CONFINEMENT IN *QCD*:NOVELTIES.

Adriano Di Giacomo

Pisa University and INFN, Italy.

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PLAN OF THE TALK

A completion of the paper A. Di Giacomo JHEP **02** 2021 208-232 .

- CONFINEMENT OF COLOR
- CONDENSATION OF MONOPOLES
- U(1) GAUGE THEORY : A TOY - MODEL
- ORDER PARAMETER FOR CONFINEMENT IN QCD
- DISCUSSION

CONFINEMENT OF COLOR

QUARKS IN THE S.M.UNDERSTOOD AT SHORT DISTANCES. NOT CLEAR WHY THEY DO NOT SHOW UP AS FREE PARTICLES.

- $\frac{n_q}{n_p} \leq 10^{-27}$ EXPECT $\approx 10^{-12}$
- $\frac{\sigma_q}{\sigma_{TOT}} \leq 10^{-15}$ EXPECT $O(1)$

NATURAL EXPLANATION : CONFINEMENT $n_q = 0$, $\sigma_q = 0$ PROTECTED BY SOME SYMMETRY BUILT-IN IN QCD.

A FUNDAMENTAL PROBLEM IN FIELD THEORY AND IN PARTICLE PHYSICS.

A POSSIBLE CANDIDATE : DUAL SUPERCONDUCTIVITY OF THE GROUND STATE ['t Hooft , Mandelstam 75].

EXPLORE BY LATTICE SIMULATIONS.

CONDENSATION OF MONOPOLES-1

ORDINARY SUPERCONDUCTOR.
– ELECTRIC CHARGE CONSERVATION SPONTANEOUSLY BROKEN. GROUND STATE A SUPERPOSITION OF STATES WITH DIFFERENT NUMBER OF COOPER PAIRS.
C CREATOR OF COOPER PAIR: $< C >$ ORDER PARAMETER.
$< C >= 0$ NORMAL. $< C >\neq 0$ SUPERCONDUCTOR
-MAGNETIC FIELD SQUEEZED INTO ABRIKOSOV FLUX TUBES WITH CONSTANT ENERGY / UNIT LENGTH.
$\longrightarrow$ MAGNETIC CHARGES CONFINED

- DUAL SUPERCONDUCTOR.

μ CREATION OPERATOR OF A MONOPOLE. $\langle\mu\rangle \neq 0$ DUAL SUPERCONDUCTOR : $\longrightarrow$ ELECTRIC CHARGES CONFINED.
$\langle\mu\rangle = 0$ NORMAL:$\longrightarrow$NO CONFINEMENT.

CONDENSATION OF MONOPOLES -2

$$\exp(ipa)|q\rangle = |q+a\rangle$$

$$\mu(\vec{x},t) = \exp(i\tfrac{m}{g}\int d^3y \vec{A}_\perp^{Cl}(\vec{x}-\vec{y})\vec{E}_\perp(\vec{y},t))$$

$$\mu(\vec{x},t)|A_\mu(\vec{x},t)\rangle = |A_\mu(\vec{x},t) + \tfrac{m}{g}\vec{A}_\perp^{Cl}(\vec{x}-\vec{y})\rangle$$

$\frac{m}{g}$ THE MAGNETIC CHARGE OF THE MONOPOLE

$U(1)$ μ IS GAUGE INVARIANT.

$$\langle\mu\rangle = \frac{1}{Z(S)}\int[dU]\exp(-\beta S)\mu \qquad \beta = \frac{2N}{g^2}\ ,\ S \propto \vec{E}_L^2 + \vec{H}_L^2$$

$$\mu = \exp(-\beta\Delta S) \longrightarrow \langle\mu\rangle = \frac{Z(S+\Delta S)}{Z(S)}$$

$$\rho(\beta) \equiv \frac{\partial \ln(\langle\mu(\beta)\rangle)}{\partial\beta} = \langle S\rangle_S - \langle S+\Delta S\rangle_{S+\Delta S}$$

$$\langle\mu(\beta)\rangle = \exp(\int_0^\beta \rho(\beta')d\beta')$$

LATTICE VERSION

$$S = \Sigma_{n,\mu\nu} \Re[P_{\mu\nu}(n) - 1]) \qquad P_{\mu\nu}(n) \qquad \text{PLAQUETTE}$$

$$P_{\mu\nu}(n) = \frac{1}{N} Tr[U_\mu(n) U_\nu(n+\hat{\mu}) U^\dagger_\mu(n+\hat{\nu}) U^\dagger_\nu(n)]$$

$$S + \Delta S = \Sigma_{n,\mu\nu} \Re[P'_{\mu\nu}(n) - 1]$$

$P'_{\mu\nu}(n) = P_{\mu\nu}(n)$ FOR ALL n, μ, ν EXCEPT

$$P'_{i0}(\vec{n}, t) = \frac{1}{N} Tr[U_i(\vec{n}, t) U_0(\vec{n} + \hat{i}) M_i(\vec{n} + \hat{i}) U^\dagger_i(\vec{n}, t+1) U^\dagger_0(\vec{n}, t)]$$

$$M_i(\vec{m}) = \exp(ig T_3 A^{Cl}_{\perp\,i}(\vec{m} - \vec{x}))$$

T_3 GENERATOR OF THE $SU(2)$ SUBGROUP WHERE MONOPOLE LIVES. FOR $U(1)$ G.T. $T_3 \to 1$.

- NUMERICALLY COMPUTE $\rho = \langle S \rangle_S - \langle S + \Delta S \rangle_{S+\Delta S}$.
- CONTROL THE THERMODYNAMIC LIMIT $V \to \infty$
- STUDY ρ ANALYTICALLY [A.D.G. JHEP **02** 2021]
  $\rho = -\sum_0^\infty \frac{(-\beta)^n}{n!} \langle\langle \Delta S^{n+1} \rangle\rangle - \langle\langle S \sum_1^\infty \frac{(-\beta)^n}{n!} \Delta S^n \rangle\rangle$

$U(1)$ GAUGE THEORY

- ANALYTIC PROOF THAT $\langle\bar{\mu}\rangle \neq 0 \quad \beta \leq \bar{\beta}_c$. [NO ELECTRIC CHARGE !][FROLICH MARCHETTI 87, CIRIGLIANO PAFFUTI 99].
- NUMERICAL SIMULATIONS : CHARGES CONFINED $\beta \leq \beta_c$ [AREA LAW OF WILSON LOOPS] DECONFINED $\beta > \beta_c$.
- PROOF THAT $\mu = \bar{\mu}$. NUMERICAL COMPUTATION OF $\rho(\beta)$ and $\langle\mu\rangle \longrightarrow \beta_c = \bar{\beta}_c$. FIG.S [A.D.G., G.PAFFUTI 97].
- A TOY MODEL: $U(1)$ HAS NO NON-TRIVIAL FIXED POINT. NO FIELD THEORY.
- THERMODYNAMIC LIMIT $V \to \infty$? STUDY ρ ANALYTICALLY IN STRONG COUPLING REGIME [A.D.G. JHEP **02** 2021 208.]

ρ - $U(1)$ GAUGE THEORY

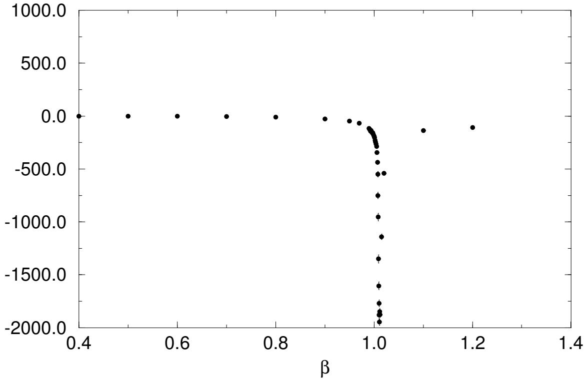

Figure: 1 ρ versus β . A.D.G. ,G.Paffuti P.R.D 56,6816 ,1997

$\langle\mu\rangle$ - $U(1)$ GAUGE THEORY

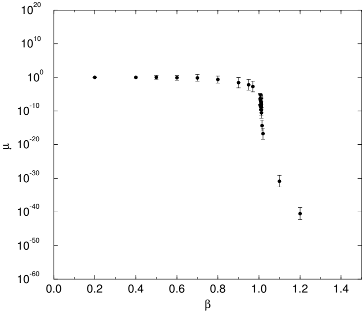

Figure: 2. $\langle\mu\rangle = \exp(\int_0^\beta \rho(\beta')d\beta')$ for U(1).

COMPUTING ρ ANALYTICALLY-1

$$\rho = -\sum_0^\infty \frac{(-\beta)^n}{n!}\langle\langle \Delta S^{n+1}\rangle\rangle - \langle\langle S\sum_1^\infty \frac{(-\beta)^n}{n!}\Delta S^n\rangle\rangle$$

- $\langle\langle .. \rangle\rangle \equiv$ CONNECTED PART OF $\langle .. \rangle_S$.
- $S = \Sigma_n \Sigma_{\mu\nu} \Re[1 - P_{\mu\nu}(n)]$
- $\Delta S = \Sigma_{\vec{n}} \Sigma_i [(C_i(\vec{n}) - 1)\Re P_{i0}(\vec{n}, 0) - S_i(\vec{n})\Im Q_{i0}(\vec{n}, 0)]$
- $C_i(\vec{n}) = \cos(\frac{g}{2} A_i^{Cl}(\vec{n} + \hat{i} - \vec{x}))$ $\quad (C_i(\vec{n}) - 1) \approx_{n\to\infty} \frac{1}{n^2}$
- $S_i(\vec{n}) = \sin(\frac{g}{2} A_i^{Cl}(\vec{n} + \hat{i} - \vec{x}))$ $\quad S_i(\vec{n}) \approx_{n\to\infty} \frac{1}{n}$
- $Q_{i0}(\vec{n}, 0) \equiv \frac{1}{N} Tr[U_i(\vec{n}, 0) U_0(\vec{n} + \hat{i}, 0) T_3 U_i^\dagger(\vec{n}, 1) U^\dagger(\vec{n}, 0)]$
  ODD UNDER CHARGE CONJ. [$U(1) : Q_{i0}(\vec{n}, 0) = P_{i0}(\vec{n}, 0)$]
  ONLY EVEN POWERS CONTRIBUTE!
- ρ INTEGRAL ON POSITIONS OF CORRELATORS OF $(1 - C_i(\vec{n}))\Re P_{i0}(\vec{n})$, $\Re P_{\mu\nu}(n)$, $S_i(\vec{n})\Im Q_{i0}(\vec{n})$.

COMPUTING ρ ANALYTICALLY-2

- CONFINED PHASE : MASS-GAP , INTEGRALS ON RELATIVE DISTANCES CONVERGENT. "KINEMATIC" LINEAR DIVERGENCE IN SUM OF THEM FROM TERMS $\propto (C_i(\vec{n}) - 1)$ AND $\propto S_i^2(\vec{n})$ CANCELS FOR $U(1)$, DOES NOT FOR HIGHER GROUPS IN WHATEVER ABELIAN PROJECTION.
- DECONFINED PHASE : NO SCALE. $\rho \propto_{V\to\infty} -K \ln V \longrightarrow \langle\mu\rangle \to 0$ IN THERMODYNAMIC LIMIT.
  $d^3n(1 - C_i(\vec{n}))\Re P_{i0}(\vec{n}) \approx \frac{d^3n}{n^2} a^4 \vec{G}_{i0}\vec{G}_{i0}$ DIMENSION -3
  $d^3nS_i(\vec{n})\Im Q_{i0}(\vec{n}) \approx \frac{d^3n}{n} a^2 \vec{G}_{i0}.$ DIMENSION 0
  ONLY TERMS WITH NO FACTOR $(1 - C_i(\vec{n}))\Re P_{i0}(\vec{n})$ CONTRIBUTE TO THE DIVERGENT PART of ρ. QUADRATIC TERM NEGATIVE DEFINITE [A.D.G. JHEP **02** 2021 208-232]
- FINITE CONTRIBUTIONS TO ρ IRRELEVANT.

DISCUSSION OF THE "KINEMATIC" DIVERGENCES

- STRONG COUPLING: $\langle O \rangle = \frac{\int [dU] \exp(-\beta S) O}{\int [dU] \exp(-\beta S)} = \sum_n O_n \beta^n$,

- $\rho_{div} = -\langle\langle \Delta S + \beta S \Delta S + \sum_{i,\vec{n}_1,\vec{n}_2} [\beta \Im Q_{0i}(\vec{n}_1) \Im Q_{0i}(\vec{n}_2)$

  $-\frac{1}{2}\beta^2 S \Im Q_{0i}(\vec{n}_1) \Im Q_{0i}(\vec{n}_2)] S_i^2(\vec{n})\rangle\rangle \qquad \vec{n} = \frac{\vec{n}_1+\vec{n}_2}{2}$

- $$\rho_{div} = \sum_{n=0}^{\infty} \frac{\beta^{2n+1}}{2n!}(n+1)$$

  $\sum_{i,\vec{n}_1,\vec{n}_2} \langle\langle [\Re P_{i0}(\vec{n}_1) \Re P_{i0}(\vec{n}_2) - \Im Q_{i0}(\vec{n}_1) \Im Q_{i0}(\vec{n}_2)] S^{2n} \rangle\rangle$

- $U(1) : Q_{i0} = P_{i0} \quad \frac{1}{2}\langle\langle [P_{i0}(\vec{n}_1) P_{i0}(\vec{n}_2) + P^*_{i0}(\vec{n}_1) P^*_{i0}(\vec{n}_2)]\rangle\rangle = 0$

  $\partial_i F_{0i} = 0$

  $\rho_{div} = 0$ TO ALL ORDERS

NO ELECTRIC CHARGES [Cfr. FROLICH MARCHETTI 87]
[OR $r_h << $ LATTICE SPACING].

FIFT ORDER CONTRIBUTION TO ρ_{div}

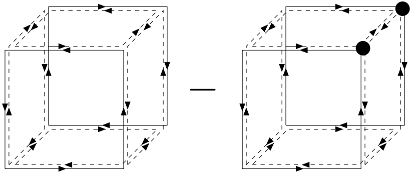

Figure: 3. β^5 CONTRIBUTION TO ρ_{div}.
DOTS DENOTE T_3 INSERTIONS.

$QCD - 1$

- FOR $SU(N)$ THE LEFT CUBE IN FIG.3 $= \frac{1}{N^4}$, THE RIGHT CUBE $= 0$ [$Tr[T_3] = 0$]. $\rho \propto_{V \to \infty} V^{\frac{1}{3}}$: $\langle\mu\rangle$ NO ORDER PARAMETER.[COSSU et al. 2007]
- MONOPOLE BREAKS $SU(N) \to SU(N)/U(1)$. LOCAL GAUGE SYMMETRY CAN NOT BE BROKEN [ELITSUR 75]
- REPLACE T_3 BY ITS PARALLEL TRANSPORT TO ∞ ,$\bar{T}_3$: THE SYMMETRY BREAKING IS NOW GLOBAL

  $Q_{i0}(\vec{n}, 0) \to \frac{1}{N} Tr[U_i(\vec{n}, 0) U_0(\vec{n} + \hat{i}, 0) \bar{T}_3 U_i^\dagger(\vec{n}, 1) U_0^\dagger(\vec{n}, 0)]$

  $$\bar{T}_3 = V_C(\vec{n}, \infty) T_3 V_C^\dagger(\vec{n}, \infty)$$

  OR

  $G_{io}(\vec{n}, 0) \to \Phi_{i0}(\vec{n}, 0) \equiv V_C^\dagger(\vec{n}, \infty) G_{io}(\vec{n}, 0) V_C(\vec{n}, \infty)$

  AND BY SIMPLE ALGEBRA

  $\partial_i \Phi_{i0}(\vec{n}, 0) = V_C^\dagger(\vec{n}, \infty) D_i G_{io}(\vec{n}, 0) V_C(\vec{n}, \infty) =$

  $gV_C^\dagger \bar{\psi} \vec{T} \gamma_0 \psi V_C = 0$ IN ABSENCE OF QUARKS.

  THE KINEMATIC DIVERGENCES CANCEL .

QCD -2

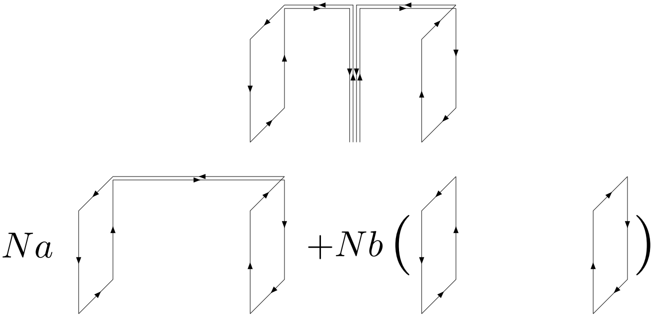

Figure: The product of two gauge-invariant fields. , $b = -a\frac{1}{N}$.

GAUGE INVARIANT CONNECTED FIELD STRENGTH CORRELATORS [STOCHASTIC VACUUM DOSCH 87, SIMONOV 88] STUDIED ON LATTICE [DI GIACOMO ,PANAGOPOULOS 92, DI GIACOMO, DOSCH, SHEVCHENKO, SIMONOV 02].

DISCUSSION

- 1) A GAUGE INVARIANT ORDER PARAMETER FOR MONOPOLE CONDENSATION CAN BE DEFINED IN QCD: MONOPOLES MUST LIVE IN THE GROUP AT ∞.
- 2) THE ORDER PARAMETER CAN BE TRADED WITH THE ELECTRIC FIELD TWO POINT GAUGE-INVARIANT CORRELATOR: AT DECONFINEMENT IT CHANGES FROM EXPONENTIAL TO POWER LAW. [A.D.G.,E. MEGGIOLARO,H. PANAGOPOULOS '97]
- 3)THE OLD PROBLEM THAT FLUX TUBES SHOULD KEEP MEMORY OF THE "ABELIAN PROJECTION" OF THE MONOPOLES, AND THEY DO NOT [GREENSITE WINCHESTER 89, A.D.G., M. MAGGIORE, S. OLEJNIK 90] IS SOLVED: MEMORY OF THE COLOR DIRECTION GETS LOST IN THE PARALLEL TRANSPORT FROM ∞ .
- 4) LATTICE COMPUTATION OF THE NEW $\langle \mu \rangle$ NEEDED.