,.$$

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The chiral anomaly

- Theory of a chiral fermion:

$$\mathcal{L}_{\mathrm{YM}} = i\overline{\psi}\gamma^{\mu}(\partial_{\mu} - it_a\mathcal{A}^a_{\mu})\psi\,.$$

- Under a general shift $\mathcal{A}^a_{\mu} \to \mathcal{A}^a_{\mu} + \delta\mathcal{A}^a_{\mu}$:

$$\delta\Gamma[\mathcal{A}] = \int d^Dx\, \delta\mathcal{A}^a_{\mu}\, J^{\mu}_a(x)_{\mathrm{cons}}\,. \qquad (1)$$

- The chiral anomaly is given by the *"failure of the effective action to be invariant under axial gauge transformations"*:

$$\mathcal{A}_{\mu} \longrightarrow U\mathcal{A}_{\mu}U^{-1} - i\partial_{\mu}U\,U^{-1}\,, \qquad U = \exp\left(i\Lambda^{\mathrm{Axial}}_a t_a\right)\,,$$

$$\delta_{\mathrm{gauge}}\Gamma[\mathcal{A}] = -\int d^Dx\, \Lambda^{\mathrm{Axial},\,a}\, G_a[\mathcal{A}]\,.$$

- Particularizing **(1)** to $\delta\mathcal{A}^a_{\mu} = (D_{\mu}\Lambda^{\mathrm{Axial}})^a$ ➞
  ➞ (non)-conservation law for the consistent current:

$$\boxed{D_{\mu}J^{\mu}_a(x)_{\mathrm{cons}} = G_a[\mathcal{A}(x)]\,.}$$

The non-abelian anomaly

- Bardeen form of the non-abelian anomaly for the symmetry group $\mathrm{U}(N_f)_L \times \mathrm{U}(N_f)_R$ **[W.A. Bardeen, PR184, '69]**:

$$\mathcal{L}_{\rm YM} = i\overline{\psi}_L\gamma^\mu(\partial_\mu - it_a\mathcal{A}^a_{L\,\mu})\psi_L + i\overline{\psi}_R\gamma^\mu(\partial_\mu - it_a\mathcal{A}^a_{R\,\mu})\psi_R\,.$$

$$\boxed{\begin{aligned} G_a[\mathcal{V},\mathcal{A}] = \frac{iN_c}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\Big\{t_a\Big[&\mathcal{V}_{\mu\nu}\mathcal{V}_{\rho\sigma} + \tfrac{1}{3}\mathcal{A}_{\mu\nu}\mathcal{A}_{\rho\sigma} - \tfrac{32}{3}\mathcal{A}_\mu\mathcal{A}_\nu\mathcal{A}_\rho\mathcal{A}_\sigma \\ &+ \tfrac{8}{3}i(\mathcal{A}_\mu\mathcal{A}_\nu\mathcal{V}_{\rho\sigma} + \mathcal{A}_\mu\mathcal{V}_{\rho\sigma}\mathcal{A}_\nu + \mathcal{V}_{\rho\sigma}\mathcal{A}_\mu\mathcal{A}_\nu)\Big]\Big\}\,. \end{aligned}}$$

where

$$\begin{aligned} \mathcal{V}_{\mu\nu} &= \partial_\mu\mathcal{V}_\nu - \partial_\nu\mathcal{V}_\mu - i[\mathcal{V}_\mu,\mathcal{V}_\nu] - i[\mathcal{A}_\mu,\mathcal{A}_\nu]\,, \\ \mathcal{A}_{\mu\nu} &= \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu - i[\mathcal{V}_\mu,\mathcal{A}_\nu] - i[\mathcal{A}_\mu,\mathcal{V}_\nu]\,. \end{aligned}$$

- This includes *triangle*, *square* and *pentagon* one-loop diagrams.
- The anomaly arises from the breaking of gauge invariance under *'axial'* gauge transformations of the effective action $\Gamma_0[\mathcal{V},\mathcal{A}]$:

$$\mathscr{V}_a(x)\Gamma_0[\mathcal{V},\mathcal{A}] = 0\,, \qquad \mathscr{A}_a(x)\Gamma_0[\mathcal{V},\mathcal{A}] = G_a[\mathcal{V},\mathcal{A}]\,. \qquad (2)$$

$(\mathscr{V}_a(x),\mathscr{A}_a(x)) \equiv$ Local generator of (vector, axial) transform.

- Computation of $\Gamma_0[\mathcal{V},\mathcal{A}]$ from a solution of **(2)** by trial and error.

Issues

The anomalous functional

- Solution of the local functional $\Gamma_0[V, A, G]$ **[Mañes,EM,Valle,MAVZ'18]**:

$$\Gamma_0[V,A,G] = -\frac{N_c}{32\pi^2}\int dt\, d^3x \sqrt{g_3}\, \epsilon^{ijk}\, \mathrm{Tr}\Big\{ \frac{32}{3} i\, V_0 A_i A_j A_k + \frac{4}{3}(A_0 A_i + A_i A_0) A_{jk} + 4(V_0 A_i + A_i V_0) V_{jk} + \frac{8}{3}(A_0^2 + 3V_0^2) A_i \partial_j a_k \Big\} + C_2 T_0^2 \int dt\, d^3x \sqrt{g_3}\, \epsilon^{ijk} \mathrm{Tr} A_i\, \partial_j a_k$$

- V_μ and A_μ are KK inv fields: $A_0 = \mathcal{A}_0$, $A_i = \mathcal{A}_i - a_i \mathcal{A}_0$, etc.
- Γ_0 can be determined also from differential geometry methods: Chern-Simons effective action $\longrightarrow$ dimensional reduction **[Jensen, Laganayagam, Yarom '14; Mañes, EM, Valle, MAVZ '18 '19].**
- C_2 related to the mixed gauge-gravitational anomaly $\sim \mathrm{Tr}\{t_a\}\, \epsilon^{\mu\nu\rho\sigma} R_{\mu\nu\lambda\kappa} R_{\rho\sigma}{}^{\lambda\kappa}$ $\longrightarrow$ One should take into account the Riemann tensor contribution in the anomaly polynomial $\sim \mathrm{Tr}\, \mathcal{F}_A R^\mu{}_\nu R^\nu{}_\mu$ **[Nair,Ray,Roy PRD86 '12].**

Currents in equilibrium and constitutive relations

- The covariant currents are defined by adding to the consistent currents the Bardeen-Zumino (BZ) polynomials:

$$J^\mu_{\rm cov} = J^\mu_{\rm cons} + J^\mu_{\rm BZ} \,,$$

$$\text{with} \quad J^\mu_{\rm BZ\,V} = -\frac{N_c}{8\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\Big\{t_a\big(\mathcal{A}_\nu\mathcal{V}_{\rho\sigma} + \mathcal{V}_{\rho\sigma}\mathcal{A}_\nu + \tfrac{8}{3}i\,\mathcal{A}_\nu\mathcal{A}_\rho\mathcal{A}_\sigma\big)\Big\},$$

$$J^\mu_{\rm BZ\,A} = -\frac{N_c}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\Big\{t_a\,(\mathcal{A}_\nu\mathcal{A}_{\rho\sigma} + \mathcal{A}_{\rho\sigma}\mathcal{A}_\nu)\Big\}.$$

- Background for $(N_f = 3)$:

$$V_\mu(\vec{x}) = V_{0\,\mu}(\vec{x})t_0 + V_{3\,\mu}(\vec{x})t_3 + V_{8\,\mu}(\vec{x})t_8 \,, \qquad t_0 = \frac{1}{\sqrt{6}}1_{2\times 2}\,, \;\; t_a = \frac{1}{2}\lambda_a$$

$$A_0 = A_{5\,0}\,t_0\,, \qquad A_i = 0\,, \qquad \mathcal{A}_i = a_i A_{5\,0}\,t_0\,.$$

- Equilibrium velocity field $u_\mu = -e^\sigma(1, a_i)$.
- Equil. baryonic, isospin, strangeness and axial chemical pot.:

$$\mu_0 = \mathcal{V}_{0\,0}\,e^{-\sigma}\,, \quad \mu_3 = \mathcal{V}_{3\,0}\,e^{-\sigma}\,, \quad \mu_8 = \mathcal{V}_{8\,0}\,e^{-\sigma}\,, \quad \mu_5 = \mathcal{A}_{5\,0}\,e^{-\sigma}\,.$$

μ_5 controls chiral imbalance **[Gatto, Ruggieri '12; Espriu et al, '13].**

Constitutive relations ($N_f = 3$)

- Non-abelian magnetic field:

$$\mathcal{B}^\mu_a = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu \mathcal{V}_{\alpha\beta\, a}\,, \quad \Longrightarrow \quad \langle J^\mu_{a\,\mathrm{V}}\rangle_{\mathrm{cov}} = \frac{N_c}{4\sqrt{6}\pi^2}\mu_5 \mathcal{B}^\mu_a\,, \ (a = 0, 3, 8)\,.$$

- Change of basis $(t_0, t_3, t_8) \to (B, Q, S)$:

$$V_\mu(\vec{x}) = \sum_{a=0,3,8} V_{a\,\mu}(\vec{x})t_a = V_{B\,\mu}(\vec{x})B + V_{3\,\mu}(\vec{x})Q + V_{S\,\mu}(\vec{x})S\,; \quad A_0 = A_{5\,0}t_0 = A_B B\,.$$

- The electromagnetic field $\mathbb{V}_\mu$ is the only (physical) propagating gauge field. Assuming also $\mu_B = \mu_S = 0$:

$$\left.\begin{array}{l} V_{3\,\mu} \equiv e\mathbb{V}_\mu \\ V_{B\,\mu} = 0 \\ V_{S\,\mu} = 0 \end{array}\right\} \quad \Longrightarrow \quad \left.\begin{array}{l} V_{0\,\mu} = 0 \\ V_{3\,\mu} = \sqrt{3}V_{8\,\mu} = e\mathbb{V}_\mu \\ \mathcal{B}^\mu_0 = 0\,, \quad \mathcal{B}^\mu_3 = \sqrt{3}\mathcal{B}^\mu_8 = e\mathcal{B}^\mu \end{array}\right\}$$

- Constitutive relation for the electromagnetic current:

$$\langle J^\mu_{\mathrm{em}}\rangle_{\mathrm{cov}} = \frac{e^2 N_c}{3\sqrt{6}\pi^2}\mu_5 \mathcal{B}^\mu \longrightarrow \nexists\ \text{CVE in vector current for } \mathrm{U}(3)_\mathrm{V}\times\mathrm{U}(3)_A\,.$$

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Wess-Zumino-Witten partition function

[Kaiser '01], [Son, Stephanov '08], [Fukushima, Mameda '12], [Brauner, Kadam '17], . . .

- Effects of the anomaly when symmetry is spontaneously broken.
- The WZW action describes the interactions of the Goldstone bosons among them and with the background fields.
- Application to QCD in the confined phase ⟶ Hadronic fluids ⟶ Fluid of pions, kaons, . . . interacting with external EM fields.
- WZW action:

$$\boxed{\Gamma^{WZW}[\xi, A] = \Gamma_0[A] - \Gamma_0[A_{-\xi}]}$$

  $\Gamma_0 \equiv$ anomalous functional in absence of symmetry breaking (computed above).
  $A_{-\xi} \equiv$ gauge field transformed with gauge parameters $\Lambda_a = -\xi_a$.
  [Wess, Zumino '71; Witten '83; Mañes '85; Chu, Ho, Zumino '96].
- $\xi_a \equiv$ Goldstone bosons

$$U(\xi) = \exp\left(2i\sum_a \xi_a t_a\right) .$$

Goldstone bosons

- Spontaneous breaking of axial symmetry:
$$\mathrm{U}(2)_L \times \mathrm{U}(2)_R \to \mathrm{U}(2)_V .$$
- The matrix of Goldstone bosons (GB)
$$U(\xi) = \exp\left(2i \sum_{a=1}^{3} \xi_a t_a\right) ,$$
includes three GBs from the broken $\mathrm{SU}(2)_A$ symmetry. The fourth GB ξ_0 is absent, as the $\mathrm{U}(1)_A$ sym is violated by non-pert effects.
- In terms of conventionally normalized GB fields $\pi^0, \pi^\pm$:
$$2\sum_{a=1}^{3} \xi_a t_a = \frac{\sqrt{2}}{f_\pi} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 & \pi^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 \end{pmatrix} ,$$
where $f_\pi \approx 92\,\mathrm{MeV}$ is the pion decay constant.
- Lagrangian to zeroth order
$$\mathcal{L} = -\frac{f_\pi^2}{4} G^{\mu\nu} \mathrm{Tr}\left\{ D_\mu U (D_\nu U)^\dagger \right\} , \qquad D_\mu U = \partial_\mu U - i[\mathcal{V}_{3\,\mu} t_3, U] .$$

Wess-Zumino-Witten partition function

- Spontaneous breaking of axial symmetry:
$$\mathrm{U}(2)_L \times \mathrm{U}(2)_R \to \mathrm{U}(2)_V.$$
Suitable gauge transformation is an axial transformation with parameters $\Lambda_a^{\mathrm{Axial}}(x) = -\xi_a(x)$.
- WZW partition function:

$$T_0 \mathcal{W}^{WZW} = \frac{N_c}{8\pi^2} \int d^3x \sqrt{g}\, V_{0\,0}\, \epsilon^{ijk} \left[-\frac{1}{2} \mathrm{Tr}\{\partial_i (R_j + L_j) t_3\} V_{3\,k} + \frac{1}{6} i\, \mathrm{Tr}\{L_i L_j L_k\} \right]$$
$$+ \frac{N_c}{16\pi^2} \int d^3x \sqrt{g}\, \epsilon^{ijk} \mathrm{Tr}\{(R_i + L_i) t_3\}$$
$$\times \left(V_{0\,0}\, \partial_j V_{3\,k} + V_{3\,0}\, \partial_j V_{0\,k} + V_{0\,0} V_{3\,0}\, \partial_j a_k \right)$$
$$+ \frac{N_c}{48\pi^2} A_{5\,0} \int d^3x \sqrt{g}\, \epsilon^{ijk} \left[\mathrm{Tr}\{(L_i - R_i) t_3\} + 2\mathrm{Tr}\{t_3^2 - U t_3 U^{-1} t_3\} V_{3\,i} \right]$$
$$\times \left(\partial_j V_{3\,k} + V_{3\,0} \partial_j a_k \right)$$

where $L_j = i\partial_j U\, U^{-1}, \quad R_j = iU^{-1}\partial_j U.$

Constitutive relations at first order in derivatives

- We define:

$$\mathcal{E}_{\mu(a)} \equiv \mathcal{V}_{\mu\nu\, a} u^\nu = T\partial_\mu \left(\mu_a/T\right) \qquad \text{(non-abelian electric field)} .$$

- Constitutive relations are expressed in terms of physical quantities of the fluid: fluid velocity, external fields, etc.
  ➞ No dependence in a_i.
- Chiral electric effect (CEE) in vector current **[Neiman,Oz, JHEP09 '11]**

$$\langle \delta J^i_{\rm em}\rangle_{\rm cov} = \frac{e^2 N_c}{12\pi^2}\left[\frac{1}{f_\pi}\epsilon^{ijk}\partial_j\pi^0\mathcal{E}_k + \frac{5}{3}\mu_5\mathcal{B}^i + \frac{2}{f_\pi^2}\mu\mu_5\pi^+\pi^-\omega^i\right] + \cdots ,$$

- Chiral electric effect in axial currents:

$$\langle \delta J^i_{0\,\rm A}\rangle_{\rm cov} = \frac{N_c}{24\pi^2 f_\pi^2}\left[ie\epsilon^{ijk}(\pi^-\partial_j\pi^+ - \pi^+\partial_j\pi^-)\mathcal{E}_k + 2e\mu\pi^+\pi^-\mathcal{B}^i - 2\mu_5^2\omega^i\right] + \cdots$$

- Explicit values for the transport coefficients of the CEE derived from an equilibrium partition function ➞ CEE is *time-reversal even* by construction ➞ CEE cannot lead to entropy production ➞ CEE is non-dissipative.

Conclusions

- We have studied non-dissipative transport effects up to 1st order in the hydrodynamic expansion in (non)abelian theories.
- Effects are induced by *external electromagnetic fields*, *vortices* and curvature in a relativistic fluid $\Longrightarrow$ Anomalous Transport.
- Equilibrium partition function method can only account for non-dissipative effects: *time reversal properties*.
- Dissipative effects (shear viscosity, electric conductivity, ...) ➞ other methods: Kubo formulae, Fluid/gravity correspondence, ...
- Anomalous effects in presence of Spontaneous Breaking of non-abelian gauge symmetries:
  - Interactions of Goldstone bosons among them and with external electromagnetic fields.
  - Application to QCD in confined phase: Fluid of pions, kaons, etc.
- Future directions:
  - Application to other sectors of the SM ➞ ElectroWeak sector.
  - Application to condensed matter systems ➞ Superfluids, Weyl semi-metals **[Basar, Kharzeev, Yee '14], [Landsteiner '14].**

Thank You!