Naturalness in a type II seesaw model and implications for physical scalars

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Abstract

In this paper we consider a minimal extension to the standard model by a scalar triplet field with hypercharge $Y = 2$. This model relies on the seesaw mechanism which provides a consistent explication of neutrino mass generation. We show from naturalness considerations that the Veltman condition is modified by virtue of the additional scalar charged states and that quadratic divergencies at one loop can be driven to zero within the allowed space parameter of the model, the latter is severely constrained by unitarity, boundedness from below and is consistent with the di-photon Higgs decay data of LHC. Furthermore, we analyse the naturalness condition effects to the masses of heavy Higgs bosons H^0, A^0, $H^{\pm}$ and $H^{\pm\pm}$, providing a drastic reduction of the ranges of variation of $m_{H^{\pm}}$ and $m_{H^{\pm\pm}}$ with an upper bounds at 288 and 351 GeV respectively, while predicting an almost degeneracy for the other neutral Higgs bosons H^0, A^0 at about 207 GeV.

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I. INTRODUCTION

After the LHC's Run 1 and beginning of Run 2, we are now more confident that the observed 125 GeV scalar boson is the long sought Higgs boson of the Standard Model (SM) [1, 2]. However, although its brilliant success in describing particle physics, still many pressing questions are awaiting convincing solutions that cannot be answered within SM. The hierarchy problem and the neutrinos oscillations are the most illustrative ones. In this context, many theoretical frameworks have been proposed and the most popular one is Supersymmetry.

The search for Supersymmetry at Run I of LHC gave a negative result. Therefore the original motivation of Susy to solve hierarchy problem by suppressing quadratic divergencies (QD) is questionnable. In this case, it is legitimate to propose other perspective to interpret and control the QD. It is known that one has to call upon new physics to deal with such problem. More specifically, the new degrees of freedom in a particular model conspire with those of the Standard Model to modify the Veltman Condition and to soften the divergencies [3–6].

In this paper, we aim to investigate the naturalness problem in the context of Type II Seesaw model, dubbed HTM, with emphasis on its effect of the HTM parameter space . More precisely, we will study how to soften the divergencies and how to gain some insight on the allowed masses of the heavy scalars in the Higgs sector. A more recent work of Kundu et al.[7] has partially discussed this issue. However, unlike the analysis in [7], our study use the most general renormalisable Higgs potential of HTM [8] and is essentially based on dimensional regularisation approach which complies with unitarity and Lorentz invariance [8]. More importantly, the phenomenological analysis takes into account the full set of theoretical constraints, including unitarity [8] and the consistent conditions of boundedness from below [8, 9].

This work is organised as follows. In section 2, we briefly review the main features of Higgs Triplet Model and present the full set of constraints on the parameters of the Higgs potential. Section 3 is devoted to the derivation of the modified Veltman condition (mVC) in HTM. The analysis and discussion of the results are performed in section 4, with emphasis on the effects of mVC on the heavy Higgs bosons, particularly on charged Higgs. Conclusion with summary of our results will be drawn in section 5.

II. SEESAW TYPE II MODEL: BRIEF REVIEW

Type II seesaw mechanism can be implemented in the Standard Model via a scalar field Δ transforming as a triplet under the $SU(2)_L$ gauge group with hypercharge $Y_\Delta = 2$. In this case the most general $SU(2)_L \times U(1)_Y$ gauge invariant Lagrangian of the HTM scalar sector is given by [8, 10]:

$$\mathcal{L} = (D_\mu H)^\dagger (D^\mu H) + Tr(D_\mu \Delta)^\dagger (D^\mu \Delta) - V(H, \Delta) + \mathcal{L}_{\text{Yukawa}} \tag{2.1}$$

The covariant derivatives are defined by,

$$D_\mu H = \partial_\mu H + igT^a W_\mu^a H + i\frac{g'}{2} B_\mu H \tag{2.2}$$

$$D_\mu \Delta = \partial_\mu \Delta + ig[T^a W_\mu^a, \Delta] + ig'\frac{Y_\Delta}{2} B_\mu \Delta \tag{2.3}$$

where H is the Higgs doublet while (W_μ^a, g), and (B_μ, g') represent the $SU(2)_L$ and $U(1)_Y$ gauge fields and couplings respectively. $T^a \equiv \sigma^a/2$, with σ^a ($a = 1, 2, 3$) are the Pauli matrices. The potential $V(H, \Delta)$ reads as,

$$\begin{aligned} V(H, \Delta) = {} & -m_H^2 H^\dagger H + \frac{\lambda}{4}(H^\dagger H)^2 + M_\Delta^2 Tr(\Delta^\dagger \Delta) + [\mu(H^T i\sigma^2 \Delta^\dagger H) + h.c.] \\ & + \lambda_1 (H^\dagger H) Tr(\Delta^\dagger \Delta) + \lambda_2 (Tr\Delta^\dagger \Delta)^2 + \lambda_3 Tr(\Delta^\dagger \Delta)^2 + \lambda_4 H^\dagger \Delta \Delta^\dagger H \end{aligned} \tag{2.4}$$

where Tr denotes the trace over 2×2 matrices. The Triplet Δ and doublet Higgs H are represented by:

$$\Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix} \quad \text{and} \quad H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{2.5}$$

with $\delta^0 = \frac{1}{\sqrt{2}}(v_t + \xi^0 + iZ_2)$ and $\phi^0 = \frac{1}{\sqrt{2}}(v_d + h + iZ_1)$.

After the spontaneous electroweak symmetry breaking, the Higgs doublet and triplet fields acquire their vacuum expectation values v_d and v_t respectively, and seven physical Higgs bosons appear, consisting of: two CP_{even} neutral scalars (h^0, H^0), one neutral pseudo-scalar A^0 and a pair of simply and doubly charged Higgs bosons $H^\pm$ and $H^{\pm\pm}$. [1]. The masse of

[1] For details on mixing angles for CP_{even}, CP_{odd} and the charged sectors, dubbed α, β and β' see [8]

these Higgs bosons are given by [8],

$$m_{h^0} = \frac{1}{2}\left(A + C - \sqrt{(A-C)^2 + 4B^2}\right) \tag{2.6}$$

$$m_{H^0} = \frac{1}{2}\left(A + C + \sqrt{(A-C)^2 + 4B^2}\right) \tag{2.7}$$

$$m^2_{H^{\pm\pm}} = \frac{\sqrt{2}\mu\upsilon_d^2 - \lambda_4\upsilon_d^2\upsilon_t - 2\lambda_3\upsilon_t^3}{2\upsilon_t} \tag{2.8}$$

$$m^2_{H^{\pm}} = \frac{(\upsilon_d^2 + 2\upsilon_t^2)\,[2\sqrt{2}\mu - \lambda_4\upsilon_t]}{4\upsilon_t} \tag{2.9}$$

$$m^2_{A^0} = \frac{\mu(\upsilon_d^2 + 4\upsilon_t^2)}{\sqrt{2}\upsilon_t} \tag{2.10}$$

The coefficients A, B and C are the entries of the CP_{even} mass matrix defined by,

$$A = \frac{\lambda}{2}v_d^2, \quad B = v_d(-\sqrt{2}\mu + (\lambda_1 + \lambda_4)v_t), \quad C = \frac{\sqrt{2}\mu\, v_d^2 + 4(\lambda_2 + \lambda_3)v_t^3}{2v_t} \tag{2.11}$$

In the remainder of this paper, we assume the light CP_{even} scalar h^0 as the observed Higgs with mass about $m_{h^0} \simeq 125$ GeV.

III. THEORETICAL AND EXPERIMENTAL CONSTRAINTS

The HTM Higgs potential parameters are not free but have to obey several constraints originating from theoretical requirements and experimental data. Thus any phenomenological studies are only reliable in the allowed region of HTM parameter space.

ρ parameter:

First, recall that the ρ parameter in HTM at the tree level is given by the formula, $\rho \simeq 1 - 2\frac{v_t^2}{v_d^2}$ which indicates a deviation from unity. Consistency with the current limit on ρ from precision measurements [11] requires that the limit $|\delta\rho| \leq 10^{-3}$ resulting in an upper limit on v_t about ≤ 5 GeV.

Masses of Higgs bosons :

Many experimental mass limits have been found for the Heavy Higgs bosons. From the LEP direct search results, the lower bounds on $m_{A^0,H^0} > 80 - 90$ GeV for models with more than one doublet in the case of the neutral scalars.

As to the singly charged Higgs mass we use the LEP II latest bounds, $m_{H^\pm} \geq 78$ GeV from direct search results, whereas the indirect limit is slightly higher $m_{H^\pm} \geq 125$ GeV [12].

Furthermore, the present lower bound from LHC is $m_{H^\pm} \leq 666$ GeV, where the excluded mass ranges established by ATLAS [13] and CMS [14] are taken into account. In the case of the doubly charged Higgs masses, the most recent experimental upper limits reported by ATLAS and CMS are respectively $m_{H^{\pm\pm}} \geq 409$ GeV [15] and $m_{H^{\pm\pm}} \geq 445$ GeV [16]. These bounds originate from analysis assuming 100% branching ratio for $H^{\pm\pm} \rightarrow l^\pm l^\pm$ decay. However, note that one can find realistic scenarios where this decay channel is suppressed with respect to $H^{\pm\pm} \rightarrow W^\pm W^{\pm(*)}$ [17] invalidating partially the LHC limits. For example, In HTM with moderate triplet' VEV, $v_t \approx 1$ GeV, the analysis of $H^{\pm\pm} \rightarrow W^\pm W^{\pm *}$ decay channel can easily overpasses the two-sign same lepton channel for $m_{H^{\pm\pm}}$ where the limit decreases up to $90 - 100$ GeV [18].

As to the theoretical constraints on the parameter space, we should take into account the perturbativity constraints on the λ_i as well as the stability of the electroweak vacuum that ensure that the potential is bounded from below (BFB). Let us first recall all the constraints obtained in [8][2]

<u>BFB</u>:

$$\lambda \geq 0 \;\;\&\;\; \lambda_2 + \lambda_3 \geq 0 \;\;\&\;\; \lambda_2 + \frac{\lambda_3}{2} \geq 0 \tag{3.1}$$

$$\&\;\; \lambda_1 + \sqrt{\lambda(\lambda_2 + \lambda_3)} \geq 0 \;\;\&\;\; \lambda_1 + \lambda_4 \sqrt{\lambda(\lambda_2 + \lambda_3)} \geq 0 \tag{3.2}$$

$$\&\;\; \lambda_3 \sqrt{\lambda} \leq |\lambda_4| \sqrt{\lambda_2 + \lambda_3} \;\text{ or }\; 2\lambda_1 + \lambda_4 + \sqrt{(\lambda - \lambda_4^2)(2\frac{\lambda_2}{\lambda_3} + 1)} \geq 0 \tag{3.3}$$

[2] Notice here that for BFB, the new BFB condition derived by [9] has been used. However, our analysis is almost insensitive to the modified BFB.

Unitarity:

$$|\lambda_1 + \lambda_4| \leq 8\pi \tag{3.4}$$

$$|\lambda_1| \leq 8\pi \tag{3.5}$$

$$|2\lambda_1 + 3\lambda_4| \leq 16\pi \tag{3.6}$$

$$|\lambda| \leq 16\pi \tag{3.7}$$

$$|\lambda_2| \leq 4\pi \tag{3.8}$$

$$|\lambda_2 + \lambda_3| \leq 4\pi \tag{3.9}$$

$$|\lambda + 4\lambda_2 + 8\lambda_3 \pm \sqrt{(\lambda - 4\lambda_2 - 8\lambda_3)^2 + 16\lambda_4^2}\,| \leq 32\pi \tag{3.10}$$

$$|3\lambda + 16\lambda_2 + 12\lambda_3 \pm \sqrt{(3\lambda - 16\lambda_2 - 12\lambda_3)^2 + 24(2\lambda_1 + \lambda_4)^2}\,| \leq 32\pi \tag{3.11}$$

$$|2\lambda_1 - \lambda_4| \leq 16\pi \tag{3.12}$$

$$|2\lambda_2 - \lambda_3| \leq 8\pi \tag{3.13}$$

We stress here that all values presented in the plots and in our subsequent analysis (section 5) are consistent with all theoretical and experimental bounds described in this section.

IV. THE MODIFIED VELTMAN CONDITION

The method to collect the quadratic divergencies in a framework of dimensional regularisation is due to Veltman [19] and needs a dimensional gymnastics since the space-time dimension to pick up the quadratic divergencies depends on the number of loops. But the idea to use the quadratic divergencies to get physical insights is much older, and goes back to the pioneering work of Stuckelberg in 1939 [20]. In the context of a renormalisable theory (which is our case), it is not clear that the deductions coming from such a concept are pertinent. Indeed it provides for relations between the masses or/and the couplings constants of the theory, which could give some credit to an underlying and hypothetical symmetry.

From a more basic level, one can imagine that the relations found by this method are indicative and could provide orders of magnitude, bounds or constraints and we will follow this line in this work. This is more or less the point of view of Chakraborty-Kundu [7] where they apply the cancellation of quadratic divergencies to the HTM, the Standard Model extended by a scalar triplet. But we differ from this work in the way to pick up the quadratic divergencies. They use the cut-off method to regularise the divergent integrals although we will use the Veltman method that we will sketch later on. Both methods have advantages and back-draws that we will not discuss here, since they provide for similar relations. We just mention that the cut-off method is intuitive but neither Lorentz nor gauge invariant. A contrario, the dimensional regularisation method is both Lorentz and gauge invariant, although less intuitive. We refer to previous works such as [21] and references therein for more details and discussions.

To check the Veltman conditions one needs to calculate the quadratic divergencies which show up in the tadpoles of the two CP-even neutral Higgs of our model, namely h^0 and H^0. Since vacuum is supposed to be CP-even, the tadpole associated with the neutral pseudoscalar field A^0 vanishes. Also, it is noticeable that no QCD contribution appears at one loop level, hence only the electro-weak part of the HTM model is concerned in this procedure. Since there is no derivative couplings, it is easy to figure out how to calculate each diagram (see the appendix for all relevant vertices required for such calculation). Aside the coupling constant, we just need to consider the propagator of the field in the loop, where the contribution of each loop can be written in a simple form in terms of the Passarino-Veltman function $A_0(m^2) = \frac{i}{16\pi^2}\int \frac{d^n q}{q^2-m^2}$ [22].

In a 4 dimensional space-time, $A_0(m^2) = m^2(\Delta_4 + 1 - \log m^2)$, up to some pure numerical factor, where Δ_4 is the pole term. In a 2 dimensional space-time, $A_0(m^2)$ is no longer mass dimensioned and it is a pure U.V. divergent number; this property is fundamental because it guarantees a gauge invariant result. This pure number will be forgotten everywhere in this section since irrelevant in the equations, together with other common factors like the dimensional dependence of the coupling constants.

For the propagator of a scalar particle in HTM Higgs sector, $(h^0, H^0, A^0, G^0, G^\pm, \eta^Z, \eta^\pm)$, we have $\frac{i}{q^2-m^2}$ leading to a $A_0(m^2)$ contribution. For a vectorial particle $(W^\pm, Z)$, we have $-i\left(\frac{T^{\mu\nu}}{q^2-m^2} + \xi\frac{L^{\mu\nu}}{q^2-\xi m^2}\right)$ leading to a $((n-1)A_0(m^2) + \xi A_0(\xi m^2))$ contribution, where $T^{\mu\nu}$ and $L^{\mu\nu}$ are the transverse and the longitudinal projectors and $n = 2$ is the space-time dimension. For a true fermionic particle (massive leptons and quarks), we have $i\frac{\gamma.q+m}{q^2-m^2}$ leading to a $mA_0(m^2)$ contribution.

To get the final results, one just has to sum up all the possible diagrams, taking into account the -1 for the fermionic loops (including Faddeev-Popov ghosts), the symmetry factor s_i of the diagram i and possibly the color factor for the quarks that we forget for conciseness. As a matter of fact, for the Higgs particle h_0 one gets:

$$T_{h_0} = \Sigma_{i=1}^{9} c_i s_i t_i - \Sigma_{fermions} c_{10} s_{10} t_{10} - \Sigma_{i=11}^{12} c_i s_i t_i$$

where the couplings c_i, the symmetry factors s_i and the propagator loops t_i are given in the appendix. This formula leads, in a 2 dimensional space time, to a quite large formula depending on many parameters of the model as well as the mixing angles.

Similarly, for the other CP-even Higgs particle H^0, one only has to change $c_i t_i$ into $C_i T_i$ as indicated in the appendix where these coefficients are listed.

To summarise, the Veltman condition implies that the quadratic divergencies of the two possible tadpoles T_{h^0} and T_{H^0} of the h^0 and H^0 CP-even neutral scalar fields vanish. Both results are not very tractable, so we do not write them. The linear combination of the fermionic coupling constants $s_\alpha c_{f\bar{f}} + c_\alpha C_{f\bar{f}}$ is zero; further it turns out that the combination $s_\alpha T_{h^0} + c_\alpha T_{H^0}$ induces simplification and one ends up with the short and nice expression:

$$T_t = 4\frac{m_W^2}{v_{sm}^2}(\frac{1}{c_w^2} + 1) + (2\lambda_1 + 8\lambda_2 + 6\lambda_3 + \lambda_4) \tag{4.1}$$

where $c_w = \cos\theta_{weinberg}$ and $v_{sm}^2 = v_d^2 + 4v_t^2$ is the square of the Standard Model vacuum expectation value (≈ 246 GeV).

Of course, it is very tempting to calculate the orthogonal combination, $c_\alpha T_{h^0} - s_\alpha T_{H^0}$, which also leads to a simple result, where the quarks contributions must be multiplied by the color factor:

$$T_d = -2Tr(I_n)\Sigma_f \frac{m_f^2}{v_d^2} + 3(\lambda + 2\lambda_1 + \lambda_4) + 2\frac{m_W^2}{v_{sm}^2}(\frac{1}{c_w^2} + 2) \tag{4.2}$$

Here, we do comment both results. The two linear combinations $s_\alpha h^0 + c_\alpha H^0$ and $c_\alpha h^0 - s_\alpha H^0$ reproduce the real neutral components of the triplet and the doublet of the HTM, after their VEV shifts. So T_t and T_d represent the quadratic divergencies of the fields ξ^0 and h, as can be shown from Eq. (2.26) of [8]. Hence, its is straightforward to understand that all mixing angles disappear in T_t and T_d.

The absence of the λ parameter in T_t is obvious since λ is the coupling constant of the pure doublet quartic interaction. In the same way, the absences of λ_2 and λ_3 are also natural in T_d since these two couplings only concern the triplet.

We are also faced up to two intriguing questions: why the fermionic part is missing in the first equation? why the μ parameter is missing in both equations ?

The answer to the first question, on one hand, we have already noticed that the cancellation originates from the combination $s_\alpha T_h + c_\alpha T_H$, that is on the form of the trilinear couplings Higgs-fermion-antifermion. On the other hand, the absence of fermionic part lies in the form of the Yukawa coupling with the triplet Δ, given by Eq. (2.5) in [8]). Indeed, a close look shows that the fermionic doublet L is no longer associated with $L^\dagger$ but with L^T, that forbids any $\xi^0 f\bar{f}$ coupling, insuring the absence of fermionic contribution at one loop order.

About the second question, it is bizarre because μ is the strength of a trilinear coupling between doublets and triplet, so a priori able to give contribution, but the interaction is linear in the triplet and a *doublet-triplet-triplet* vertex is excluded, that explains the lack of μ in T_d. As to T_t, the solution is again in the form of the interaction where we have the doublet H and the transposed doublet H^T but not $H^\dagger$: before breaking of the symmetries and the vev shifts, a coupling such as $\delta^{0*} - \varphi^0 - \varphi^{0*}$ is forbidden. Then after breaking and shifting the fields, we end up with two vertices, $\xi^0 - h - h$ and $\xi^0 - Z_1 - Z_1$, with opposite values $\pm\frac{\mu}{2\sqrt{2}}$, where h and Z_1 are the real and imaginary parts of the shifted field φ^0. This feature suffices to cancel these two contributions in a 2 dimensional space-time, when the loop does not depend on the propagator mass as explained before. In some sense, the quadratic divergencies record the physics before breaking and shifting, and this is probably not a pure one-loop effect.

Finally, it is worth to notice that Veltman Condition in Standard Model [21] is recovered when, we remove the couplings λ_1 and λ_4 in the doublet formula T_d, so discarding any mixing between doublet and triplet.

V. ANALYSIS: IMPLICATIONS FOR THE PHYSICAL SCALAR MASSES

In this section we will focus our analysis on the modified Veltman condition (mVC) given in Eqs. (25 − 26). Our aim is twofold: first we will show that one loop quadratic divergencies are softened and go to zero within HTM space parameter. The latter is consistent with theoretical constraints given in Eqs. (12 − 24), namely unitarity, BFB as well as absence of tachyon in the potential (inducing a constraint on μ parameter). It also complies with the observed Higgs at 125 GeV and with LHC measurements for Higgs decay to several channels

($\gamma\gamma$, WW, ZZ, $\tau\tau$, and $b\bar{b}$) [23]. Here we only show results with the photon mode that has the best mass resolution. The second aim of our analysis is to gain more insight on the masses of the heavy Higgs bosons and the effect of naturalness on their allowed range of variations.

We plot in Fig.1, the scatter plot in the (λ_1, λ_4). The figure shows the excluded regions of space parameter by unitarity (red), by combined set of BFB and unitarity (green). If in addition, we impose consistency withe ATLAS and CMS combined data within 2σ on the diphoton decay mode, (with a signal strength $R_{\gamma\gamma} = 1.15 \pm 0.25$ [24]), the blue area is also discarded. Furthermore, when the Veltman condition for the doublet and triplet field enters into the game, we see a drastic reduction of the space parameter to a relatively small allowed region marked in brown.

Fig.2 illustrates the doubly charged Higgs mass $m_{H^{\pm\pm}}$ as a function of λ_1 resulting from a scan over different values of λ_4 and for $\mu = v_t = 1$. We find the only relevant parameter region where all of the constraints are imposed, is the area marked in grey which encodes cancellation of quadratic divergencies. This area is delineated by λ_1 in the range$[-0.3, 2.3]$ corresponding to $m_{H^{\pm\pm}}$ varying from 90 to 351 GeV. For these mass values, λ_4 is limited to lie in a reduced interval between -2.6 and 1.8.

The remarkable feature of effects of the modified Veltman condition on the doubly charged Higgs masse is clearly indicated in Fig.3 showing the $R_{\gamma\gamma}$ ratio (signal strength) as a function of the doubly charged Higgs mass $m_{H^{\pm\pm}}$ for different values of λ_1. First we consider the case when when mVC are absent (red plot). We see that, small values of λ_1 less than 1 favour low $m_{H^{\pm\pm}}$ varying between 90 and 230 GeV, while for larger λ_1, its range of variation is significantly enlarged with an upper limit about 515 GeV. When mVC is turned on (green plot), we show that the $m_{H^{\pm\pm}}$ intervals get smaller and smaller for λ_1 getting larger. We also note that unlike the lower limit which is unaffected by mVC, the upper bound is very sensitive to mVC and experiment a drastic reduction to 351 GeV. Similar analysis is seen in Fig.4 for the simply charged Higgs mass. Here, the same behaviour is reproduced: the lower mass, 160 GeV, is almost insensitive to the constraints including mVC, while the upper limit shows a substantial reduction from 392 GeV to 288 GeV when the conditions mVC are considered. These overall resulting ranges of $m_{H^\pm}$ and $m_{H^{\pm\pm}}$ are compatible with the LHC exclusion limits on the charged Higgs bosons ($H^\pm$, $H^{\pm\pm}$) .

VI. CONCLUSION

We have studied the naturalness problem In the type II seesaw model. We have shown that the Veltman condition is modified by virtue of the additional scalar charged states and that quadratic divergencies at one loop can be driven to zero within at the allowed parameter space of the model. Furthermore, when a set of constraints, including unitarity, consistent boundedness from below, is combined with requirement of compatibility with the diphoton Higgs decay data of LHC for the observed Higgs at 125 GeV, the resulting parameter region is severely constrained. If in addition, the modified Veltman condition is also imposed, the effects on the masses of heavy Higgs bosons H^0, A^0, $H^\pm$ and $H^{\pm\pm}$ are analysed. The analysis has shown a drastic reduction of the mass spectrum of $m_{H^\pm}$ and $m_{H^{\pm\pm}}$ with an upper bounds at 288 and 351 GeV respectively. These limits are still consistent with LHC measurements . Besides, we have found an almost mass degeneracy of CP odd Higgs and heavy CP even Higgs, with $m_{H^0} \simeq m_{A^0} \simeq 207$ GeV. [3]

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[3] An diphoton excess around 750 GeV has been reported by ATLAS and CMS with a local significance of 3.6σ and 2.6σ while the global significances are 2σ and 1.2σ respectively [25, 26]. Although several interpretations of this intriguing signal in terms of Higgs-like resonances have been proposed, this excess could be just another statistical fluctuation which will be washed away with more data.

APPENDIX : FEYNMAN RULES

In this appendix, we give the couplings used to calculate the tadpoles of the two neutral CP-even Higgs h^0 and H^0. Since we are interested in the one-loop contributions, only three-leg couplings are useful. Further, within this restricted class, we look for vertices such as $h^0 F_i \bar{F}_i$ or $H^0 F_i \bar{F}_i$, where F_i stands for any quantum field of our model: scalar and vectorial bosons, fermions, Goldstone fields G_i and Faddeev-Popov ghost fields η_i. To be precise, we have used the well-known linear R_ξ gauge where the gauge fixing Lagrangians are $\frac{-1}{2\xi_Z}(\partial_\mu Z^\mu - \xi_Z m_Z G_0)^2$ for the neutral sector and $\frac{-1}{2\xi_W}(\partial_\mu W^\mu_\pm - \xi_W m_W G_\pm)^2$ for the charged sector.

We note $c_{F_i \bar{F}_i}$ ($C_{F_i \bar{F}_i}$) the couplings to the Higgs h^0 (H^0). Since the field F_i fixes the propagator, we also give the values t_i (T_i) of the loop due to the propagator of the F_i particle which gain a factor 2 in case of charged fields, and the symmetry factor s_i.

$$
\begin{aligned}
c_1 \equiv c_{h_0 h_0} &= \frac{-3i}{2}(c_\alpha^3 \lambda v_d + 2c_\alpha(\lambda_1+\lambda_4)s_\alpha^2 v_d + 4(\lambda_2+\lambda_3)s_\alpha^3 v_t + 2c_\alpha^2 s_\alpha(-\sqrt{2}\mu + (\lambda_1+\lambda_4)v_t)) \\
C_1 \equiv C_{H_0 H_0} &= \frac{3i}{2}(2c_\alpha^2(\lambda_1+\lambda_4)s_\alpha v_d + \lambda s_\alpha^3 v_d - 4c_\alpha^3(\lambda_2+\lambda_3)v_t + 2c_\alpha s_\alpha^2(\sqrt{2}\mu - (\lambda_1+\lambda_4)v_t)) \\
t_1 &= iA_0(m_{h_0}^2) \\
T_1 &= iA_0(m_{H_0}^2) \\
s_1 &= \frac{1}{2}
\end{aligned}
\tag{6.1}
$$

$$
\begin{aligned}
c_2 \equiv c_{G_0 G_0} &= -\frac{i}{2}(-4\sqrt{2}\mu c_\alpha c_\beta s_\beta + 2s_\beta^2(c_\alpha(\lambda_1+\lambda_4)v_d + 2(\lambda_2+\lambda_3)s_\alpha v_t) + c_\beta^2(2\sqrt{2}\mu s_\alpha + c_\alpha \lambda v_d \\
&\quad +2(\lambda_1+\lambda_4)s_\alpha v_t)) \\
C_2 \equiv C_{G_0 G_0} &= -\frac{i}{2}(s_\alpha(4\sqrt{2}c_\beta \mu s_\beta - c_\beta^2 \lambda v_d - 2(\lambda_1+\lambda_4)s_\beta^2 v_d) + 2c_\alpha(2(\lambda_2+\lambda_3)s_\beta^2 v_t \\
&\quad +c_\beta^2(\sqrt{2}\mu + (\lambda_1+\lambda_4)v_t))) \\
t_2 = T_2 &= iA_0(\xi_Z m_Z^2) \\
s_2 &= \frac{1}{2}
\end{aligned}
\tag{6.2}
$$

$$
\begin{aligned}
c_3 &\equiv c_{G_+G_-} = -\frac{i}{2}(s_{\beta'}^2(c_\alpha(2\lambda_1+\lambda_4)v_d+4(\lambda_2+\lambda_3)s_\alpha v_t)+c_{\beta'}^2(c_\alpha\lambda v_d+2\lambda_1 s_\alpha v_t)-c_{\beta'}s_{\beta'}\\
&\qquad (4c_\alpha\mu-\sqrt{2}\lambda_4(s_\alpha v_d+c_\alpha v_t))),\\
C_3 &\equiv C_{G_+G_-} = -\frac{i}{2}(s_{\beta'}^2((2\lambda_1+\lambda_4)s_\alpha v_d-4c_\alpha(\lambda_2+\lambda_3)v_t)+c_{\beta'}^2(\lambda s_\alpha v_d-2c_\alpha\lambda_1 v_t)-c_{\beta'}s_{\beta'}\\
&\qquad (4\mu s_\alpha+\sqrt{2}\lambda_4)(c_\alpha v_d-s_\alpha v_t))),\\
t_3 &= T_3 = 2\times iA_0(\xi_W m_W^2),\\
s_3 &= \frac{1}{2},
\end{aligned}
\tag{6.3}
$$

$$
\begin{aligned}
c_4 &\equiv c_{H_0H_0} = -\frac{i}{2}(2c_\alpha^3(\lambda_1+\lambda_4)v_d+c_\alpha(3\lambda-4(\lambda_1+\lambda_4))s_\alpha^2 v_d+2s_\alpha^3(-\sqrt{2}\mu+(\lambda_1+\lambda_4)v_t)\\
&\qquad +4\,c_\alpha^2\,s_\alpha(\sqrt{2}\mu-(\lambda_1-3(\lambda_2+\lambda_3)+\lambda_4)v_t))\\
C_4 &\equiv C_{h_0h_0} = \frac{i}{2}(c_\alpha^2(3\lambda-4(\lambda_1+\lambda_4))s_\alpha v_d+2(\lambda_1+\lambda_4)s_\alpha^3 v_d+2c_\alpha^3(\sqrt{2}\mu-(\lambda_1+\lambda_4)v_t)\\
&\qquad -4\,c_\alpha\,s_\alpha^2(\sqrt{2}\mu-(\lambda_1-3(\lambda_2+\lambda_3)+\lambda_4)v_t)),\\
t_4 &= iA_0(m_{H_0}^2),\\
T_4 &= iA_0(m_{h_0}^2),\\
s_4 &= \frac{1}{2},
\end{aligned}
\tag{6.4}
$$

$$
\begin{aligned}
c_5 &\equiv c_{A_0A_0} = -\frac{i}{2}(c_\alpha(4\sqrt{2}c_\beta\mu s_\beta+2c_\beta^2(\lambda_1+\lambda_4)v_d+\lambda s_\beta^2 v_d)+2s_\alpha(2c_\beta^2(\lambda_2+\lambda_3)v_t+s_\beta^2(\sqrt{2}\mu\\
&\qquad +(\lambda_1+\lambda_4)v_t))),\\
C_5 &\equiv C_{A_0A_0} = \frac{i}{2}(4\sqrt{2}c_\beta\mu s_\alpha s_\beta+2c_\beta^2((\lambda_1+\lambda_4)s_\alpha v_d-2c_\alpha(\lambda_2+\lambda_3)v_t)+s_\beta^2(\lambda s_\alpha v_d-2c_\alpha(\sqrt{2}\mu\\
&\qquad +(\lambda_1+\lambda_4)v_t))),\\
t_5 &= T_5 = iA_0(m_{A_0}^2),\\
s_5 &= \frac{1}{2},
\end{aligned}
\tag{6.5}
$$

$$
\begin{aligned}
c_6 &\equiv c_{H_+H_-} = -\frac{i}{2}(c_{\beta'}^2(c_\alpha(2\lambda_1+\lambda_4)v_d+4(\lambda_2+\lambda_3)s_\alpha v_t)+s_{\beta'}^2(c_\alpha\lambda v_d+2\lambda_1 s_\alpha v_t)+c_{\beta'}s_{\beta'}(4c_\alpha\mu\\
&\qquad -\sqrt{2}\lambda_4(s_\alpha v_d+c_\alpha v_t))),\\
C_6 &\equiv C_{H_+H_-} = \frac{i}{2}(c_{\beta'}^2((2\lambda_1+\lambda_4)s_\alpha v_d-4c_\alpha(\lambda_2+\lambda_3)v_t)+s_{\beta'}^2(\lambda s_\alpha v_d-2c_\alpha\lambda_1 v_t)+c_{\beta'}s_{\beta'}(4\mu s_\alpha\\
&\qquad +\sqrt{2}\lambda_4)(c_\alpha v_d-s_\alpha v_t))),\\
t_6 &= T_6 = 2\times iA_0(m_{H_\pm}^2),\\
s_6 &= \frac{1}{2},
\end{aligned}
\tag{6.6}
$$

$$
\begin{aligned}
c_7 &\equiv c_{H_{++}H_{--}} = -i(c_\alpha \lambda_1 v_d + 2\lambda_2 s_\alpha v_t), \\
C_7 &\equiv C_{H_{++}H_{--}} = i(\lambda_1 s_\alpha v_d - 2c_\alpha \lambda_2 v_t), \\
t_7 &= T_7 = 2 \times iA_0(m^2_{H_{\pm\pm}}), \\
s_7 &= \frac{1}{2},
\end{aligned}
\tag{6.7}
$$

$$
\begin{aligned}
c_8 &\equiv c_{ZZ} = (iem_W(c_\alpha c_{\beta'} + 2\sqrt{2} s_\alpha s_{\beta'}))/(c_w^2 s_w), \\
C_8 &\equiv C_{ZZ} = (-iem_W(c_{\beta'} s_\alpha - 2\sqrt{2} c_\alpha s_{\beta'}))/(c_w^2 s_w), \\
t_8 &= T_8 = -i((n-1)A_0(m_Z^2) + \xi_Z A_0(\xi_Z m_Z^2), \\
s_8 &= \frac{1}{2},
\end{aligned}
\tag{6.8}
$$

$$
\begin{aligned}
c_9 &\equiv c_{W_+W_-} = iem_W(c_\alpha c_{\beta'} + \sqrt{2} s_\alpha s_{\beta'})/s_w, \\
C_9 &\equiv C_{W_+W_-} = -iem_W(c_{\beta'} s_\alpha - \sqrt{2} c_\alpha s_{\beta'})/s_w, \\
t_9 &= T_9 = 2 \times (-i((n-1)A_0(m_W^2) + \xi_W A_0(\xi_W m_W^2)), \\
s_9 &= \frac{1}{2},
\end{aligned}
\tag{6.9}
$$

$$
\begin{aligned}
c_{10} &\equiv c_{f\bar{f}} = \frac{-i}{2} e(c_\alpha / c_{\beta'}) m_f / (m_W s_w), \\
C_{10} &\equiv C_{f\bar{f}} = \frac{i}{2} e(s_\alpha / c_{\beta'}) m_f / m_W s_w), \\
t_{10} &= T_{10} = i m_f A_0(m_f^2) Tr(I_n), \\
s_{10} &= 1,
\end{aligned}
\tag{6.10}
$$

$$
\begin{aligned}
c_{11} &\equiv c_{\eta_Z \bar{\eta_Z}} = \frac{-i}{2} em_W(c_\alpha c_{\beta'} + 2\sqrt{2} s_\alpha s_{\beta'})\xi_Z)/(c_w^2 s_w), \\
C_{11} &\equiv C_{\eta_Z \bar{\eta_Z}} = \frac{i}{2} em_W(c_{\beta'} s_\alpha - 2\sqrt{2} c_\alpha s_{\beta'})\xi_Z/(c_w^2 s_w), \\
t_{11} &= T_{11} = iA_0(\xi_Z m_Z^2), \\
s_{11} &= 1,
\end{aligned}
\tag{6.11}
$$

$$
\begin{aligned}
c_{12} &\equiv c_{\eta_\pm \bar{\eta_\pm}} = \frac{-i}{2} em_W(c_\alpha c_{\beta'} + \sqrt{2} s_\alpha s_{\beta'})\xi_W / s_w, \\
C_{12} &\equiv C_{\eta_\pm \bar{\eta_\pm}} = \frac{i}{2} em_W(c_{\beta'} s_\alpha - \sqrt{2} c_\alpha s_{\beta'})\xi_W / s_w, \\
t_{12} &= T_{12} = 2 \times iA_0(\xi_W m_W^2), \\
s_{12} &= 1,
\end{aligned}
\tag{6.12}
$$

m_Φ	**Unitarity**	**Unitarity + BFB**	**Unitarity + BFB +** $R_{\gamma\gamma}$	**Unitarity + BFB +** $R_{\gamma\gamma}$ **+ mVC**
H^0	$[206.8-207.3]$ GeV	$[206.8-207]$ GeV	$[206.8-207]$ GeV	206.8 GeV
A^0	206.8 GeV	206.8 GeV	206.8 GeV	206.8 GeV
$H^\pm$	$[160-474]$ GeV	$[160-474]$ GeV	$[160-392]$ GeV	$[161-288]$ GeV
$H^{\pm\pm}$	$[90-637]$ GeV	$[90-637]$ GeV	$[90-513]$ GeV	$[90-351]$ GeV

TABLE I. Higgs bosons masses allowed intervals in the Higgs triplet model resulting from various constraints, including the modified Veltman conditions

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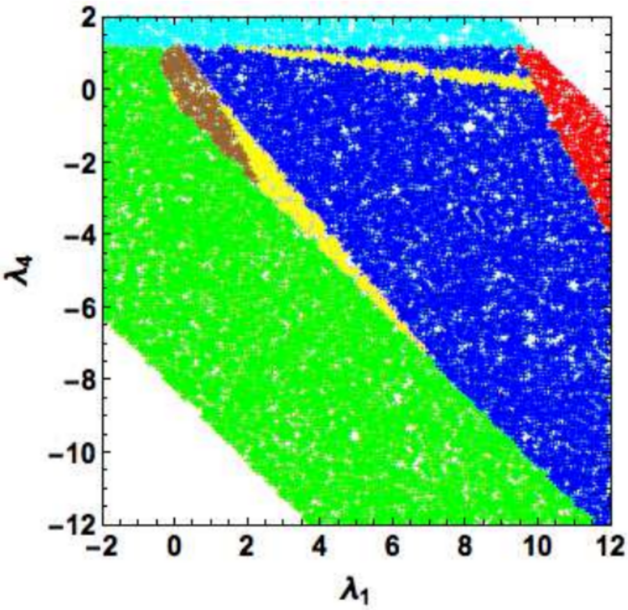

FIG. 1. The allowed regions in (λ_1, λ_4) plans after imposing theoretical and experimental constraints. (cyan) : Excluded by μ constraints, (red) : Excluded by μ+Unitarity constraints, (green) : Excluded by μ+Unitarity+BFB constraints, (blue) : Excluded by μ+Unitarity+BFB+$R_{\gamma\gamma}$ constraints, (yellow) : Excluded by μ+Unitarity+BFB $R_{\gamma\gamma}$& $T_d = 0 \wedge T_t = 0$ constraints. Only the brown area obeys ALL constraints. Our inputs are $\lambda = 0.52$, $-2 \leq \lambda_1 \leq 12$, $\lambda_2 = -\frac{1}{6}$, $\lambda_3 = \frac{3}{8}$, $-12 \leq \lambda_4 \leq 2$, $v_t = 1$ GeV and $\mu = 1$ GeV.

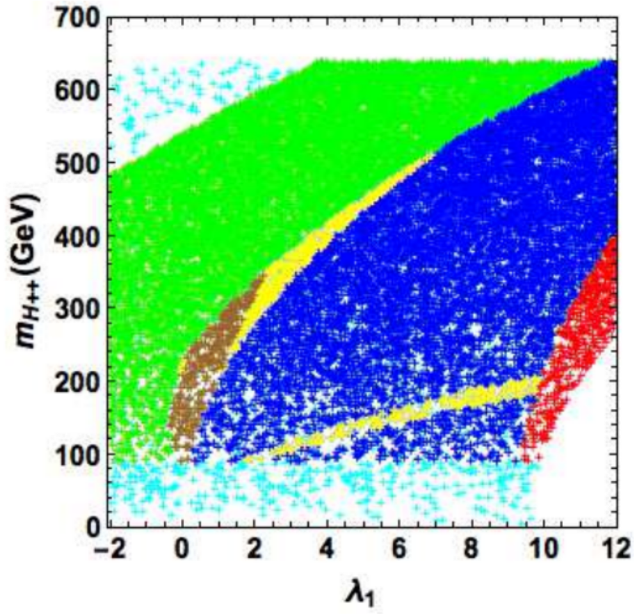

FIG. 2. The allowed regions in $(\lambda_1, m_{H^{\pm\pm}})$ plans after imposing theoretical and experimental constraints. Our inputs are $\lambda = 0.52$, $-2 \leq \lambda_1 \leq 12$, $\lambda_2 = -\frac{1}{6}$, $\lambda_3 = \frac{3}{8}$, $-12 \leq \lambda_4 \leq 2$, $v_t = 1$ GeV and $\mu = 1$ GeV.

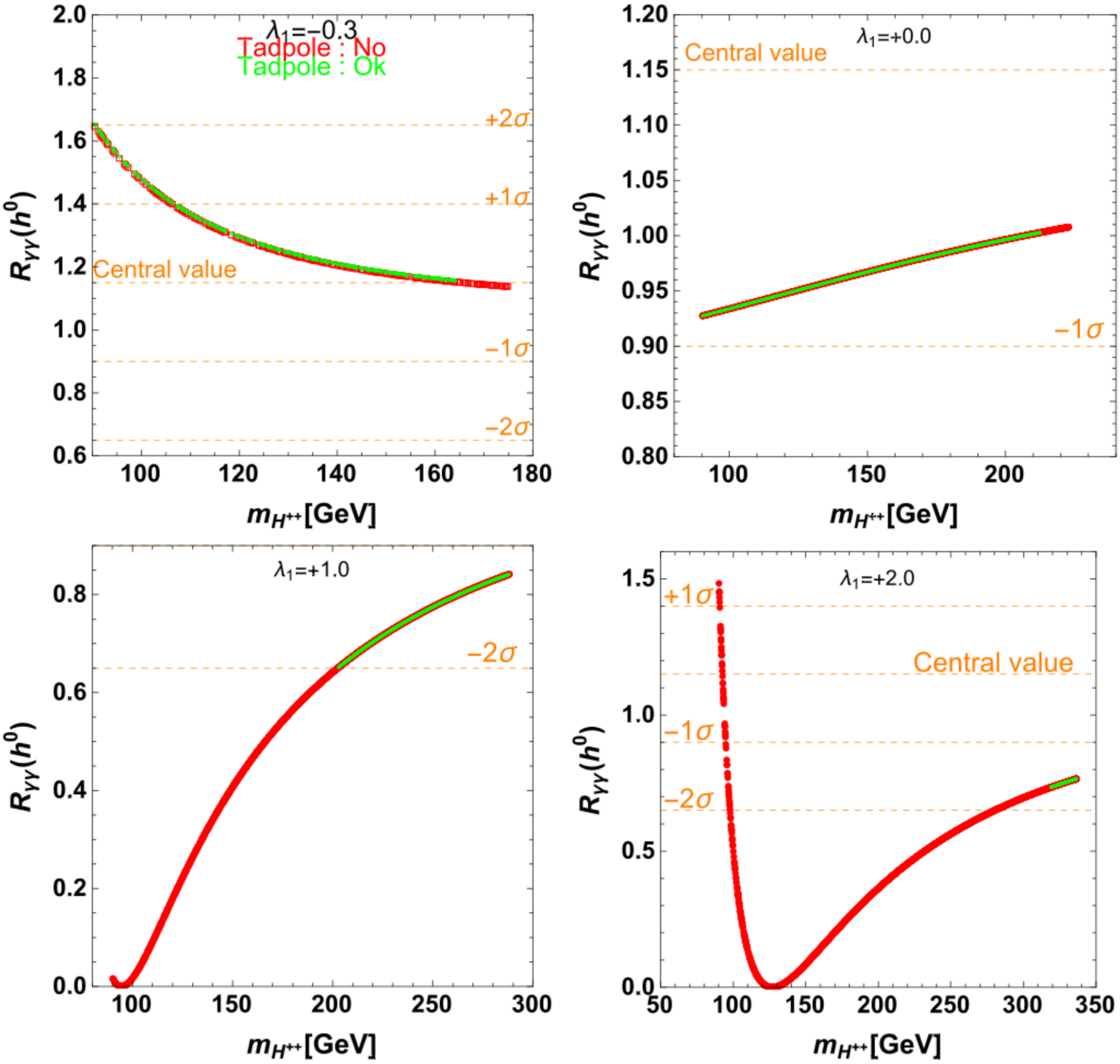

FIG. 3. $R_{\gamma\gamma}(h^0)$ as a function of $m_{H^{\pm\pm}}$ for various values of λ_1 with and without Veltman conditions ($T_d = 0 \wedge T_t = 0$). We scan over the HTM parameters as : $\lambda = 0.52$, $\lambda_2 = -\frac{1}{6}$, $\lambda_3 = \frac{3}{8}$, $-10 \leq \lambda_4 \leq 2$, $v_t = 1$ GeV and $\mu = 1$ GeV.

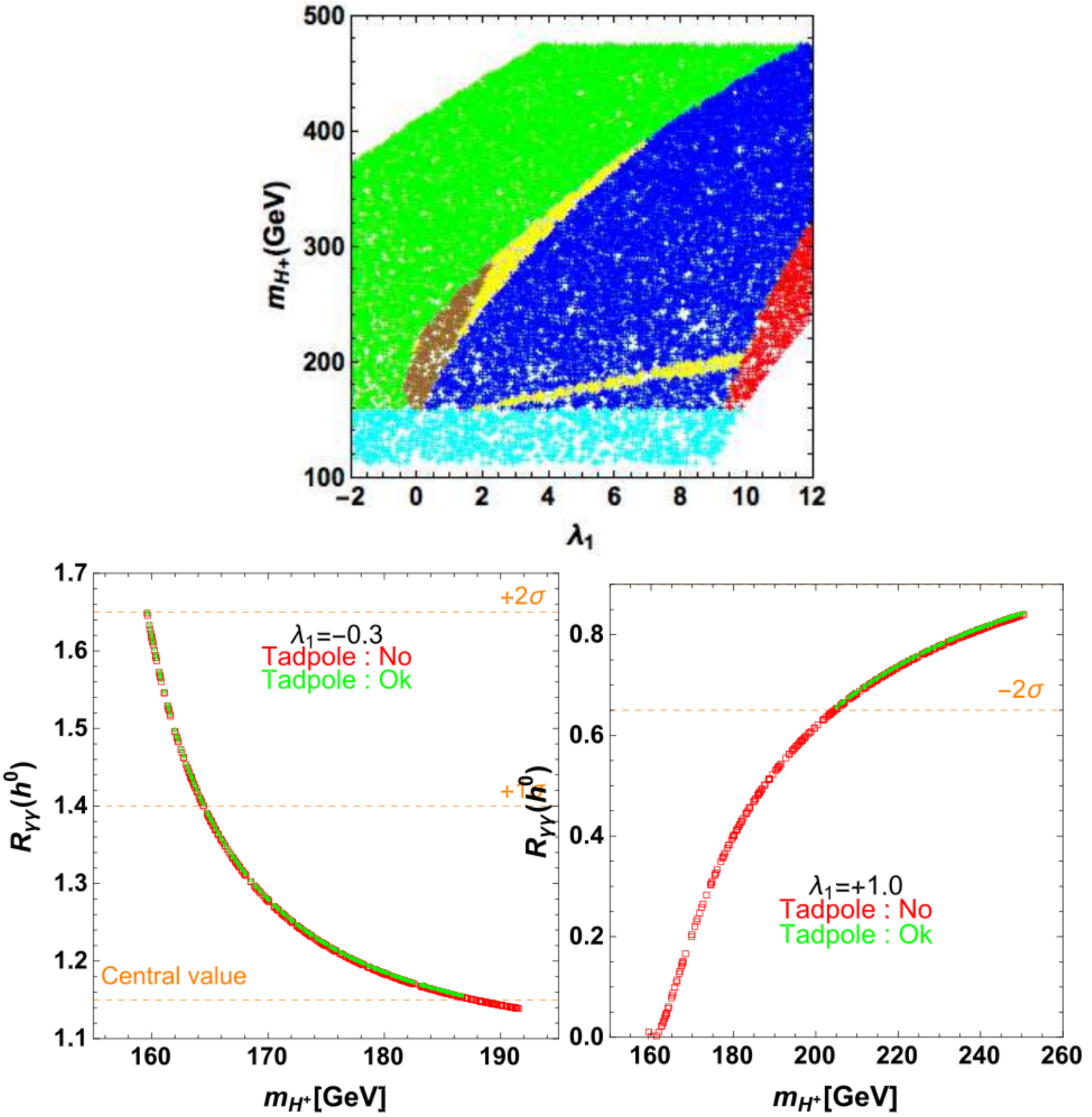

FIG. 4. Upper slide : The allowed regions in $(\lambda_1, m_{H^\pm})$ plans after imposing theoretical and experimental constraints. Lower slide : $R_{\gamma\gamma}(h^0)$ as a function of $m_{H^\pm}$ for $\lambda_1 = -0.3$ (left) and $\lambda_1 = +1.0$ (right) with and without Veltman conditions ($T_d = 0 \wedge T_t = 0$). Our inputs are $\lambda = 0.52$, $\lambda_2 = -\frac{1}{6}$, $\lambda_3 = \frac{3}{8}$, $-12 \leq \lambda_4 \leq 2$ and $\mu = v_t = 1$ GeV.