

# 1+ XTZ States Within Sum Rules

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# A VERY BRIEF SUMMARY ON EXOTIC HADRONS



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## Hadrons successfully explained by Quark Model

Gell-Mann & Zweig in 1964

*PHYS. LETT. 8 (1964)*

*CERN-TH-201 & 412 (1964)*

## First experimental evidence of a tetraquark state, the $S(1930)$

*announced in 1974 by Brookhaven National Laboratory*

*A.S. CARROLL ET AL. (BNL COLLAB.), PHYS. REV. LETT. 32 (1974)*

## Within the string model, the tetraquark state could explain the $S(1930)$ bump

Rossi & Veneziano in 1977

*NUCL. PHYS. B 123 (1977)*

## The existence of a pentaquark state has been conjectured

Montanet, Rossi & Veneziano in 1980

*PHYS. REPT. 63 (1980)*

# A VERY BRIEF SUMMARY ON EXOTIC HADRONS

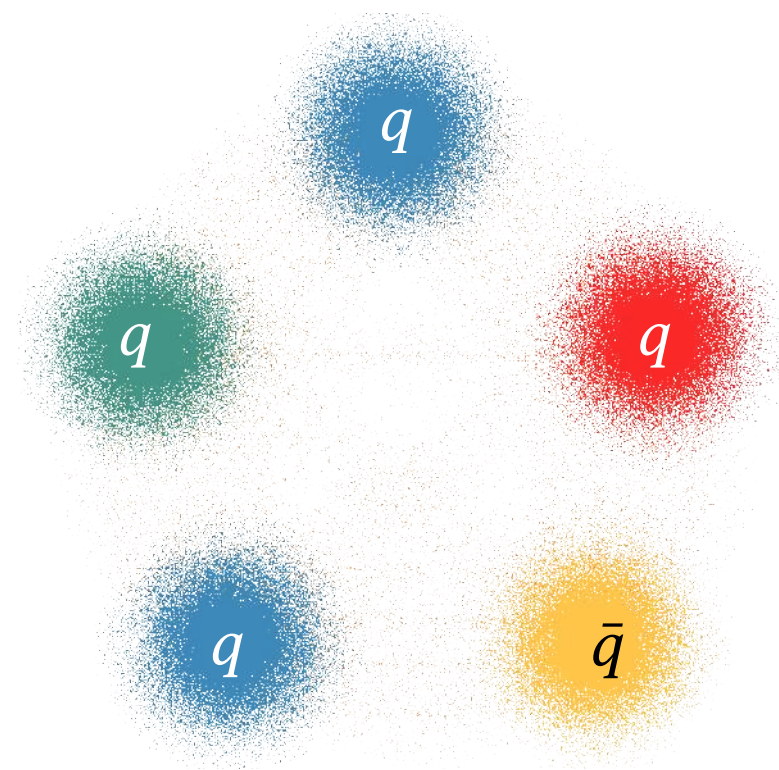
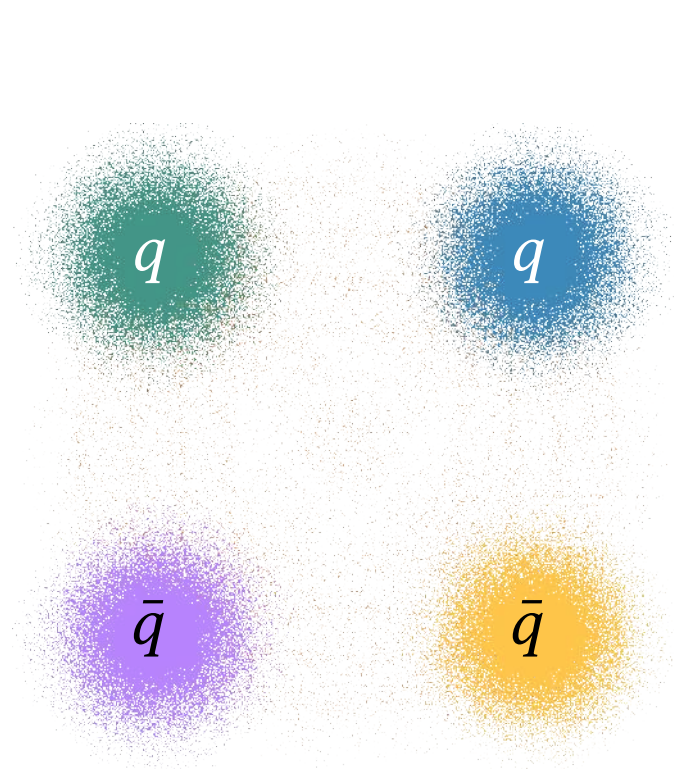
After almost 30 years...

**New experimental data are announced on the existence of a tetraquark state  $X_{(3872)}$  and a pentaquark state  $\Theta_{(1540)}$**

*both announced in 2003 by Belle Collaboration*

*S.K. CHOI ET AL. [BELLE COLLAB.], PHYS. REV. LETT. 91 (2003) 262001*

*T. NAKANO ET AL. [BELLE COLLAB.], PHYS. REV. LETT. 91 (2003) 261601*



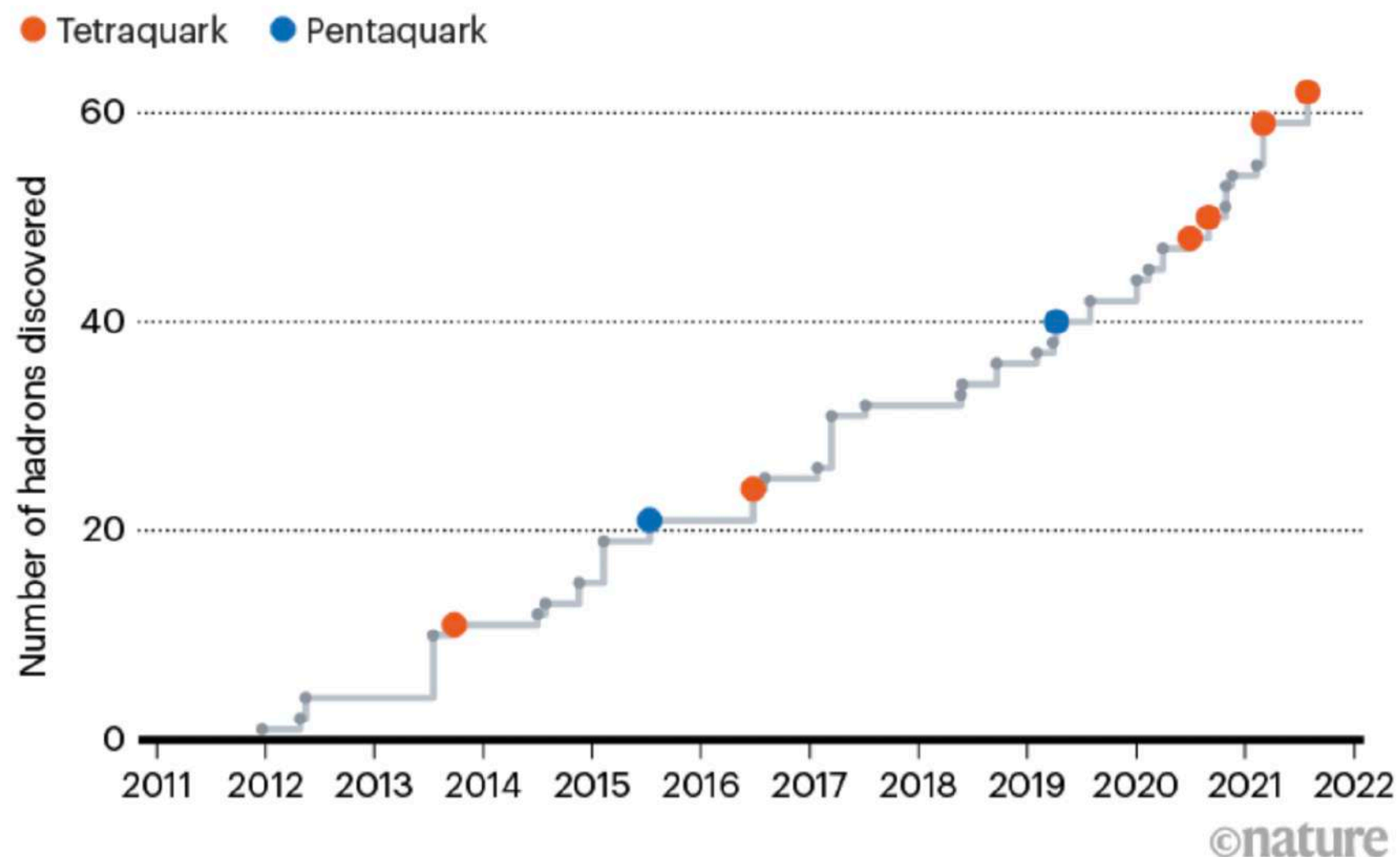
*announced*

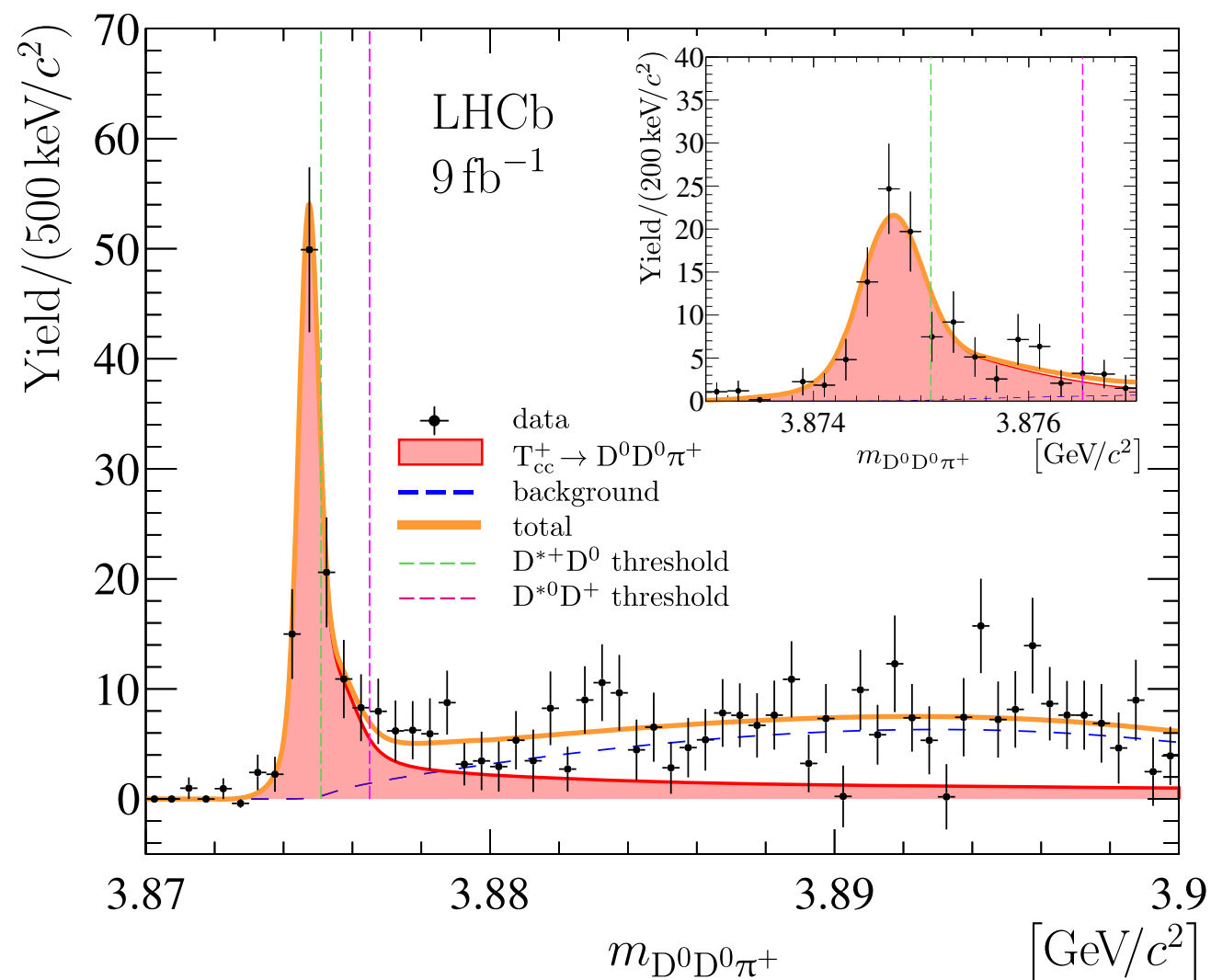
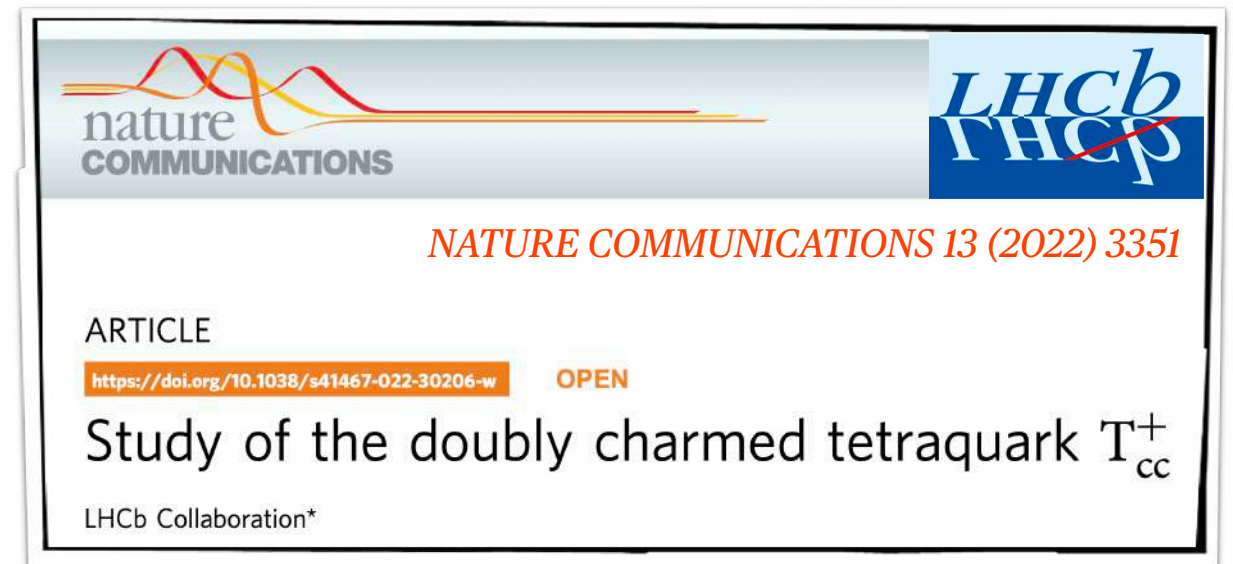
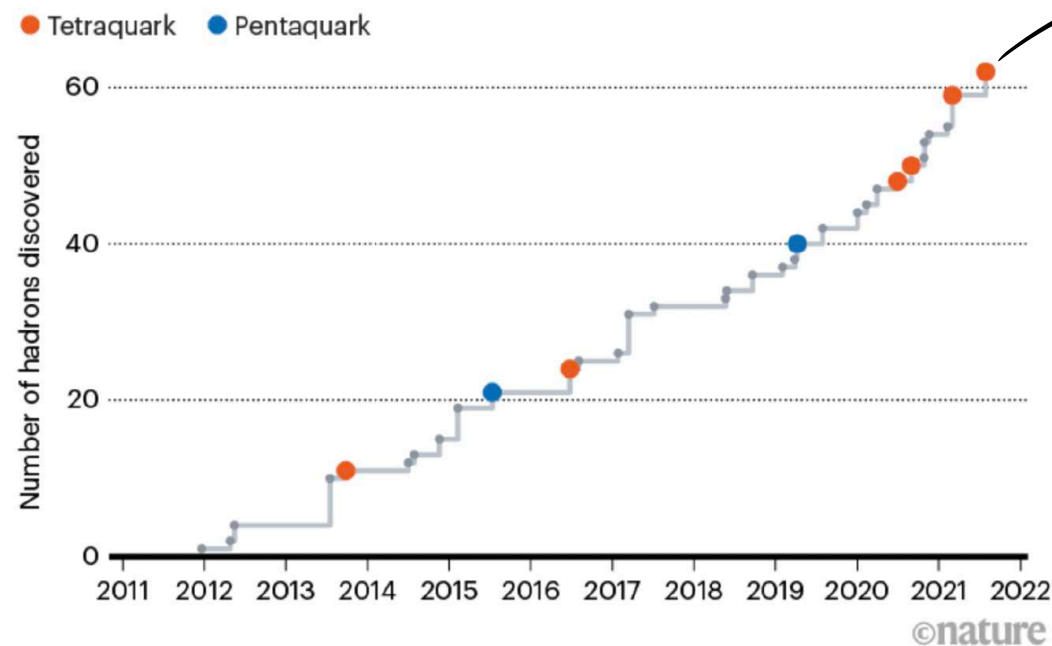
# A VERY BRIEF SUMMARY ON EXOTIC HADRONS



**Since 2013, LHCb collaboration announced the observation of several candidates for exotic hadrons**

*LHCb COLLAB., NATURE COMMUNICATIONS 13 (2022) 3351*





- LHCb Collab. announces the presence of a  $1^+$  state in the  $D^0 D^0 \pi^+$  decays.
- Absence of a signal in the  $D^0 D^+$  and  $D^+ D^0 \pi^+$  mass distributions.
- Strong argument for interpreting such a state as the **isoscalar  $T_{cc}$  tetraquark, with  $J^P = 1^+$** .
- Mass around **3875 MeV**.

# THE METHOD IN QCD



# QCD SUM RULES

**M.A. Shifman, A.I. Vainshtein, V.I. Zakharov**

*NUCL. PHYS. B 147 (1979)*

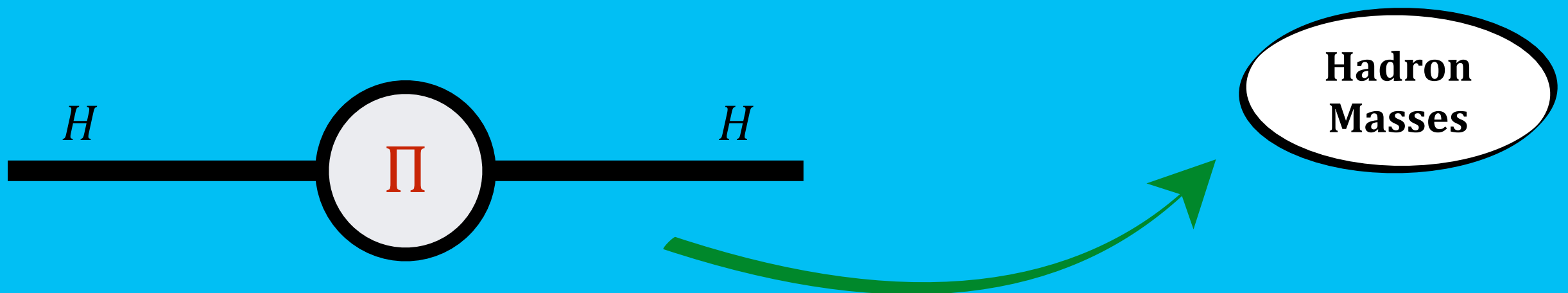
Excellent reviews on the method can be found on...

- ❖ **P. Pascual and R. Tarrach**, *"QCD: renormalization for practitioner"*, Springer (1984)
- ❖ **L. J. Reinders, H. Rubinstein and S. Yazaki**, *"Hadron Properties from QCD Sum Rules"*, Phys. Rept. 127 (1985)
- ❖ **S. Narison**, *"QCD Spectral Sum Rules"*, World Sci. Lect. Notes Phys. 26 (1989)
- ❖ **S. Narison**, *"QCD as a Theory of Hadrons"*, Cambridge Monogr. Part. Phys. Nucl. Phys. Cosmol. 17 (2004)
- ❖ **B.L. Ioffe**, *"QCD at Low Energies"*, Prog. Part. Nucl. Phys. 56 (2006)
- ❖ **H.G. Dosch**, *"Nonperturbative methods in quantum chromodynamics"*, Prog. Part. Nucl. Phys. 33 (1994)
- ❖ **E. de Rafael**, *"An Introduction to Sum Rules in QCD"*, hep-ph/9802448 (1998)
- ❖ **F.J Yndurain**, *"The Theory of Quark and Gluon Interactions"*, 3rd edition, Springer (1999)



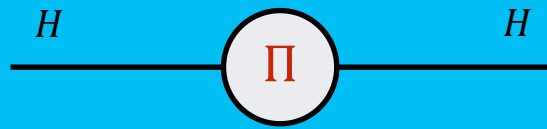
# The QCD inverse Laplace sum rules (LSR) approach

## 2-point Correlator Function



# The QCD inverse Laplace sum rules (LSR) approach

We shall be concerned with the two-point correlator:



$$\Pi_{\mathcal{H}}^{\mu\nu}(q^2) = i \int d^4x e^{iqx} \langle 0 | \mathcal{T} \mathcal{O}_{\mathcal{H}}^{\mu}(x) (\mathcal{O}_{\mathcal{H}}^{\nu}(0))^{\dagger} | 0 \rangle$$

- The local hadronic operators introduce the characteristics of the hadron  $H$ .
- It obeys Finite Energy Inverse Laplace Transform Sum Rule (LSR) and their ratios:

$$\mathcal{L}_n^c |_{\mathcal{H}}(\tau, \mu) = \int_0^{t_c} dt t^n e^{-t\tau} \frac{1}{\pi} \text{Im} \Pi_{\mathcal{H}}^{(1,0)}(t, \mu)$$

$(2M_c + m_q + m_{q'})^2$

$$\mathcal{R}_{\mathcal{H}}^c(\tau) = \frac{\mathcal{L}_1^c |_{\mathcal{H}}}{\mathcal{L}_0^c |_{\mathcal{H}}}$$

- At the **optimization point**, we deduce the ground state mass of the hadron

$$\mathcal{R}_{\mathcal{H}}^c(\tau_0) = M_{\mathcal{H}}^2$$

# The precision technique: Double Ratio of Sum Rule (DRSR)

*S. NARISON, PHYS. LETT. B 210 (1988)*

$$r_{\mathcal{H}'/\mathcal{H}}(\tau_0) \equiv \sqrt{\frac{\mathcal{R}_{\mathcal{H}'}^c}{\mathcal{R}_{\mathcal{H}}^c}} = \frac{M_{\mathcal{H}'}}{M_{\mathcal{H}}}$$

- In general, free from systematics errors.
- Provided that  $\mathcal{R}_{\mathcal{H}}^c$  and  $\mathcal{R}_{\mathcal{H}'}^c$  must **optimize** at the same values of  $\tau$  and  $t_c$ .

# OPTIMIZATION



# The stability criteria for extracting the optimal results

- **$\tau$  - stability:** region around of a minimum or inflexion points corresponding to a complete dominance of the lowest ground-state contribution.
- **$t_c$  - stability:** we take the values until it to be around the mass of the first excitation state.
- **$\mu$  - stability:** used to fix in a rigorous optimal way, the arbitrary subtraction constant appearing in the perturbative calculation and in the QCD input renormalized parameters.

**Physical observables should not depend on these parameters.**

## NLO PT corrections

NLO PT corrections justify the use of **running heavy quark mass**.

# QCD PARAMETERS

Parameters	Values	Sources	Refs.
$\alpha_s(M_Z)$	0.1181(16)(3)	$M_{\chi_{0c,b}} - M_{\eta_{c,b}}$	<i>NARISON</i>
$\bar{m}_c(m_c)$ [MeV]	1266(6)	$D, B_c \oplus J/\psi, \chi_{c1}, \eta_c$	<i>NARISON</i>
$\bar{m}_b(m_b)$ [MeV]	4196(8)	$B_c \oplus \Upsilon$	<i>NARISON</i>
$\hat{\mu}_q$ [MeV]	253(6)	Light	<i>NARISON</i>
$\hat{m}_s$ [MeV]	114(6)	Light	<i>NARISON</i>
$\kappa \equiv \langle \bar{s}s \rangle / \langle \bar{d}d \rangle$	0.74(6)	Light-Heavy	<i>ALBUQUERQUE, NARISON, NIELSEN</i>
$M_0^2$ [GeV <sup>2</sup> ]	0.8(2)	Light-Heavy	<i>NARISON, DOSCH, IOFFE, PIVOVAROV</i>
$\langle \alpha_s G^2 \rangle$ [GeV <sup>4</sup> ]	$6.35(35)10^{-2}$	Light-Heavy	<i>NARISON</i>
$\langle g^3 G^3 \rangle / \langle \alpha_s G^2 \rangle$	$8.2(1.0)$ [GeV <sup>2</sup> ]	$J/\psi$	<i>NARISON</i>
$\rho \alpha_s \langle \bar{q}q \rangle^2$ [GeV <sup>6</sup> ]	$5.8(9)10^{-4}$	Light, $\tau$ -decay	<i>NARISON, DOSCH, TARRACH</i>

The full list of references can be found in  
**Nucl. Phys. A 1023 (2022)**

# RESULTS

# THE $T_{cc}$ STATE

*PHYS. LETT. B 649 (2007)*

Since, the pioneering work of **Navarra, Nielsen and Lee**, the mass and coupling of  $T_{cc}$  and its beauty analogue have been extracted from LSR by different groups.

*1. Z.-G. WANG*

*2. AGAEV, AZIZI, H. SUNDU*

*3. TANG, WAN, MALTMAN, QIAO*

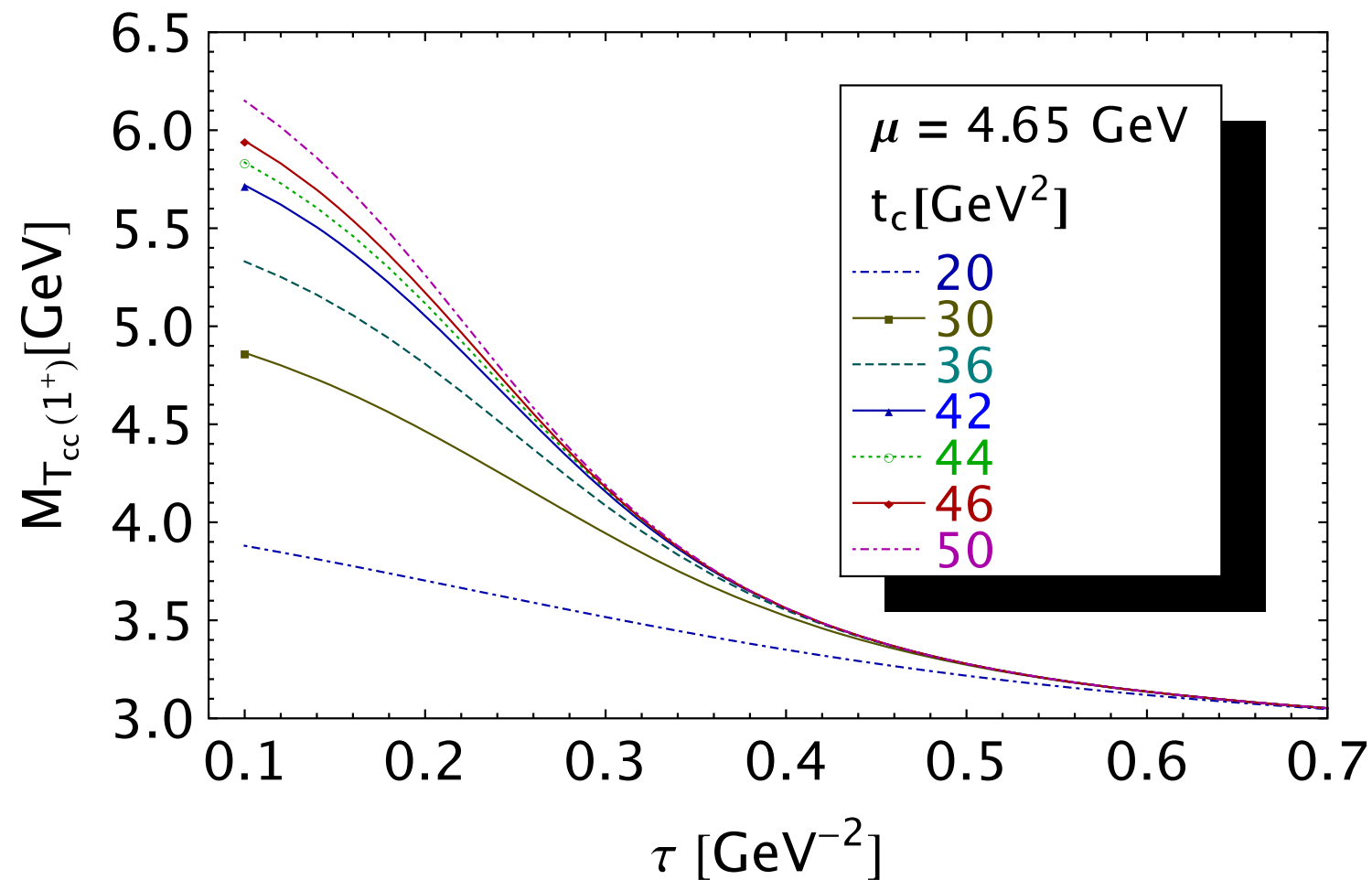
*4. DU, CHEN, ZHU*

We improve and extend the analysis using **LSR** and **DRSR** by including the factorized NLO PT contributions and controlling the different sources of the errors.

**Tetraquark:** 
$$\mathcal{O}_T^{1+} = \frac{1}{\sqrt{2}} \epsilon_{ijk} \epsilon_{mnk} \left( c_i^T C \gamma^\mu c_j \right) \left[ \left( \bar{u}_m \gamma_5 C \bar{d}_n^T \right) - \left( \bar{d}_m \gamma_5 C \bar{u}_n^T \right) \right]$$



# THE $T_{cc}$ STATE



## LSR Analysis

stability region for the set

$$\tau = \mathbf{0.31 - 0.34 \text{ GeV}^{-2}}$$

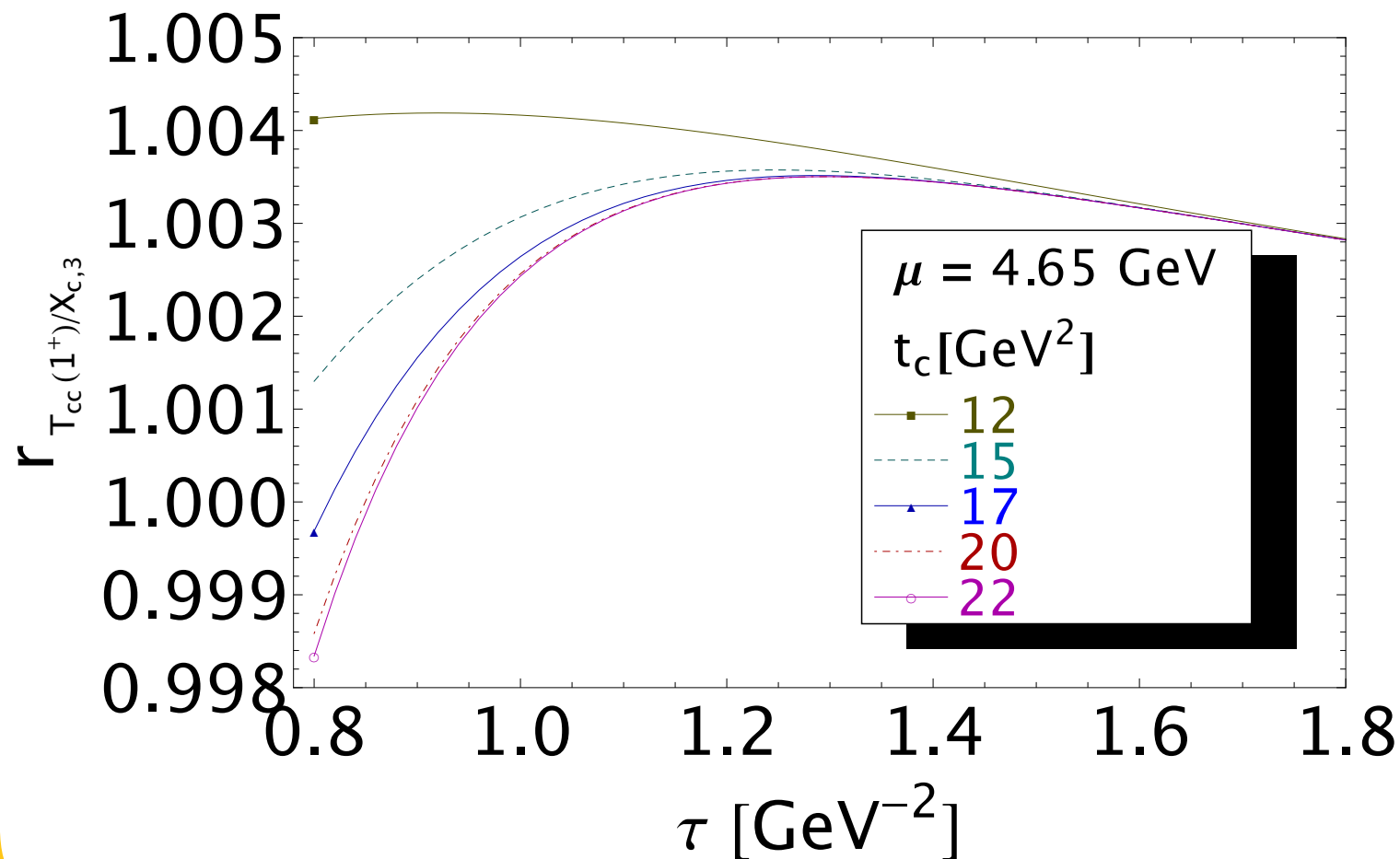
$$t_c = \mathbf{30 - 46 \text{ GeV}^2}$$

From which we deduce:

$$M_{T_{cc}}(1^+) = 3885(123) \text{ MeV}$$

This mass can be compared with the experimental value **3875 MeV**.

# THE $T_{cc}$ STATE



## DRSR Analysis

Ratio between  $T_{cc}$  and  $X_{3872}$

stability region for the set

$$\tau = \mathbf{1.24 - 1.30 \text{ GeV}^{-2}}$$

$$t_c = \mathbf{15 - 20 \text{ GeV}^2}$$

From which we deduce:

$$r_{T_{cc}^{1^+}/X_c} = 1.0035(10)$$

Taking the experimental mass of  $X_{3872}$ :  **$3871.65 \pm 0.06 \text{ MeV}$**

$$M_{T_{cc}}(1^+) = 3886(4) \text{ MeV}$$

## THE $T_{cc}$ STATE

$$M_{T_{cc}}(1^+) = 3885(123) \text{ MeV}$$

LSR Analysis

$$M_{T_{cc}}(1^+) = 3886(4) \text{ MeV}$$

DRSR Analysis

- Both results are in agreement, but we obtain a very accurate one with DRSR
- This value is comparable with the recent LHCb data:  $T_{cc}(1^+) = 3875 \text{ MeV}$
- $(9 \pm 4) \text{ MeV}$  **above** the  $D^*D$  threshold of  $3877 \text{ MeV}$ .

## THE $T_{ccsu}$ STATE

**Tetraquark:**  $\mathcal{O}_{T_{us}^{1+}} = \epsilon_{ijk} \epsilon_{mnk} \left( c_i C \gamma^\mu c_j^T \right) \left( \bar{u}_m \gamma_5 C \bar{s}_n^T \right)$

We can perform the SU(3) mass ratios between the  $T_{ccsu}$  and  $T_{cc}$

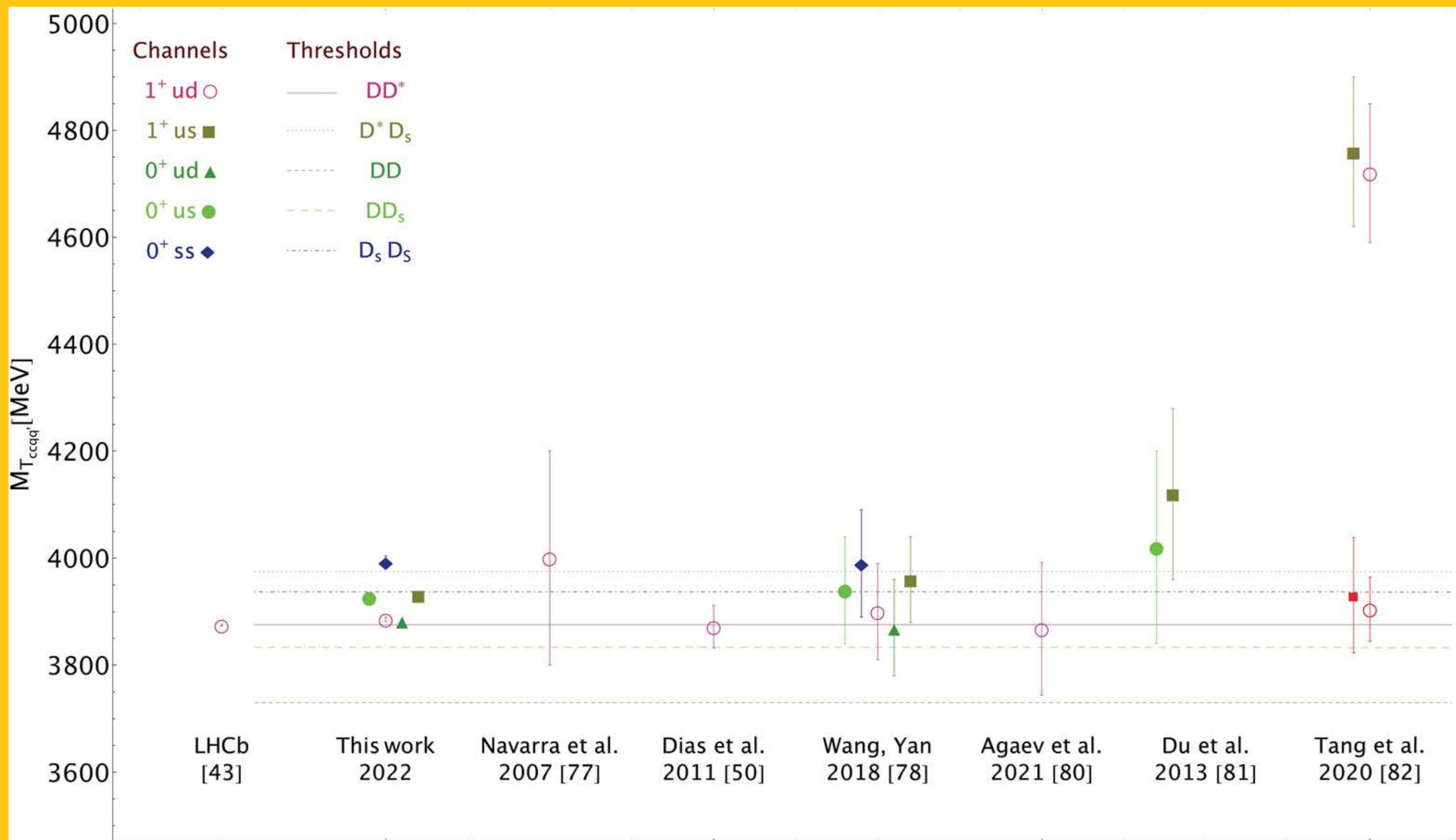
$$r_{T_{cc\bar{s}\bar{u}}/T_{cc}(1^+)} = 1.0115(13) \quad \Rightarrow \quad M_{T_{cc\bar{s}\bar{u}}}(1^+) = 3931(7) \text{ MeV}$$

### DRSR Analysis

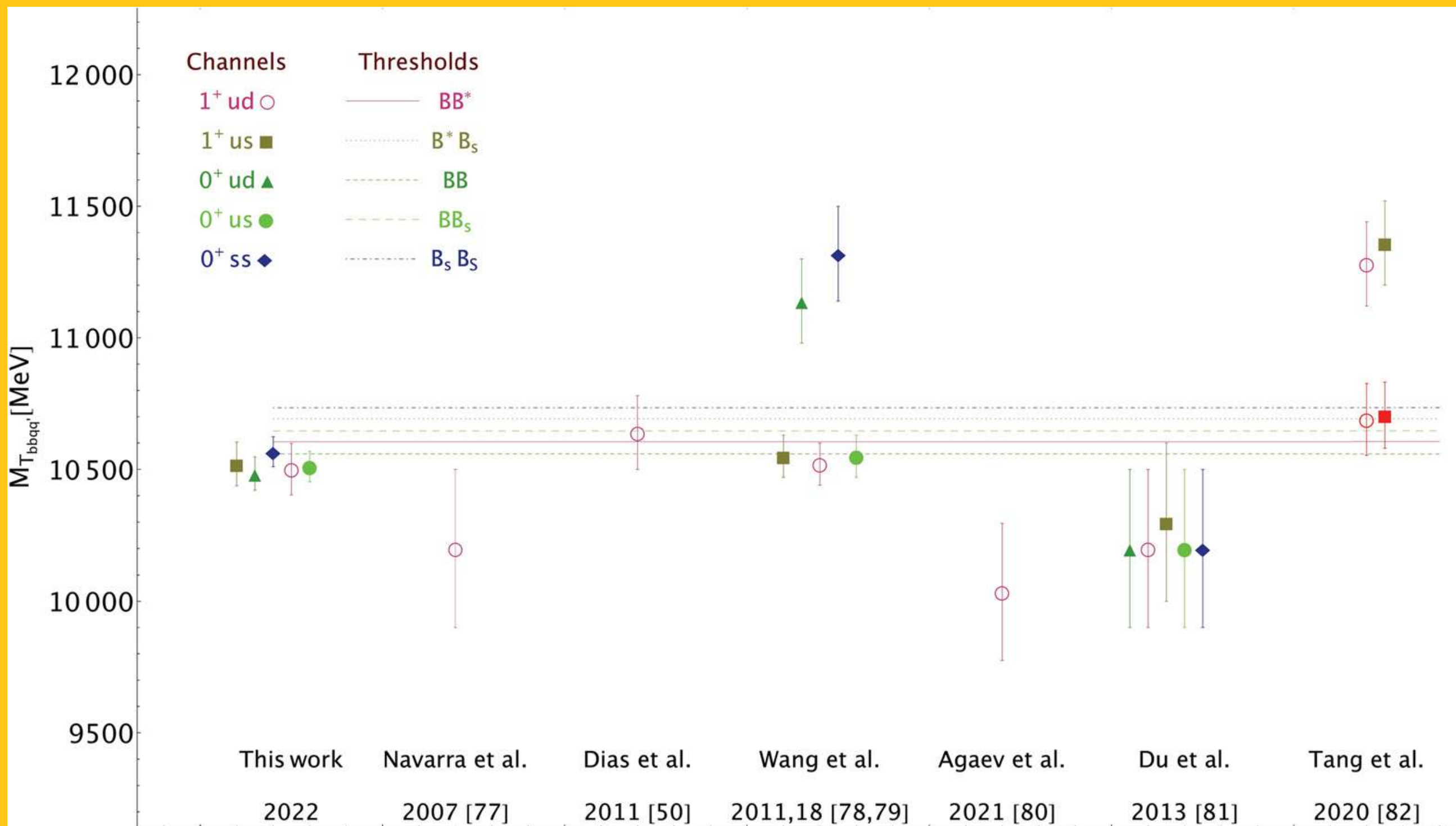
- This value is a prediction for the SU(3) breaking mass symmetry.
- We notice that the mass value is **below** the  $D^* D_s$  threshold of 3975 MeV.
- *Then we do not expect strong decay channels for this state.*
- LHCb Collaboration could provide some new results on the  $D^* D_s$  decay channel in a future publications.



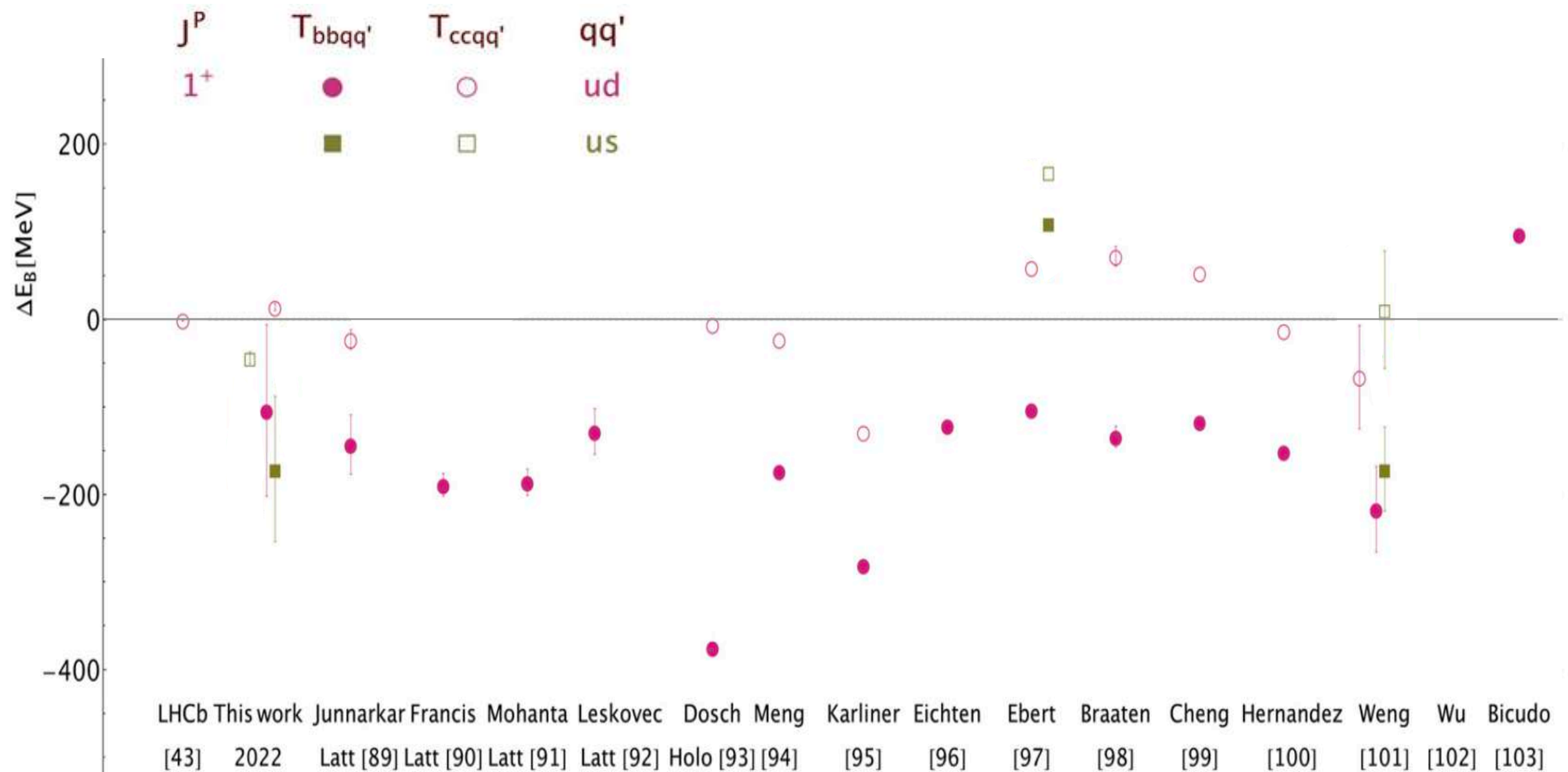
# QCD SUM RULES RESULTS FOR $T_{cc}$



# QCD SUM RULES RESULTS FOR $T_{bb}$



# Confronting DRSR results with Lattice and Quark Models



# CONCLUSIONS



# CONCLUSIONS

- We do an extensive analysis on axialvector  $(1^+)$   $T_{QQ}$ -like state.
- We confront our results with the ones from different approaches.
- The  $T_{cc}$  state is expected to be around the physical threshold.
- While the  $T_{bb}$  state is below the threshold.
- We do predictions for SU(3) breakings states and we hope LHCb could test them in a near future publications.
- In general, our results for the masses of different states are grouped around the physical thresholds.
- For more details, please check it out...

R.M. Albuquerque, S. Narison and D. Rabetiarivony  
**Nucl. Phys. A 1023 (2022)**

***THANK YOU FOR YOUR ATTENTION!***

