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ABSTRACT:

The modifications of the littoral of Morondava remain important and the decline of the coastline represents a major risk for the populations and could be pressures of social and economic order, studies have already been raised since 1914, oceanography, geology, sedimentology. Our work is dedicated to dynamic studies and finite element modeling of the physical phenomena of oceanic currents governed by Navier Stokes equations according to the conditions of tidal currents as well as currents associated with waves, the swells that derive their energy from moderate winds and river estuary on the coastal area of Morondava.

Key words: Morondava, ocean, modeling, dynamics, Navier Stokes, finite element

1. INTRODUCTION

Morondava, a town in southwestern Madagascar, is attacked by the ocean, especially since the 1950s. At the beginning of the 1951 revival of a phenomenon listed as early as 1914, the Hellot Canal (one of the Morondava River's distributaries) to the Betania Cove. Since nothing is done, neither the rip raps nor the ears, the city is slowly nibbled by the sea. In 1987, record year, the erosion is backing the shoreline of 20 m. Since 1951 the city has declined from 200 to 400 m. In 1936, all the flow of the Morondava passed by the South arm (the Kabatomena) leaving the North arm, and thus the Canal Hellot completely dry. In 1950 is dug a channel of 1000m long and 20m wide, between Betania and Port Bebe, it opens 700m from the sea in the Betania Pass. [1]

2. GENERAL CHARACTERS OF THE REGION

In 1956, the channel is redone along a new axis substantially parallel to the coast, it is wider and opens only 400m from the sea in the Betania Pass [Figure 1]. In 1952, a violent cyclone fell on the city destroying especially the harbor facilities of the coal dike.

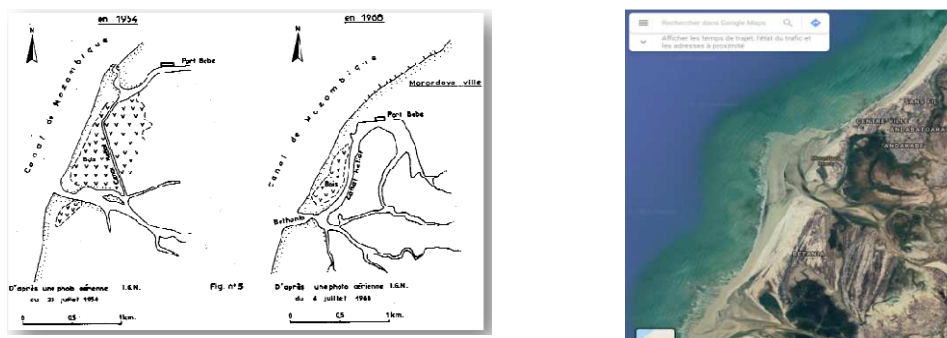


Figure 1 - Evolutions of the developments of the Hellot Canal and its mouth [2] [3]



Figure 2: Vigilance alert very strong tide from 21 to 24 December 2018 [4]

Morondava, alert very strong tide is expected from 21 to 24 December 2018. A new cycle with a very high coefficient, from 91 to 98, is expected from 21 to 24 December 2018. These very high coefficients increase the risk of submersion. The Commune recommends the utmost caution and vigilance, especially the evening of Tuesday 22 and the days of Wednesday and Thursday 23 and 24 January (coefficients of more than 100 conjugated with gusts of high winds of 43 km/h on average and heights waves up to 3.8 meters). Residents closest to the coast are urged to take precautions to protect property and people.

- **The winds**

The study of the LCHF on site of the city of Morondava leads to retain the following characteristic values [5]:
The Winds: Two main régimes:

- of West South: $V_{moy} = 4,7m/s$ relatively constant
- of West North: $V_{moy} = 4 m/s$ disrupted by the passages uncertain of the cyclones.

Hydrographic level: It is located 2.40 m below the NGM zero. We have not recorded any information on local variations of this level since the development of coastal erosion of Morondava.

- **The tide**

Table 1: Characteristic values of the tide (Morondava)

Tide	Coefficient	PM (m)	BM (m)	Amplitude (m)
Weaker neap tide	20	+ 2,68	+ 2,32	0,36
Average neap tide	45	+ 3,20	+ 1,76	1,44
Average tide	70	+ 3,72	+ 1,17	2,55
Spring tide Average	25	+ 4,25	+ 0,60	3,65
Spring tide extraordinary	100	+ 4,77	+ 0,02	

- **The currents**

Table 2: Tidal Currents (Morondava)

Direction	South West Regime		North West Regime
	Flood	Ebb	Flood and Ebb
	North East	South West	No reversal Always South West
Speed (m/s)	0,10 à 0,20	max à 0,40 if favorable winds	max 0,40

Table 3: Coastal currents

	South West Regime	North West Regime
Direction	East - North East	South - South West
Speed (m/s)	moy 0,26 max 0,37	moy 0,13 max 0,26

- **The swell**

Table 4 : The swell

	Swell West - South West	Swell of North West
Périodes	moy 11,4 s max 14 s à 18 s	moy 9,1 s max 14 s à 17 s
Amplitudes	moy 0,5 m - 0,9 m HS 0,8 m H max 2,20 m	moy 0,3 à 0,6 m HS 0,48 m H max 1,4 m
Direction	moy 265° (à l'isobathe - 4 m)	moy 313° (aux isobathes - 5 et -6 m)
Limits of the sector origin	250° - 290°	280° - 340°

- **The data by windguru [6]**

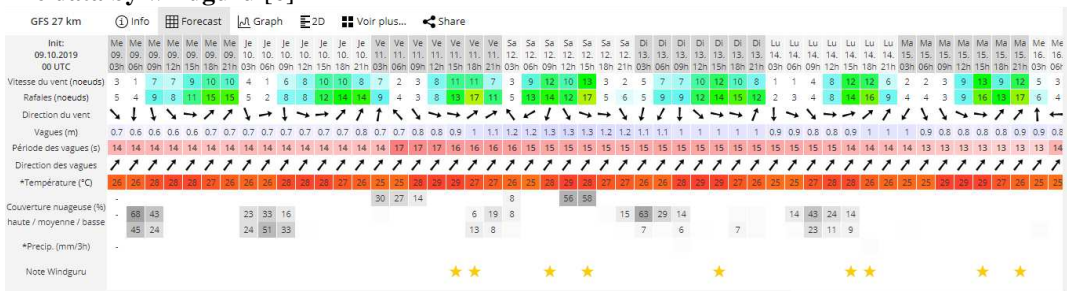


Figure 3 : Waves

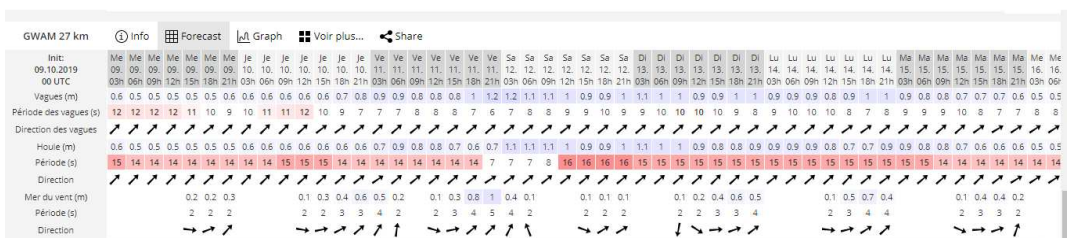


Figure 4 : The swell

This is actually the numerical wave prediction model used to simulate wave behavior every week. We will see a little further the parameters that are available on this subject.

In the same way as for atmospheric models, the mesh is then indicated (50 km). Finally, the rest corresponds to the date and time - on the Coordinated Universal Time (UTC) scale - where the simulation of the wave model was performed. Then in the upper left corner comes the forecast model used for the atmosphere as we saw earlier. This is the example of the US model GFS 27km. [6]. It can be noted that the data have similarities with the data provided by LCHF

- **Sedimentary characteristics** [2][5]
 - The slope of the foreshore varies between 3 and 4% on average between 0 m and 5 m but it increases to about 7% above the coast +3 m at the ears and tusks of the city (LCHF, 1974).
 - The front beaches have a slope that changes from 0.33% between -5 and -10 m to 0.6% between 0 and -5 m.
 - Sands have a median that varies between 0.20 and 0.25 mm; which places them in the category of very fine sands at the limit of fine sands.

3. METHODOLOGIE

3.1. Navier Stokes equations

The Navier Stokes equations is [7]

- Conservation of momentum

$$\rho \frac{\partial u}{\partial t} + \rho u \cdot \nabla u = -\nabla p + \mu \nabla^2 u + F$$

- Conservation of mass (continuity equation)

$$\frac{\partial \rho}{\partial t} + \rho \nabla \cdot u = 0$$

is solved, with ρ as the fluids density,

- u the velocity of the fluid,
- p for the pressure,
- μ as the fluids dynamic viscosity
- and F as the sum of the external forces

3.2. Sedimentary Transport

- **LCHF transport formula** [8]

Using the formulation of Sauvage of Saint-Marc and Vincent:

$$Q = \frac{K(\gamma_0, d_{50})}{\rho} (\rho_s - \rho) g H_0^2 T f(\alpha_0) = \frac{K(\gamma_0, d_{50})}{\rho} g H_0^2 T f(\alpha_0)$$

Hence, considering that the denominator is a constant close to unity ($\rho \sim 1025 \text{ kg/m}^3$, $\rho_s \sim 2500 \text{ kg/m}^3$ et $c \sim 0,7$) which can be integrated in the constant K :

$$Q = K(\gamma_0, d_{50}) g H_0^2 T f(\alpha_0)$$

The work of the Central Laboratory of Hydraulics of France (LCHF) and J. Larras led them to adopt for the constant of proportionality:

$$K(\gamma_0, d_{50}) = \frac{1,810^{-6}}{\gamma_0 \sqrt{d_{50}}}$$

α_0 : the angle between the direction of the swell to the large and the normal to the coast

d_{50} : the median diameter of the sand grains

γ_0 : the wave steepness of the swell to the large

H_0 : the height crest to hollow of the surge to the large

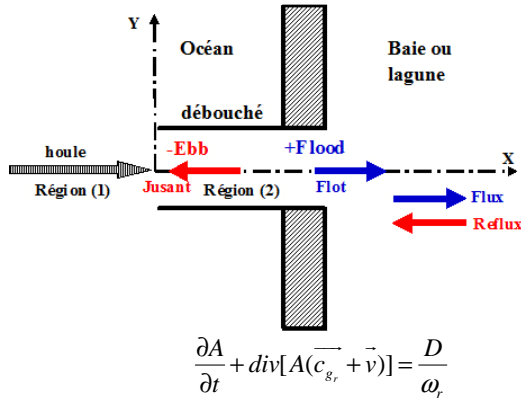
and for the obliquity function:

$$f(\alpha) = \sin\left(\frac{7\alpha}{4}\right)$$

Note that the presence of camber at the denominator seems to favor the behavior observed on reduced models.

3.3. Estuary study

A gravity wave in a tidal inlet interacts with the tidal current. On a current of ebb current, which opposes the propagation of the swell in the estuary (opposing current): the arching of the wave increases (because its amplitude H increases and its wavelength L decreases) and the wave can thus reach the break. This breaking causes a problem with the river navigation and an estuarine erosion (by transport of sediment). [9]



- \bar{E} : energy density (en J/m^2)
- E : the energy per linear meter of the ridge line (J/m)
- L : the wavelength of the swell
- ω_r : the angular pulsation relative to the current ($= 2\pi/T_r$) (en s^{-1})
- H : peak amplitude - hollow of the swell (in m)
- ρ : density of the fluid (Kg/m^3)
- g : gravity acceleration = $9,81m/s^2$
- \bar{c}_{gr} : the relative group speed (in m/s)
- $\bar{c}_{gr} + v$: absolute group speed (in m/s)
- D : rate of energy dissipation by breaking (in W/m^2)

$$\text{With } A \equiv \frac{1}{8} \rho g \frac{H^2}{\omega_r} = \frac{\bar{E}}{\omega_r} \text{ et } \bar{E} = \frac{E}{L} = \frac{\rho g H^2}{8}$$

Often in estuary the flow is 1D

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial X} \left[\frac{\bar{E}(c_{gr} + \bar{u})}{\omega_r} \right] = \frac{D}{\omega_r} \quad \bullet X : \text{the axis of the estuary with origin at the mouth}$$

$$\bullet \bar{u} : \text{uniform current on the draft (medium)}$$

At the swell - scale with respect to the tidal wave it can be admitted that the wave - tidal interaction regime that the regime is permanent:

The parameterization of the term (D) proposed by Thornton and Guza (1983) is of the following form

$$D_{\text{breaking}} = -\frac{3\sqrt{\pi}}{16} \rho g \frac{B^3}{\gamma^4 h^5} \frac{\omega_r}{2\pi} H_{rms}^7 (W / m^2)$$

$B \approx 1$ & $\gamma \approx 0,42$ are two empirical parameters related to wave breaking

One can admit that the régime of interaction swell - tide that the régime is permanent

$$\frac{\partial}{\partial X} \left[\frac{\bar{E}(c_{gr} + \bar{u})}{\omega_r} \right] = \frac{D}{\omega_r}$$

The angular frequency of the wave in a reference frame in translation at the speed \bar{u}

$$\omega_r = \omega + k_r \bar{u}$$

or ($\omega = 2\pi f$) is the absolute frequency

The relative wavelength is calculated by

$$L_r = \frac{2\pi}{k_r}$$

In shallow water the dispersion equation is reduced to:

$$k_r = \frac{\omega}{u + \sqrt{gh}}$$

where the relative angular frequency is given by:

$$\omega_r = \sqrt{gk_r \tanh(k_r h)}$$

The relative group speed is given by:

$$c_{gr} = \frac{1}{2} \frac{\omega_r}{k_r} \left(1 + \frac{2k_r h}{\sinh(2k_r h)} \right)$$

4. RESULTAT

4.1. Navier Stokes

To solve the Equations of Navier Stokes, these equations are modelised by finit element. Solving them, for a particular set of boundary conditions (such as inlets, outlets, and the feature of coasts), predicts the fluid velocity and its pressure in a given geometry

The fluid velocity is specified at the inlet and pressure prescribed at the outlet

A no-slip boundary condition (i.e., the velocity is set to zero) is specified at the feature of coast

- The boundary conditions

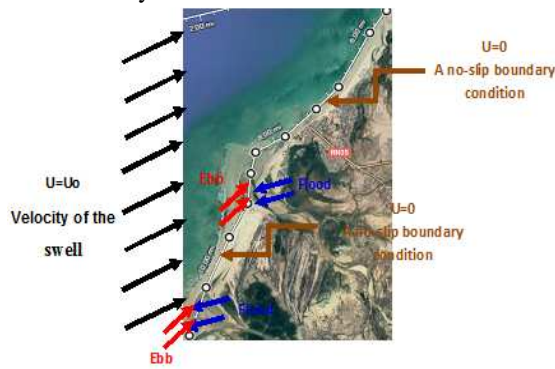


Figure 5: Boundary conditions

- Distribution of the marine current near Morondava: in their modeling, so the distributions of the velocity of the current to the surroundings of the region

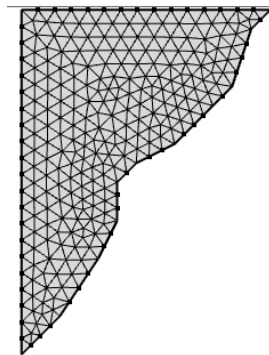


Figure 6: Mesh refinement

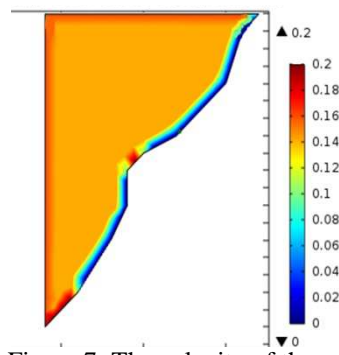


Figure 7: The velocity of the current to the surroundings of the region

4.2. The estuary : example the application

- A swell of amplitude ($H = 2,2m$) in water of depth (10 m) and period (11,4s) where the current is very low (0m/s)
- Enters a estuary of medium depth (5m) and where the ebb tide is the order of (-0,4m/s)

Pour ($h = 10m$) on a : ($kh \ll 1$)

Or $\omega^2 \approx gk \tanh(kh)$ avec $\tanh(kh) \approx kh - \frac{1}{3}(kh)^3 + O([kh]^5)$

$$\omega^2 = gk.kh \left(1 - \frac{1}{3}k^2h^2 \right)$$

The wavelength $L_1 = 107.068$ m, indeed

$$L_1 = \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi h_1}{L_1}\right)$$

$$= \frac{9,81 \times 11,4^2}{2 \times 3.14} \tanh\left(\frac{2 \times 3,14 \times 10}{107,068}\right) = 107,068m \text{ is verified}$$

$$\gamma = \frac{H_1}{L_1} = \frac{2,2}{107,068} = 0,020547$$

The celerity is $c_1 = 9,392m/s$ car

$$C_1 = \frac{gT}{2\pi} \tanh\left(\frac{2\pi h_1}{L_1}\right) = 9,392m/s$$

$$= \frac{9,81 \times 11,4}{2 \times 3.14} \tanh\left(\frac{2 \times 3,14 \times 10}{107,068}\right) = 9,392m/s$$

Group speed is ($c_g = 10,59m$ $sg_1 =$) $\ll 2k_1h_1 = 1,08455 \gg$ because

$$C_{g1} = \frac{1}{2} \left(1 + \frac{2k_1h_1}{\sinh(2k_1h_1)} \right) c_1 = 8,467m/s$$

$$= \frac{1}{2} \left(1 + \frac{2 \times 0,0586 \times 10}{\sinh(2 \times 0,0586 \times 10)} \right) \times 9,392 = 8,467m/s$$

In the estuary where ($h_2 = 5m$) et ($u_2 = -0,4m/s$) we have: $L_{r2} = 72,95m$

We have the dispersion relation with the current:

$$\frac{2\pi}{T} - \frac{2\pi}{L_{r2}} u_2 = \sqrt{g \frac{2\pi}{L_{r2}} \tanh\left(\frac{2\pi h_2}{L_{r2}}\right)} = 0,585311$$

$$\frac{2\pi}{T} - \frac{2\pi}{L_{r2}} u_2 = \frac{2 \times 2,14}{11,4} - \frac{2 \times 3,14}{72,95} (-0,4) = 0,585311 \sqrt{g \frac{2\pi}{L_{r2}} \tanh\left(\frac{2\pi h_2}{L_{r2}}\right)} = \sqrt{9,81 \times \frac{2 \times 3,14}{72,95} \tanh\left(\frac{2 \times 3,14 \times 6}{72,95}\right)} = 0,585321$$

The relationship above is verified

The relative angular frequency is given by:

$$\omega_{r2} = \sqrt{g k_{r2} \tanh(k_{r2} h_2)} = 0,58532167$$

$$\frac{2\pi}{T_{r2}} = 0,58532167$$

$$\Rightarrow T_{r2} = 10,7291431 \text{ s}$$

($T_{r2} = 10,7291431 \text{ s}$) in the estuary instead of ($T = 11,4 \text{ s}$) without current ebb

The relative speed of the wave in the estuary

$$C_{r2} = \frac{L_{r2}}{T_{r2}} = 6,7992 \text{ m/s}$$

Group speed is in the estuary

$$k_{r2} = \frac{2\pi}{L_{r2}} = 0,08608 \quad c_{gr2} = \frac{1}{2} \frac{\omega_{r2}}{k_{r2}} \left(1 + \frac{2k_{r2} h_2}{\sinh(2k_{r2} h_2)}\right) = 6,413 \text{ m/s}$$

The amplitude of the wave in the estuary

$$H_2 = H_1 \sqrt{\left(\frac{c_{g1}}{c_{g2} + \bar{u}_2}\right)_1 \left(\frac{1}{1 + \frac{\bar{u}_2}{c_{g2}}}\right)} = 2,6960 \text{ m}$$

We then deduce the wave steepness of the wave in the estuary $\gamma_2 = \frac{H_2}{L_{r2}} = 0,03695$

4.3. Transport Sédimentaire

$$Q = K(\gamma_0, d_{50}) g H_0^2 T f(\alpha_0)$$

Outside of the estuary		At the estuary	
d_0	0,2	d_0	0,2
H_0	2,2	H_0	2,69609
T	11,4	T	10,7291431
L_0	107,068	L_0	107,068
Direction 265 degré	1,48277778	Direction 265 degré	1,48277778
Q	1,73 10⁶ m³/year	Q	1,36 10⁶ m³/year

From the other researchers: to the level of Dabara (Hellot channel), the transportation thin sands in suspension is of 1,66 millions of m³ per year. In period of rises 2,8 millions of m³ [Carnus F., Rabehasinina A., Raharijaona N. (1998) et Razafimbelo M. R.I.] , there are some similarities between the calculations and the observation by other researchers

To conclude: We did the modelling and calculations, one can always improve them

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