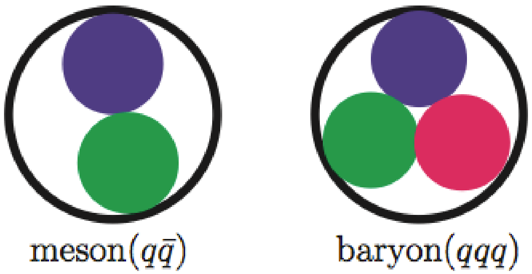
Glueballs and hybrid states within QCD sum rules

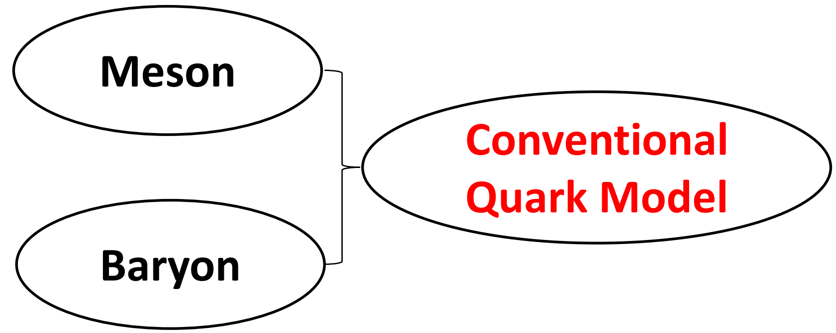
Hua-Xing Chen

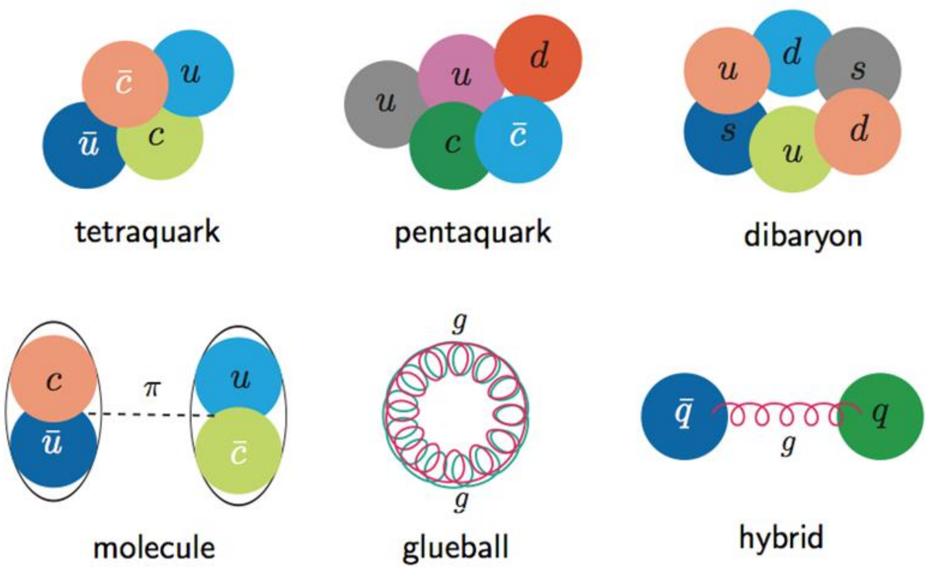
Southeast University (CN)

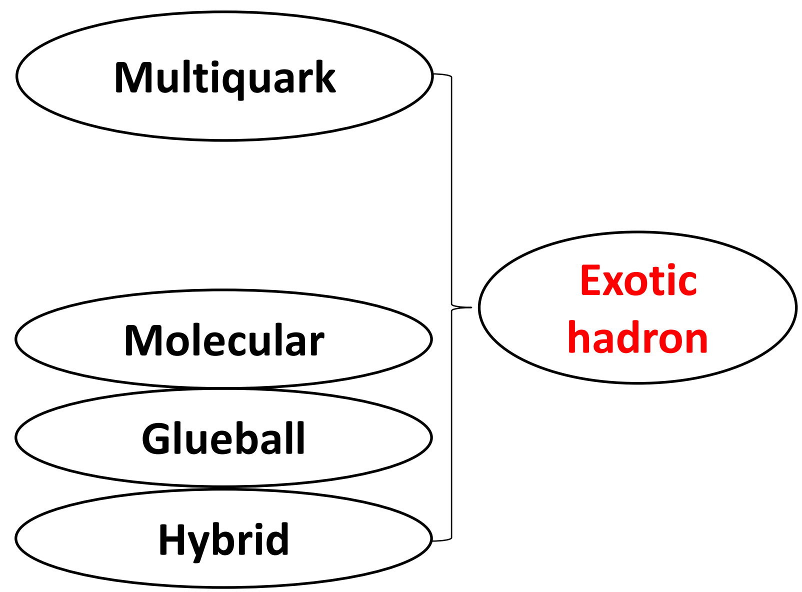
Collaborators: Niu Su, Wei Chen, Shi-Lin Zhu

Hadron spectrum









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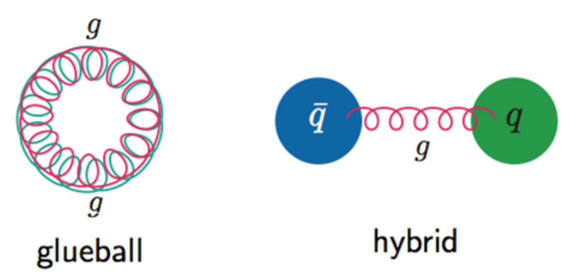

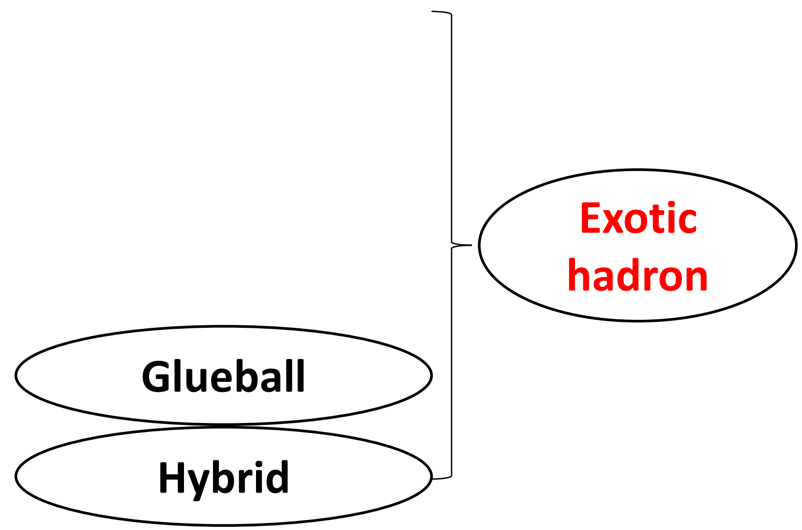

Previous Studies

MIT bag model: A. Chodos et al., Phys. Rev. D 9, 3471 (1974);
R. L. Jaffe and K. Johnson, Phys. Lett. 60B, 201 (1976).

Flux tube model: N. Isgur and J. E. Paton, Phys. Rev. D 31, 2910 (1985).

Coulomb Gauge: A. Szczepaniak et al., Phys. Rev. Lett. 76, 2011 (1996);
F. J. Llanes-Estrada, P. Bicudo and S. R. Cotanch, Phys. Rev. Lett. 96, 081601 (2006).

Glueball trajectories: I. Szanyi et al., Nucl. Phys. A 998, 121728 (2020).

Lattice QCD: K. G. Wilson, Phys. Rev. D 10, 2445 (1974);
Y. Chen et al., Phys. Rev. D 73, 014516 (2006);
V. Mathieu, N. Kochelev and V. Vento, IJMPE 18, 1 (2009);
E. Gregory et al., JHEP 1210, 170 (2012).

QCD sum rules: V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, NPB165, 67 (1980);
S. Narison, Z. Phys. C 26, 209 (1984);
S. Narison, Nucl. Phys. B 509, 312 (1998);
J. I. Latorre, S. Narison and S. Paban, Phys. Lett. B 191, 437 (1987);
E. Bagan and T. G. Steele, Phys. Lett. B 243, 413 (1990);
G. Hao, C. F. Qiao and A. L. Zhang, Phys. Lett. B 642, 53 (2006);
C. F. Qiao and L. Tang, Phys. Rev. Lett. 113, 221601 (2014);
A. Pimikov, H. J. Lee, N. Kochelev and P. Zhang, Phys. Rev. D 95, 071501(R) (2017);
A. Pimikov, arXiv:2205.12948.

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QCD sum rule approach

- Construct relativistic glueball currents using:

$$\boldsymbol{G^a_{\mu\nu}} \text{ and } \boldsymbol{\widetilde{G}^a_{\mu\nu}}$$

- Perform QCD sum rule calculations

- Compare with Lattice QCD calculations

Non-relativistic operators

R. L. Jaffe, K. Johnson and Z. Ryzak
Annals Phys. 168, 344 (1986)

$$\boldsymbol{E_i = G^a_{i0}} \quad \text{and} \quad \boldsymbol{B_i = -\frac{1}{2}\epsilon_{ijk}G^a_{jk}}$$

C=- operators

J^{PC}	Operator
1^{+-}	$d_{abc}(\vec{E}_a \cdot \vec{E}_b)\,\vec{B}_c$
1^{--}	$d_{abc}(\vec{E}_a \cdot \vec{E}_b)\,\vec{E}_c$
2^{+-}	$d_{abc}[E^i_a[E^i_a(\vec{B}_b \times \vec{E}_c)^j + (i \leftrightarrow j)]$
2^{--}	$d_{abc}[B^i_a(\vec{E}_b \times \vec{B}_c)^j + (i \leftrightarrow j)]$
3^{--}	$d_{abc}[E^i_a E^j_b E^k_c - \frac{1}{5}\vec{E}_a \cdot \vec{E}_b(\delta^{ij}E^k_c + \delta^{jk}E^i_c + \delta^{ik}E^j_c)]$
3^{+-}	$d_{abc}[B^i_a B^j_b B^k_c - \frac{1}{5}\vec{B}_a \cdot \vec{B}_b(\delta^{ij}B^k_c + \delta^{jk}B^i_c + \delta^{ik}B^j_c)]$

Relativistic currents

$G^{a}_{\mu\nu}$ and $\tilde{G}^{a}_{\mu\nu}$

$$
\begin{aligned}
\tilde{J}_1^{\alpha\beta} &= d^{abc} g_s^3 G_a^{\mu\nu} G_{b,\mu\nu} \tilde{G}_c^{\alpha\beta} \,, \\
J_1^{\alpha\beta} &= d^{abc} g_s^3 G_a^{\mu\nu} G_{b,\mu\nu} G_c^{\alpha\beta} \,, \\
\tilde{J}_2^{\alpha_1\alpha_2,\beta_1\beta_2} &= d^{abc} \mathcal{S}[g_s^3 G_a^{\alpha_1\beta_1} \tilde{G}_b^{\alpha_2\mu} G_{c,\mu}^{\beta_2} - \{\alpha_2 \leftrightarrow \beta_2\}], \\
J_2^{\alpha_1\alpha_2,\beta_1\beta_2} &= d^{abc} \mathcal{S}[g_s^3 \tilde{G}_a^{\alpha_1\beta_1} G_b^{\alpha_2\mu} \tilde{G}_{c,\mu}^{\beta_2} - \{\alpha_2 \leftrightarrow \beta_2\}], \\
\tilde{J}_3^{\cdots} &= d^{abc} \mathcal{S}[g_s^3 \tilde{G}_a^{\alpha_1\beta_1} \tilde{G}_b^{\alpha_2\beta_2} \tilde{G}_c^{\alpha_3\beta_3}] \,, \\
J_3^{\cdots} &= d^{abc} \mathcal{S}[g_s^3 G_a^{\alpha_1\beta_1} G_b^{\alpha_2\beta_2} G_c^{\alpha_3\beta_3}] \,,
\end{aligned}
$$

Non-relativistic operators

Relativistic currents

$E_i = G^a_{i0}$ and $B_i = -\frac{1}{2}\epsilon_{ijk}G^a_{jk}$

$G^a_{\mu\nu}$ and $\tilde{G}^a_{\mu\nu}$

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3^{--}	$d_{abc}[E^i_a E^j_b E^k_c - \frac{1}{5}\vec{E}_a \cdot \vec{E}$
3^{+-}	$d_{abc}[B^i_a B^j_b B^k_c - \frac{1}{5}\vec{B}_a \cdot \vec{B}$

$$\tilde{J}_1^{\alpha\beta} = d^{abc} g_s^3 G_a^{\mu\nu} G_{b,\mu\nu} \tilde{G}_c^{\alpha\beta}\,,$$

$$J_1^{\alpha\beta} = d^{abc} g_s^3 G_a^{\mu\nu} G_{b,\mu\nu} G_c^{\alpha\beta}\,,$$

$$\tilde{J}_2^{\alpha_1\alpha_2,\beta_1\beta_2} = d^{abc}\mathcal{S}[g_s^3 G_a^{\alpha_1\beta_1} \tilde{G}_b^{\alpha_2\mu} G_{c,\mu}^{\beta_2} - \{\alpha_2 \leftrightarrow \beta_2\}],$$

$$J_2^{\alpha_1\alpha_2,\beta_1\beta_2} = d^{abc}\mathcal{S}[g_s^3 \tilde{G}_a^{\alpha_1\beta_1} G_b^{\alpha_2\mu} \tilde{G}_{c,\mu}^{\beta_2} - \{\alpha_2 \leftrightarrow \beta_2\}],$$

$$\tilde{J}_3^{\cdots} = d^{abc}\mathcal{S}[g_s^3 \tilde{G}_a^{\alpha_1\beta_1} \tilde{G}_b^{\alpha_2\beta_2} \tilde{G}_c^{\alpha_3\beta_3}]\,,$$

$$J_3^{\cdots} = d^{abc}\mathcal{S}[g_s^3 G_a^{\alpha_1\beta_1} G_b^{\alpha_2\beta_2} G_c^{\alpha_3\beta_3}]\,,$$

Non-relativistic operators

Relativistic currents

$E_i = G^a_{i0}$ and $B_i = -\frac{1}{2}\epsilon_{ijk}G^a_{jk}$

$G^a_{\mu\nu}$ and $\tilde{G}^a_{\mu\nu}$

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i,j $\quad (\vec{E}\cdot\vec{E} + \vec{B}\cdot\vec{B})\,\vec{E}$

$$\tilde{J}_1^{\alpha\beta} = d^{abc} g_s^3 G_a^{\mu\nu} G_{b,\mu\nu} \tilde{G}_c^{\alpha\beta} \,,$$
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Non-relativistic operators

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$G^a_{\mu\nu}$ and $\tilde{G}^a_{\mu\nu}$

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$i,j \quad (\vec{E}\cdot\vec{E}+\vec{B}\cdot\vec{B})\,\vec{E}$

$0,i \quad (\vec{E}\cdot\vec{E}+\vec{B}\cdot\vec{B})\,\vec{B}$

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$$J_3^{\cdots} = d^{abc} \mathcal{S}[g_s^3 G_a^{\alpha_1\beta_1} G_b^{\alpha_2\beta_2} G_c^{\alpha_3\beta_3}] ,$$

$$\langle 0|J_1^{\alpha\beta}|X_{1--}\rangle = i f_{1--}\,\epsilon^{\alpha\beta\mu\nu}\epsilon_\mu p_\nu ,$$
$$\langle 0|J_1^{\alpha\beta}|X_{1+-}\rangle = i f_{1+-}(p^\alpha\epsilon^\beta - p^\beta\epsilon^\alpha) ,$$

Non-relativistic operators

$$E_i = G^a_{i0} \text{ and } B_i = -\frac{1}{2}\epsilon_{ijk}G^a_{jk}$$

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Relativistic currents

$$G^a_{\mu\nu} \text{ and } \tilde{G}^a_{\mu\nu}$$

$$i,j \qquad (\vec{E}\cdot\vec{E} + \vec{B}\cdot\vec{B})\,\vec{E}$$
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$$J_3^{\cdots} = d^{abc}\mathcal{S}[g_s^3 G_a^{\alpha_1\beta_1} G_b^{\alpha_2\beta_2} G_c^{\alpha_3\beta_3}],$$

$$\langle 0 | J_1^{\alpha\beta} | X_{1^{--}} \rangle = i f_{1^{--}} \epsilon^{\alpha\beta\mu\nu} \epsilon_\mu p_\nu ,$$

QCD Sum Rules

- In sum rule analyses, we consider two-point correlation functions:

$$\Pi\left(q^2\right) \stackrel{\text{def}}{=} i \int d^4 x e^{iqx} \langle 0|T\eta(x)\eta^+(0)\,|0\rangle$$

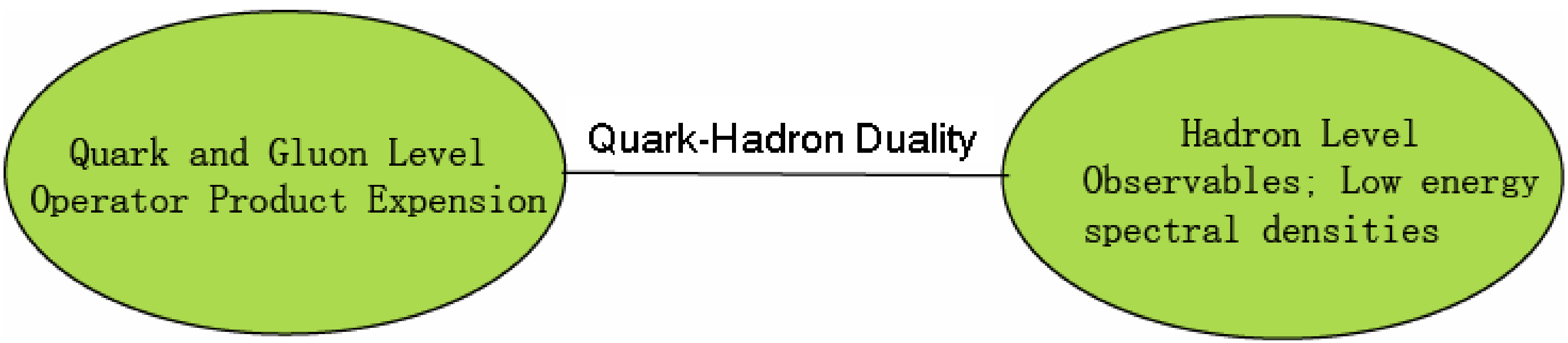
$$\approx \Sigma_n \langle 0|\eta|n\rangle\langle n|\eta^+|0\rangle$$

where η is the current which can couple to hadronic states.

- By using the dispersion relation, we can obtain the spectral density

$$\Pi\left(q^2\right) = \int_{s_<}^{\infty} \frac{\rho(s)}{s - q^2 - i\varepsilon} ds$$

- In QCD sum rule, we can calculate these matrix elements from QCD (OPE) and relate them to observables by using dispersion relation.



SVZ sum rule

Quark and Gluon Level

(Convergence of OPE)

$$\Pi_{OPE}(q^2) \xrightarrow[s\,=\,-q^2]{\text{dispersion relation}} \rho_{OPE}(s) = a_n\, s^n + a_{n-1}\, s^{n-1}$$

Quark-Hadron Duality

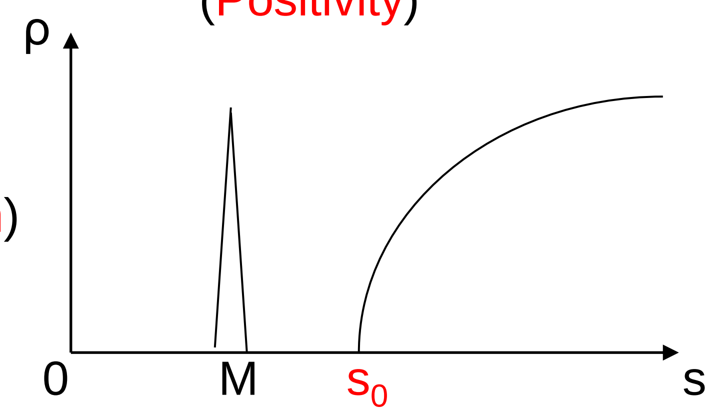
Hadron Level

$$\Pi_{phys}(q^2) = f_x^2 \frac{1}{q^2 - M^2} \longleftrightarrow \rho_{phys}(s) = \lambda_x^2 \delta(s - M_x^2) + \cdots$$

(for boson case)

(Positivity)

(Sufficient amount of Pole contribution)



Nielsen et al, Phys. Rept. 497, 41 (2010); Albuquerque et al, JPG46, 093002 (2019).

SVZ sum rule

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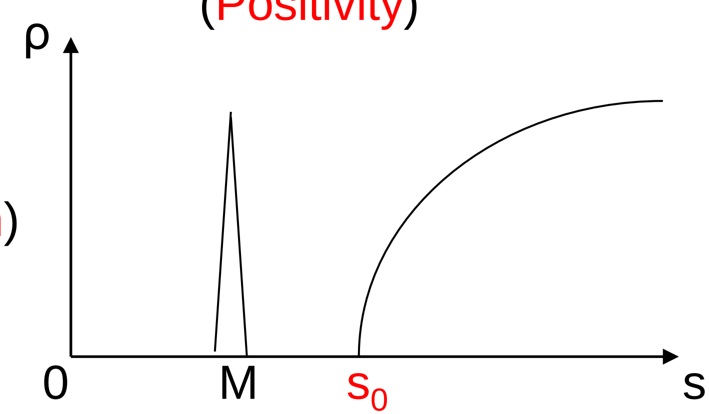

QCD Sum Rules

- Borel transformation to suppress the higher order terms:

$$\Pi(M_B^2) \equiv f^2 e^{-M^2/M_B^2} = \int_{s_<}^{s_0} e^{-s/M_B^2} \rho(s) ds$$

- Two free parameters

M_B , s_0

We need to choose certain region of (M_B, s_0).

- **Criteria**

1. Stability
2. Convergence of OPE
3. Positivity of spectral density
4. Sufficient amount of pole contribution

QCD sum rule results

$$G_{\mu\nu}^{a} = \partial_{\mu}A_{\nu}^{a} - \partial_{\nu}A_{\mu}^{a} + g_s f^{abc}A_{\mu}^{b}A_{\nu}^{c}$$

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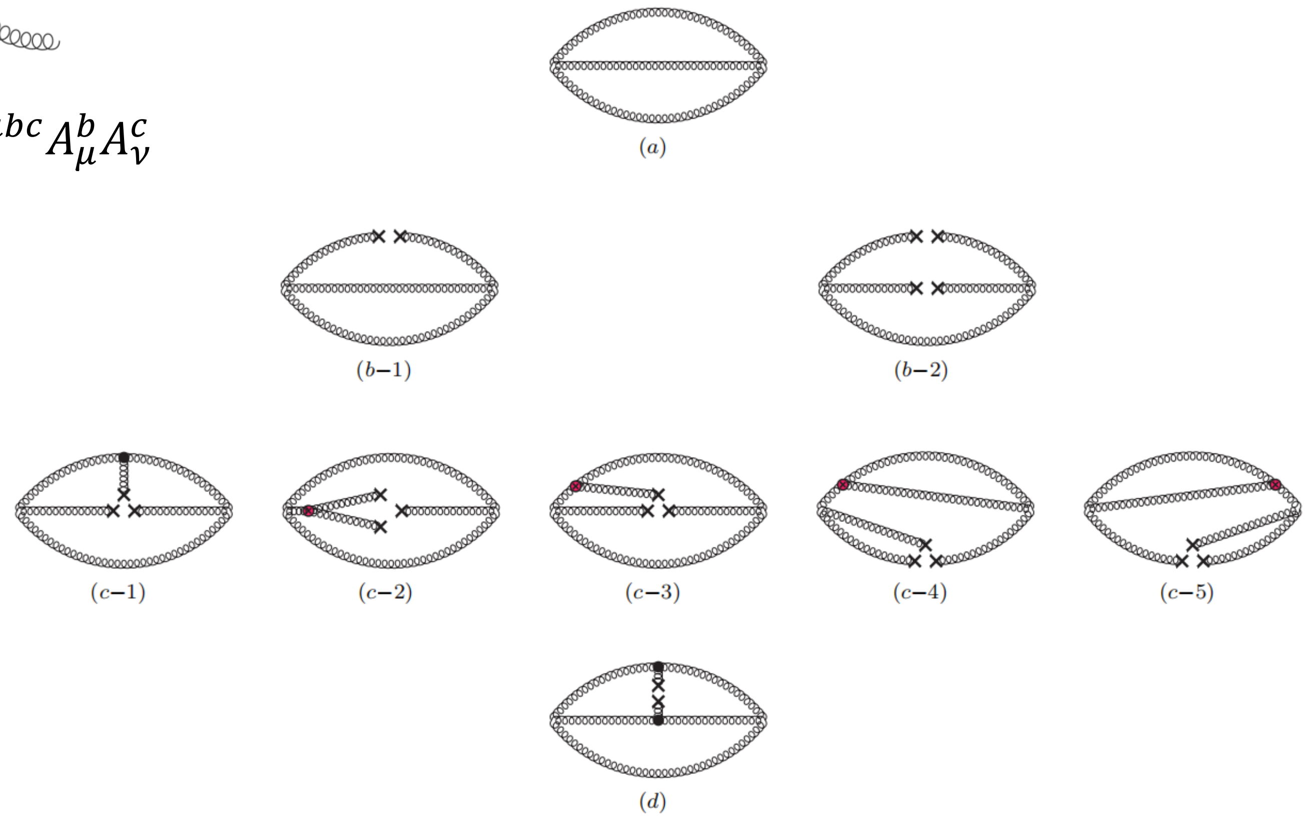
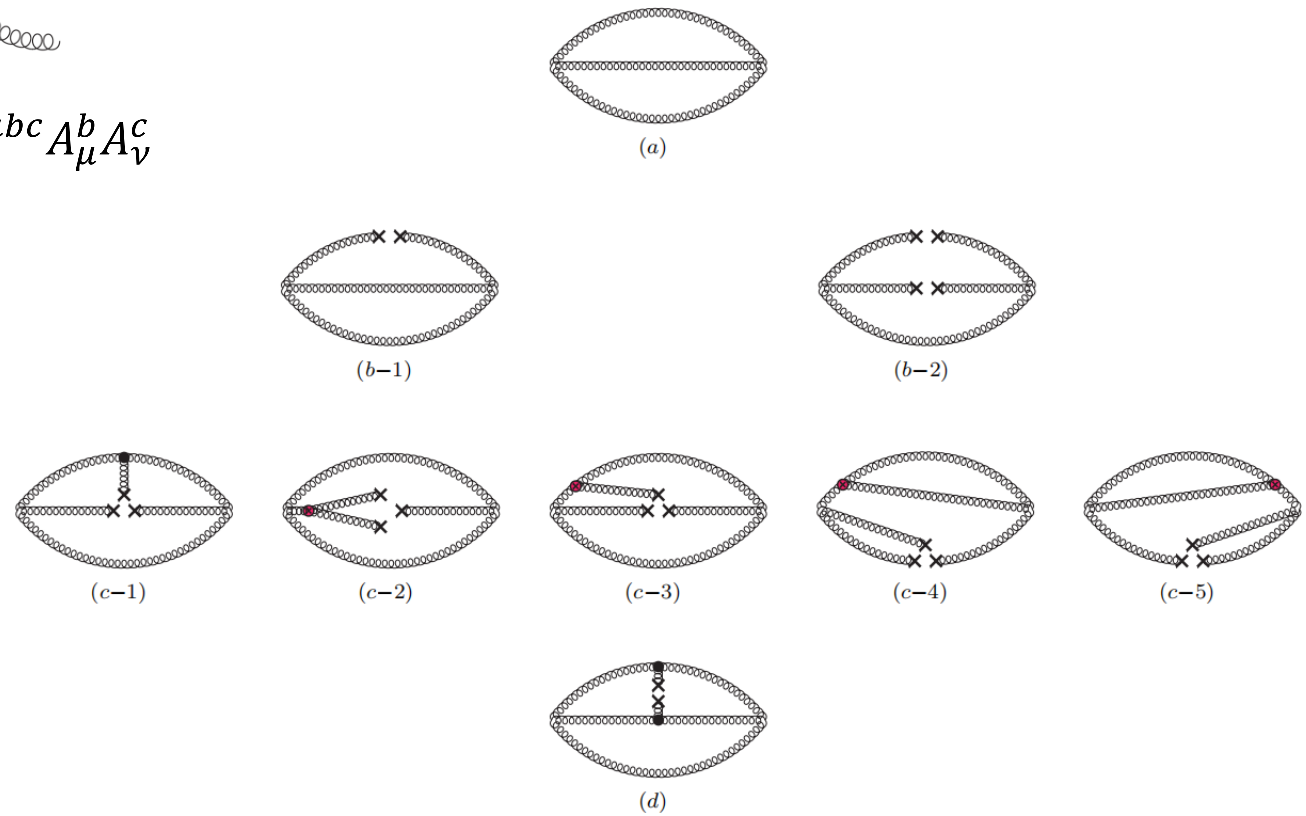
$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s f^{abc} A_\mu^b A_\nu^c$$



QCD sum rule results

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu$$



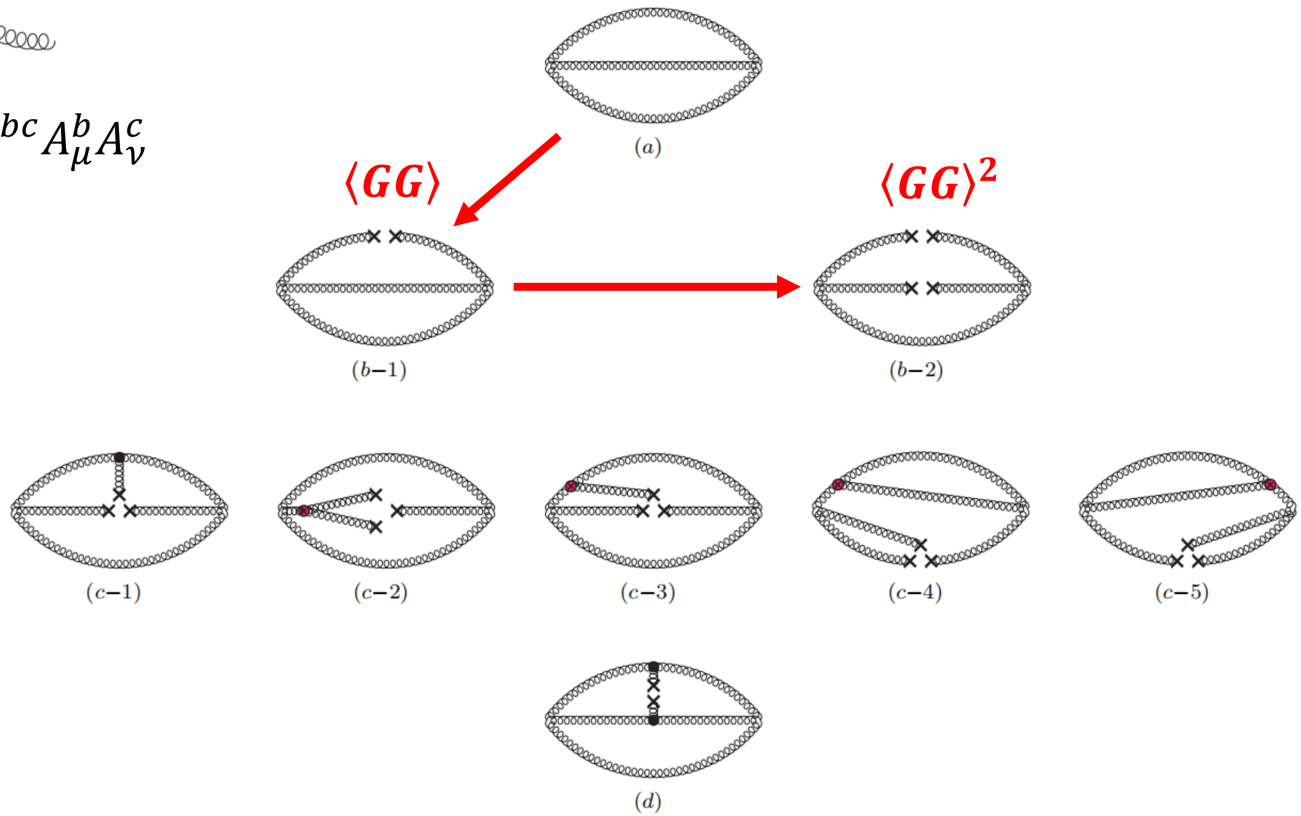
g_s^6

g_s^7

g_s^8

QCD sum rule results

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu$$



QCD sum rule results

$$J_1^{\alpha\beta} = d^{abc} g_s^3 G_a^{\mu\nu} G_{b,\mu\nu} G_c^{\alpha\beta}$$

$$\rho_{1--}(s) = \frac{4\alpha_s^3}{81\pi} s^4 - \frac{10\alpha_s^2 \langle g_s^2 GG \rangle}{9} s^2 + \frac{25\alpha_s^3 \langle g_s^2 GG \rangle}{36\pi} s^2 + \frac{35\alpha_s^2 \langle g_s^3 G^3 \rangle}{27} s$$

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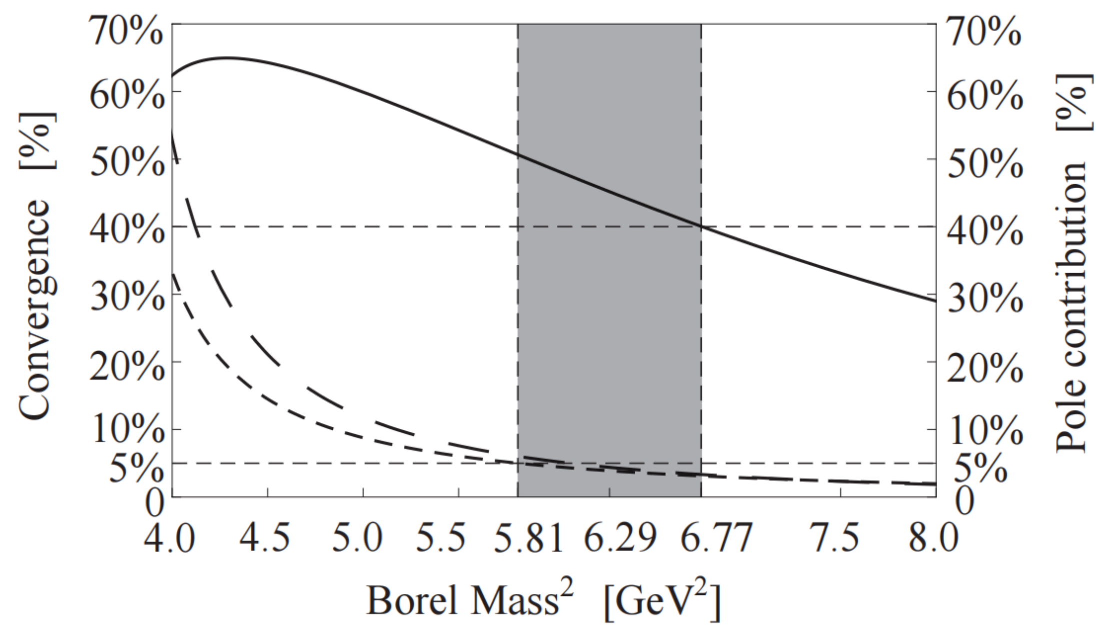
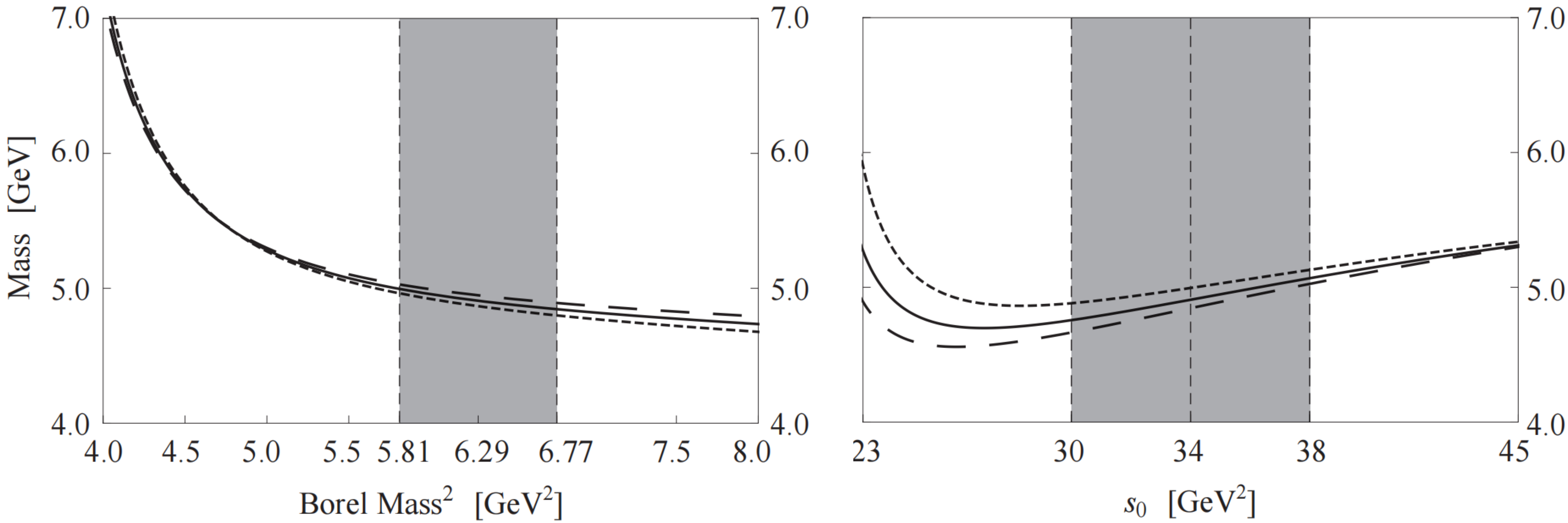
$$\mathrm{CVG}'_A \equiv \left| \frac{\Pi^{g_s^{n=8}}(s_0, M_B^2)}{\Pi(s_0, M_B^2)} \right| \leq 5\%\,,$$

$$\mathrm{CVG}'_B \equiv \left| \frac{\Pi^{\mathrm{D}=6}(s_0, M_B^2)}{\Pi(s_0, M_B^2)} \right| \leq 10\%\,.$$

$$\text{Pole contribution} \equiv \left| \frac{\Pi(s_0, M_B^2)}{\Pi(\infty, M_B^2)} \right| \geq 40\%$$

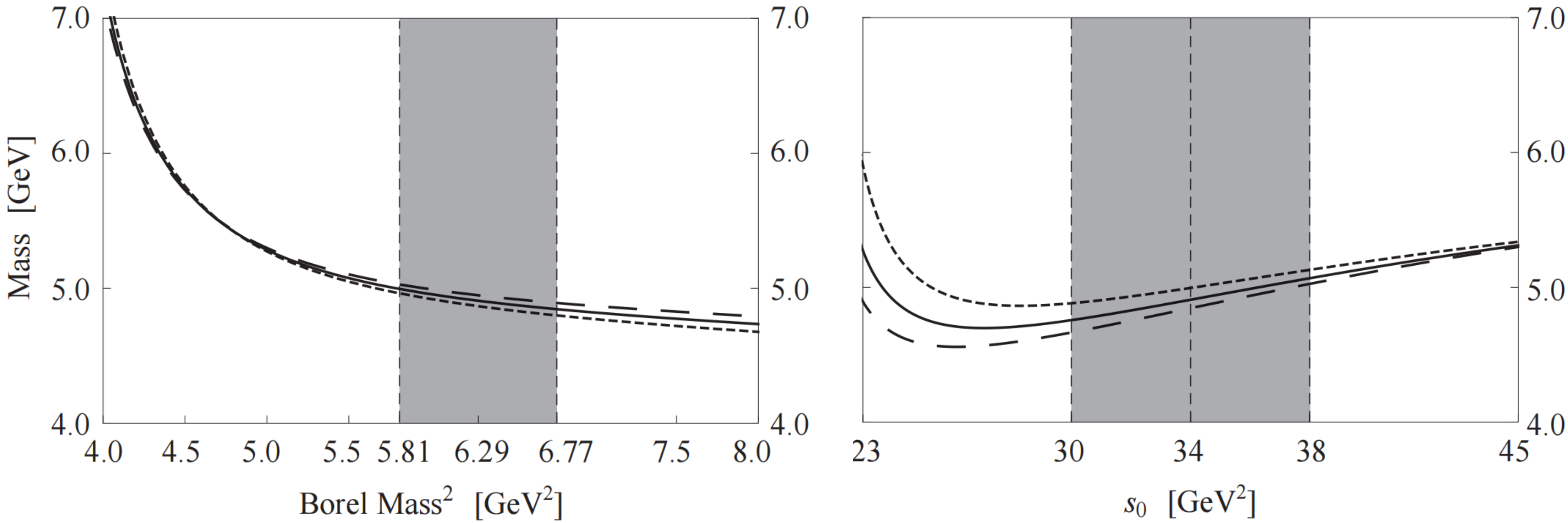


QCD sum rule results



$$\text{Mass}_{|GGG;1^{--}\rangle} = 4.91^{+0.20}_{-0.18}\ \text{GeV}$$

QCD sum rule results



$$\text{Mass}_{|GGG;1^{--}\rangle} = 4.91^{+0.20}_{-0.18}\ \text{GeV}$$

$$\langle \alpha_s GG \rangle = (6.35 \pm 0.35) \times 10^{-2}\ \text{GeV}^4\,,$$
$$\langle g_s^3 G^3 \rangle = \langle \alpha_s GG \rangle \times (8.2 \pm 1.0)\ \text{GeV}^2\,.$$

S. Narison, Int. J. Mod. Phys. A 33, 1850045 (2018)

QCD sum rule results: Two- and three-gluon glueballs

Glueball	Current	s_0^{min} [GeV^2]	Working Regions		Pole [%]	Mass [GeV]
			s_0 [GeV^2]	M_B^2 [GeV^2]		
$\|GG; 0^{++}\rangle$	J_0	7.8	9.0 ± 1.0	3.70–4.19	40–48	$1.78^{+0.14}_{-0.17}$
$\|GG; 2^{++}\rangle$	$J_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	8.5	10.0 ± 1.0	3.99–4.60	40–50	$1.86^{+0.14}_{-0.17}$
$\|GG; 0^{-+}\rangle$	$\tilde{J}_0$	8.2	9.0 ± 1.0	3.28–3.70	40–47	$2.17^{+0.11}_{-0.11}$
$\|GG; 2^{-+}\rangle$	$\tilde{J}_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	8.1	10.0 ± 1.0	3.27–4.20	40–55	$2.24^{+0.11}_{-0.11}$
$\|GGG; 0^{++}\rangle$	η_0	31.6	33.0 ± 3.0	7.25–7.61	40–44	$4.46^{+0.17}_{-0.19}$
$\|GGG; 2^{++}\rangle$	$\eta_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	16.0	35.0 ± 3.0	4.77–9.04	40–90	$4.18^{+0.19}_{-0.42}$
$\|GGG; 0^{-+}\rangle$	$\tilde{\eta}_0$	17.0	33.0 ± 3.0	4.48–8.13	40–88	$4.13^{+0.18}_{-0.36}$
$\|GGG; 2^{-+}\rangle$	$\tilde{\eta}_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	33.1	35.0 ± 3.0	8.10–8.53	40–44	$4.29^{+0.20}_{-0.22}$
$\|GGG; 1^{+-}\rangle$	$\xi_1^{\alpha\beta}$	9.0	34.0 ± 4.0	3.16–9.09	40–99	$4.01^{+0.26}_{-0.95}$
$\|GGG; 2^{+-}\rangle$	$\xi_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	32.7	35.0 ± 4.0	7.53–8.09	40–46	$4.42^{+0.24}_{-0.29}$
$\|GGG; 3^{+-}\rangle$	$\xi_3^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}$	30.2	33.0 ± 4.0	7.69–8.40	40–47	$4.30^{+0.23}_{-0.26}$
$\|GGG; 1^{--}\rangle$	$\tilde{\xi}_1^{\alpha\beta}$	31.2	34.0 ± 4.0	5.81–6.77	40–51	$4.91^{+0.20}_{-0.18}$
$\|GGG; 2^{--}\rangle$	$\tilde{\xi}_2^{\alpha_1\alpha_2,\beta_1\beta_2}$	19.7	36.0 ± 4.0	5.80–9.47	40–81	$4.25^{+0.22}_{-0.33}$
$\|GGG; 3^{--}\rangle$	$\tilde{\xi}_3^{\alpha_1\alpha_2\alpha_3,\beta_1\beta_2\beta_3}$	35.8	38.0 ± 4.0	6.15–7.22	40–49	$5.59^{+0.33}_{-0.22}$

QCD sum rule results

Lattice QCD results

Glueball	QCD sum rules	Ref. [11]	Ref. [12]	Ref. [13]	Ref. [14]
$\|GG;0^{++}\rangle$	$1.78^{+0.14}_{-0.17}$	$1.71 \pm 0.05 \pm 0.08$	$1.73 \pm 0.05 \pm 0.08$	$1.48 \pm 0.03 \pm 0.07$	1.80 ± 0.06
$\|GG;2^{++}\rangle$	$1.86^{+0.14}_{-0.17}$	$2.39 \pm 0.03 \pm 0.12$	$2.40 \pm 0.03 \pm 0.12$	$2.15 \pm 0.03 \pm 0.10$	2.62 ± 0.05
$\|GG;0^{-+}\rangle$	$2.17^{+0.11}_{-0.11}$	$2.56 \pm 0.04 \pm 0.12$	$2.59 \pm 0.04 \pm 0.13$	$2.25 \pm 0.06 \pm 0.10$	–
$\|GG;2^{-+}\rangle$	$2.24^{+0.11}_{-0.11}$	$3.04 \pm 0.04 \pm 0.15$	$3.10 \pm 0.03 \pm 0.15$	$2.78 \pm 0.05 \pm 0.13$	3.46 ± 0.32
$\|GGG;0^{++}\rangle$	$4.46^{+0.17}_{-0.19}$	–	$2.67 \pm 0.18 \pm 0.13$	$2.76 \pm 0.03 \pm 0.12$	3.76 ± 0.24
$\|GGG;2^{++}\rangle$	$4.18^{+0.19}_{-0.42}$	–	–	$2.88 \pm 0.10 \pm 0.13$	–
$\|GGG;0^{-+}\rangle$	$4.13^{+0.18}_{-0.36}$	–	$3.64 \pm 0.06 \pm 0.18$	$3.37 \pm 0.15 \pm 0.15$	4.49 ± 0.59
$\|GGG;2^{-+}\rangle$	$4.29^{+0.20}_{-0.22}$	–	–	$3.48 \pm 0.14 \pm 0.16$	–
$\|GGG;1^{+-}\rangle$	$4.01^{+0.26}_{-0.95}$	$2.98 \pm 0.03 \pm 0.14$	$2.94 \pm 0.03 \pm 0.14$	$2.67 \pm 0.07 \pm 0.12$	3.27 ± 0.34
$\|GGG;2^{+-}\rangle$	$4.42^{+0.24}_{-0.29}$	$4.23 \pm 0.05 \pm 0.20$	$4.14 \pm 0.05 \pm 0.20$	–	–
$\|GGG;3^{+-}\rangle$	$4.30^{+0.23}_{-0.26}$	$3.60 \pm 0.04 \pm 0.17$	$3.55 \pm 0.04 \pm 0.17$	$3.27 \pm 0.09 \pm 0.15$	3.85 ± 0.35
$\|GGG;1^{--}\rangle$	$4.91^{+0.20}_{-0.18}$	$3.83 \pm 0.04 \pm 0.19$	$3.85 \pm 0.05 \pm 0.19$	$3.24 \pm 0.33 \pm 0.15$	–
$\|GGG;2^{--}\rangle$	$4.25^{+0.22}_{-0.33}$	$4.01 \pm 0.05 \pm 0.20$	$3.93 \pm 0.04 \pm 0.19$	$3.66 \pm 0.13 \pm 0.17$	4.59 ± 0.74
$\|GGG;3^{--}\rangle$	$5.59^{+0.33}_{-0.22}$	$4.20 \pm 0.05 \pm 0.20$	$4.13 \pm 0.09 \pm 0.20$	$4.33 \pm 0.26 \pm 0.20$	–

QCD sum rule results

Lattice QCD results

		quenched			unquenched
Glueball	QCD sum rules	Ref. [11]	Ref. [12]	Ref. [13]	Ref. [14]
$\lvert GG; 0^{++}\rangle$	$1.78^{+0.14}_{-0.17}$	$1.71 \pm 0.05 \pm 0.08$	$1.73 \pm 0.05 \pm 0.08$	$1.48 \pm 0.03 \pm 0.07$	1.80 ± 0.06
$\lvert GG; 2^{++}\rangle$	$1.86^{+0.14}_{-0.17}$	$2.39 \pm 0.03 \pm 0.12$	$2.40 \pm 0.03 \pm 0.12$	$2.15 \pm 0.03 \pm 0.10$	2.62 ± 0.05
$\lvert GG; 0^{-+}\rangle$	$2.17^{+0.11}_{-0.11}$	$2.56 \pm 0.04 \pm 0.12$	$2.59 \pm 0.04 \pm 0.13$	$2.25 \pm 0.06 \pm 0.10$	–
$\lvert GG; 2^{-+}\rangle$	$2.24^{+0.11}_{-0.11}$	$3.04 \pm 0.04 \pm 0.15$	$3.10 \pm 0.03 \pm 0.15$	$2.78 \pm 0.05 \pm 0.13$	3.46 ± 0.32
$\lvert GGG; 0^{++}\rangle$	$4.46^{+0.17}_{-0.19}$	–	$2.67 \pm 0.18 \pm 0.13$	$2.76 \pm 0.03 \pm 0.12$	3.76 ± 0.24
$\lvert GGG; 2^{++}\rangle$	$4.18^{+0.19}_{-0.42}$	–	–	$2.88 \pm 0.10 \pm 0.1$[illegible]	–
$\lvert GGG; 0^{-+}\rangle$	$4.13^{+0.18}_{-0.36}$	–	$3.64 \pm 0.06 \pm 0.18$	$3.37 \pm 0.15 \pm 0.15$	4.49 ± 0.59
$\lvert GGG; 2^{-+}\rangle$	$4.29^{+0.20}_{-0.22}$	–	–	$3.48 \pm 0.14 \pm 0.16$	–
$\lvert GGG; 1^{+-}\rangle$	$4.01^{+0.26}_{-0.95}$	$2.98 \pm 0.03 \pm 0.14$	$2.94 \pm 0.03 \pm 0.14$	$2.67 \pm 0.07 \pm 0.12$	3.27 ± 0.34
$\lvert GGG; 2^{+-}\rangle$	$4.42^{+0.24}_{-0.29}$	$4.23 \pm 0.05 \pm 0.20$	$4.14 \pm 0.05 \pm 0.20$	–	–
$\lvert GGG; 3^{+-}\rangle$	$4.30^{+0.23}_{-0.26}$	$3.60 \pm 0.04 \pm 0.17$	$3.55 \pm 0.04 \pm 0.17$	$3.27 \pm 0.09 \pm 0.15$	3.85 ± 0.35
$\lvert GGG; 1^{--}\rangle$	$4.91^{+0.20}_{-0.18}$	$3.83 \pm 0.04 \pm 0.19$	$3.85 \pm 0.05 \pm 0.19$	$3.24 \pm 0.33 \pm 0.15$	–
$\lvert GGG; 2^{--}\rangle$	$4.25^{+0.22}_{-0.33}$	$4.01 \pm 0.05 \pm 0.20$	$3.93 \pm 0.04 \pm 0.19$	$3.66 \pm 0.13 \pm 0.17$	4.59 ± 0.74
$\lvert GGG; 3^{--}\rangle$	$5.59^{+0.33}_{-0.22}$	$4.20 \pm 0.05 \pm 0.20$	$4.13 \pm 0.09 \pm 0.20$	$4.33 \pm 0.26 \pm 0.20$	–

QCD sum rule results: Double-gluon hybrid states

$ggq\bar{q}$	s_0^{min} [GeV^2]	Working Regions		Pole [%]	Mass [GeV]
		M_B^2 [GeV^2]	s_0 [GeV^2]		
0^{-+}	24.4	5.34–5.78	27 ± 5.0	40–48	$4.25^{+0.18}_{-0.20}$
0^{++}	34.9	6.12–6.92	38 ± 8.0	40–50	$5.61^{+0.18}_{-0.20}$
1^{--}	20.0	4.60–4.91	22 ± 4.0	40–47	$3.74^{+0.18}_{-0.20}$
1^{+-}	32.1	5.51–6.31	35 ± 7.0	40–50	$5.46^{+0.19}_{-0.22}$
2^{--}	16.8	4.39–4.81	19 ± 4.0	40–49	$3.51^{+0.21}_{-0.26}$
2^{+-}	6.4	1.61–1.78	7 ± 2.0	40–48	$2.26^{+0.23}_{-0.30}$
2^{-+}	16.8	4.39–4.81	19 ± 4.0	40–49	$3.51^{+0.23}_{-0.30}$
2^{++}	20.0	5.39–5.76	22 ± 4.0	40–46	$3.74^{+0.23}_{-0.30}$

QCD sum rule results: Double-gluon hybrid states

$ggq\bar{q}$	s_0^{min} [GeV^2]	Working Regions		Pole [%]	Mass [GeV]
		M_B^2 [GeV^2]	s_0 [GeV^2]		
0^{-+}	24.4	5.34–5.78	27 ± 5.0	40–48	$4.25^{+0.18}_{-0.20}$
0^{++}	34.9	6.12–6.92	38 ± 8.0	40–50	$5.61^{+0.18}_{-0.20}$
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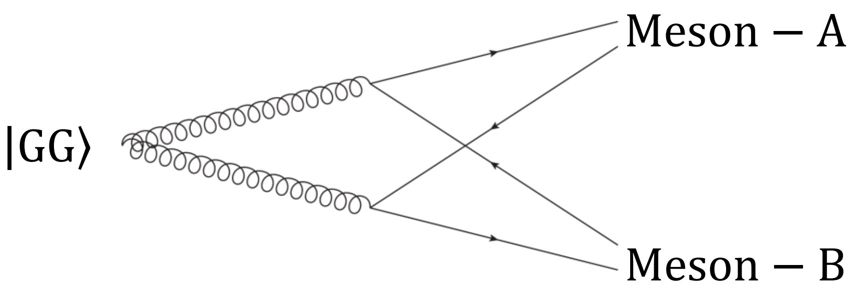
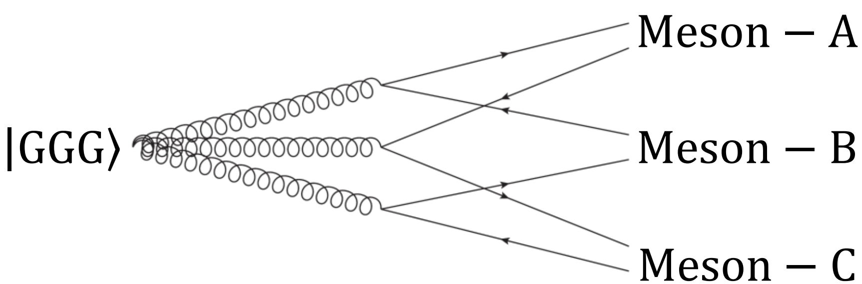
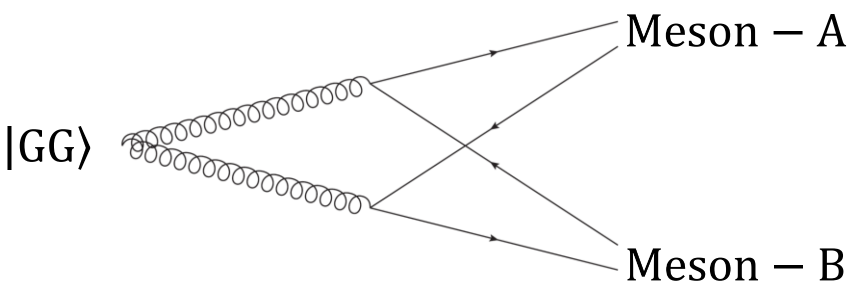
2^{-+}	16.8	4.39–4.81	19 ± 4.0	40–49	$3.51^{+0.23}_{-0.30}$
2^{++}	20.0	5.39–5.76	22 ± 4.0	40–46	$3.74^{+0.23}_{-0.30}$

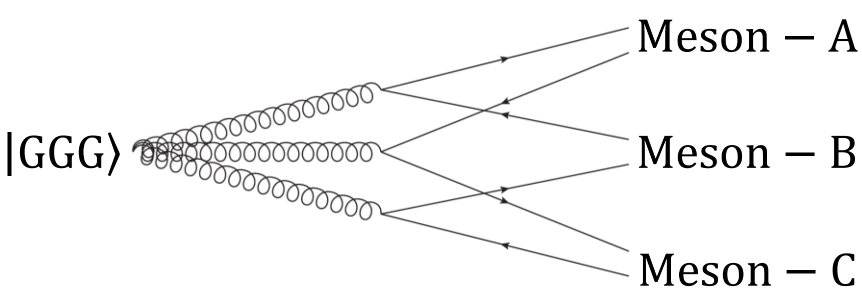
Decay analyses





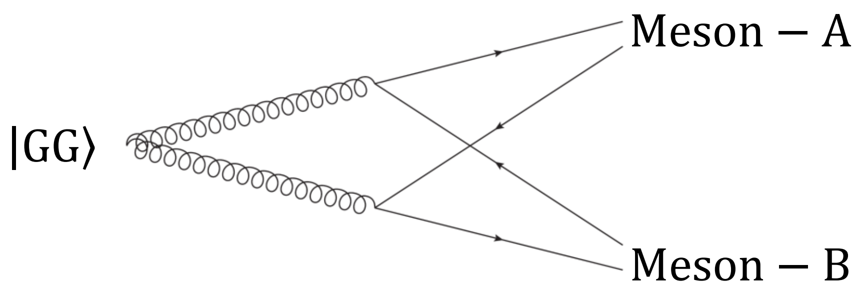
Decay analyses

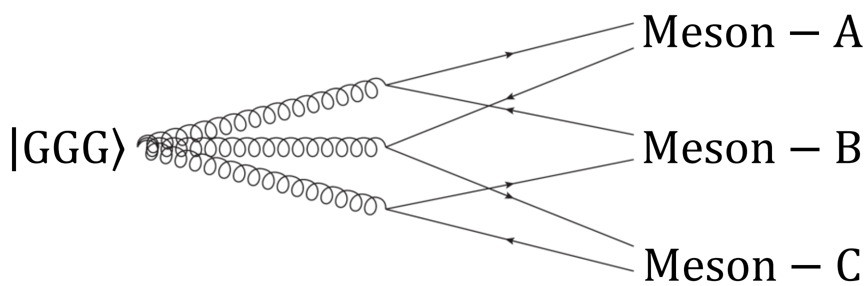




0^{-+}	$\rightarrow$	VVP, VVV	(S-wave)
0^{++}	$\rightarrow$	VPP, VVP, VVV	(P-wave)
1^{--}	$\rightarrow$	VPP, VVP, VVV	(S-wave)
1^{+-}	$\rightarrow$	PPP, VPP, VVP, VVV	(P-wave)
$2^{-\pm}$	$\rightarrow$	VVP, VVV	(S-wave)
$2^{+\pm}$	$\rightarrow$	VPP, VVP, VVV	(P-wave)
3^{--}	$\rightarrow$	VVV	(S-wave)
3^{+-}	$\rightarrow$	VVP, VVV	(P-wave)

Decay analyses





0^{-+}	$\rightarrow$	VVP, VVV	(S-wave)
0^{++}	$\rightarrow$	VPP, VVP, VVV	(P-wave)
1^{--}	$\rightarrow$	VPP, VVP, VVV	(S-wave)
1^{+-}	$\rightarrow$	PPP, VPP, VVP, VVV	(P-wave)
$2^{-\pm}$	$\rightarrow$	VVP, VVV	(S-wave)
$2^{+\pm}$	$\rightarrow$	VPP, VVP, VVV	(P-wave)
3^{--}	$\rightarrow$	VVV	(S-wave)
3^{+-}	$\rightarrow$	VVP, VVV	(P-wave)

Summary

- We study mass spectra of two- and three-gluon glueballs as well as double-gluon hybrid states through QCD sum rules.
- Our QCD sum rule results for the glueballs are generally consistent with the lattice QCD calculations.
- Honestly speaking, we still know little about glueballs and hybrid states.

Thank you very much!